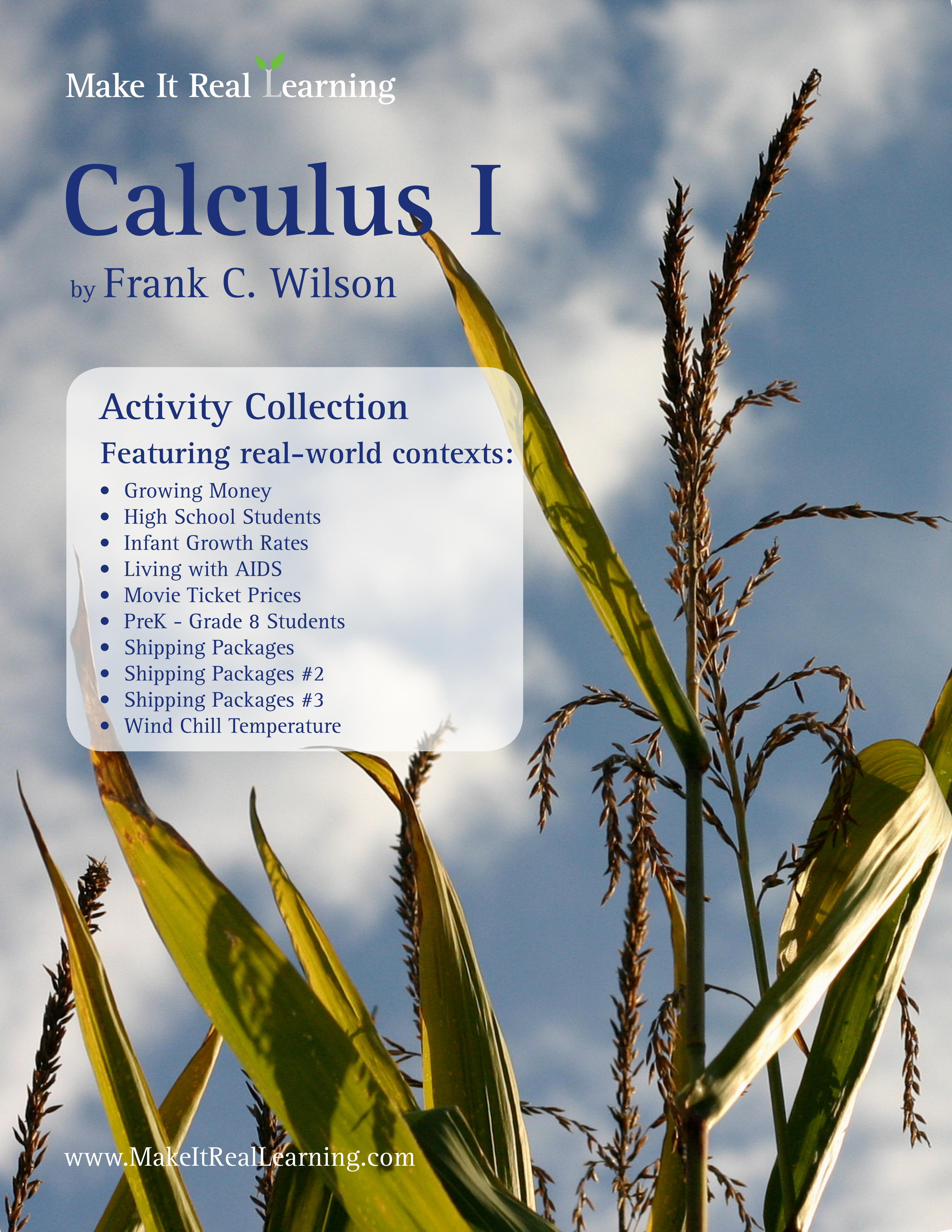

Make It Real Learning

Calculus I

by Frank C. Wilson

Activity Collection

Featuring real-world contexts:

- Growing Money
- High School Students
- Infant Growth Rates
- Living with AIDS
- Movie Ticket Prices
- PreK - Grade 8 Students
- Shipping Packages
- Shipping Packages #2
- Shipping Packages #3
- Wind Chill Temperature

www.MakeItRealLearning.com

Cover art: Blaine C. Wilson

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Introduction

"When am I ever going to use this?" It is a question that has plagued teachers and learners for decades. Now, with the help of the Make It Real Learning workbook series, you can answer the question.

The *Calculus I* workbook focuses on real-world situations that may be effectively analyzed using derivatives and other rates of change. From analyzing the growth of an infant to maximizing the volume of a package, learners get to use mathematics in meaningful ways. Rest assured that each activity integrates real world information not just "realistic" data. These are real organizations (e.g. US Postal Service) and real world issues (e.g. AIDS, movie ticket prices).

The mathematical objectives of each activity are clearly specified on the Activity Objectives page following this introduction. Through the workbook series, we have consistently sought to address the content and process standards of the National Council of Teachers of Mathematics.

There are multiple ways to use the activities in a teaching environment. Many teachers find that the activities are an excellent tool for stimulating mathematical discussions in a small group setting. Due to the challenging nature of each activity, group members are motivated to brainstorm problem solving strategies together. The interesting real world contexts motivate them to want to solve the problems. The activities may also be used for individual projects and class-wide discussions.

As a ready-resource for teachers, the workbook also includes completely worked out solutions for each activity. To make it easier for teachers to assess student work, the solutions are included on a duplicate copy of each activity.

We hope you enjoy the activities! We continue to increase the number of workbooks in the Make It Real Learning workbook series. Please visit www.MakeItRealLearning.com for the most current list of activities. Thanks!

Frank C. Wilson
Author

Calculus I Activity Objectives

Activity Title	Mathematical Objectives
Growing Money: Using Partial Derivatives (p. 6)	Find and evaluate the first partial derivatives of a multivariable function Interpret the practical meaning of partial derivatives
High School Students: Working with Rates of Change (p. 10)	Find and evaluate the derivative of a polynomial Use technology to determine relative extrema Estimate an inflection point from a graph Calculate and interpret the meaning of rates of change
Infant Growth Rates: Using Derivatives (p. 14)	Calculate the derivative of logarithmic and cubic functions Evaluate derivatives and interpret the meaning of the results Recognize the relationship between the slope of a graph and its derivative
Living with AIDS: Working with Derivatives (p. 18)	Interpret the practical meaning of the concavity of a graph Determine the sign of the first and second derivatives from a graph Find and evaluate the first and second derivatives of a quadratic function Estimate the derivative at a given value from a table
Movie Ticket Prices: Using Calculus to Analyze Change (p. 22)	Find, graph, and interpret the meaning of the derivative Shade the region representing the definite integral Calculate the area of a region represented by a definite integral Interpret the meaning of a definite integral Use an integral to find the average value of a function
PreK - Grade 8 Students: Working with Rates of Change (p. 26)	Find and evaluate the derivative of a cubic function Use the derivative to determine the slope of a function Estimate the coordinates of an inflection point from a graph Find the coordinates of an inflection point from an equation
Shipping Packages: Optimizing Functions with Calculus (p. 30)	Create a graphical representation of a verbal description Create a volume equation and calculate its derivative Determine the box dimensions that maximize volume
Shipping Packages #2: Optimizing Functions with Calculus (p. 34)	Create a graphical representation of a verbal description Create a volume equation and calculate its derivative Determine the box dimensions that minimize a volume function
Shipping Packages #3: Optimizing Functions with Calculus (p. 38)	Create a graphical representation of a verbal description Create a volume equation and calculate its derivative Determine the tube dimensions that maximize volume
Wind Chill Temperature: Using Calculus with Multivariable Functions (p. 42)	Evaluate a multivariable function at a given point Find and interpret the meaning of the first partial derivatives

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Growing Money

Using Partial Derivatives

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to determine the effect of changes in the variables of the compound interest formula.

The compound interest formula, $A = P\left(1+\frac{r}{n}\right)^{nt}$, is used by banks and other financial institutions to determine the future value of a savings account. In the formula, A is the account value (in dollars) after t years, P is the initial value (in dollars), r is the stated annual interest rate (as a decimal), and n is the compounding frequency (the number of time interest is calculated each year).

With an annual interest rate of 9% compounded monthly, the formula becomes

$$A = P(1.0075)^{12t}$$

1. Determine $\frac{\partial A}{\partial P}$ and $\frac{\partial A}{\partial t}$ including units.

2. Evaluate $\frac{\partial A}{\partial P}$ when $P = 500$ and $t = 4$. Then interpret the real world meaning of the result.

3. Evaluate $\frac{\partial A}{\partial t}$ when $P = 500$ and $t = 4$. Then interpret the real world meaning of the result.

4. If an investor wants her investment to increase by approximately $100 between the fourth and fifth years, how much does she need to invest initially? (Round your solution to the nearest dollar.)

5. Two different investors place money in a savings account paying 9% compounded monthly. The first investor deposits $2000 and the second investor deposits $1000. Each investor wants to know what effect a $1 increase in his initial deposit will have on the future value between the tenth and eleventh year.

6. Two different investors place money in a savings account paying 9% compounded monthly. The first investor deposits $2000 and the second investor deposits $1000. Each investor plans to hold the money in savings for five years and wants to know what effect an additional year increase in the savings term will have on the future value of her investment. Use partial derivatives to determine the answer.

Growing Money

Using Partial Derivatives

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to determine the effect of changes in the variables of the compound interest formula.

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$$A = P(1.0075)^{12t}$$

1. Determine $\frac{\partial A}{\partial P}$ and $\frac{\partial A}{\partial t}$ including units.

$$\frac{\partial A}{\partial P} = (1.0075)^{12t}\,\frac{\textit{dollars of future value}}{\textit{dollar of investment}}$$

$$\begin{aligned}\frac{\partial A}{\partial t} &= P\ln(1.0075)(1.0075)^{12t}(12)\frac{\textit{dollars of future value}}{\textit{year}}\\ &\approx 0.08966P(1.0075)^{12t}\ \textit{dollars per year}\end{aligned}$$

2. Evaluate $\frac{\partial A}{\partial P}$ when $P = 500$ and $t = 4$. Then interpret the real world meaning of the result.

$$\begin{aligned}\frac{\partial A}{\partial P} &= (1.0075)^{12(4)}\,\frac{\textit{dollars of future value}}{\textit{dollar of investment}}\\ &\approx 1.43\ \textit{dollar of future value per dollar of investment}\end{aligned}$$

When the initial investment is $500 and the length of the savings period is four years, a $1 increase in the initial investment corresponds with an approximate increase in the future value of $1.43.

3. Evaluate $\frac{\partial A}{\partial t}$ when $P = 500$ and $t = 4$. Then interpret the real world meaning of the result.

$$\begin{aligned}\frac{\partial A}{\partial t} &\approx 0.08966(500)(1.0075)^{12(4)}\ \textit{dollars per year}\\ &\approx 64.17\ \textit{dollars per year}\end{aligned}$$

When the initial value is $500 and the length of the savings period is four years, a 1 year increase in the savings period corresponds with an approximate increase in future value of $64.17.

4. If an investor wants her investment to increase by approximately \$100 between the fourth and fifth years, how much does she need to invest initially? (Round your solution to the nearest dollar.)

$$\begin{aligned} 100 &\approx 0.08966P(1.0075)^{12(4)} \\ 100 &\approx 0.1283P \\ P &\approx 779 \end{aligned}$$

She needs to invest \$779 initially in order for her investment to increase by approximately \$100 between the fourth and fifth years.

5. Two different investors place money in a savings account paying 9% compounded monthly. The first investor deposits \$2000 and the second investor deposits \$1000. Each investor wants to know what effect a \$1 increase in his initial deposit will have on the future value between the tenth and eleventh year.

Recall that $\frac{\partial A}{\partial P} = (1.0075)^{12t} \frac{\text{dollars of future value}}{\text{dollar of investment}}$. *This function tells us what effect a \$1 increase in the amount initially invested will have on the future value of the investment. Notice that this partial derivative is a function of time,* t. *Therefore, the amount of money initially invested,* P, *does not affect the effect of a \$1 increase in the amount initially invested. We simply need to evaluate* $\frac{\partial A}{\partial P}$ *at* $t = 10$.

$$\begin{aligned} \frac{\partial A}{\partial P} &= (1.0075)^{12(10)} \\ &\approx 2.45 \end{aligned}$$

For both investors, a \$1 increase in the initial amount invested corresponds with a \$2.45 approximate increase in future value between the tenth and eleventh years.

6. Two different investors place money in a savings account paying 9% compounded monthly. The first investor deposits \$2000 and the second investor deposits \$1000. Each investor plans to hold the money in savings for five years and wants to know what effect an additional year increase in the savings term will have on the future value of her investment. Use partial derivatives to determine the answer.

$$\begin{aligned} \frac{\partial A}{\partial t} &\approx 0.08966(1000)(1.0075)^{12(5)} \text{ dollars per year} \\ &\approx 140.38 \text{ dollars per year} \end{aligned}$$

$$\begin{aligned} \frac{\partial A}{\partial t} &\approx 0.08966(2000)(1.0075)^{12(5)} \text{ dollars per year} \\ &\approx 280.76 \text{ dollars per year} \end{aligned}$$

For the investor with a \$1000 initial investment, a one year increase in the savings term will increase the future value by about \$140.38. For the investor with a \$2000 initial deposit, the future value increases by \$280.76.

High School Students

Working with Rates of Change

"When am I ever going to use this?"

Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

Based on projections from 2006–07 to 2016–17, the number of students in grades 9 through 12 in the United States may be modeled by

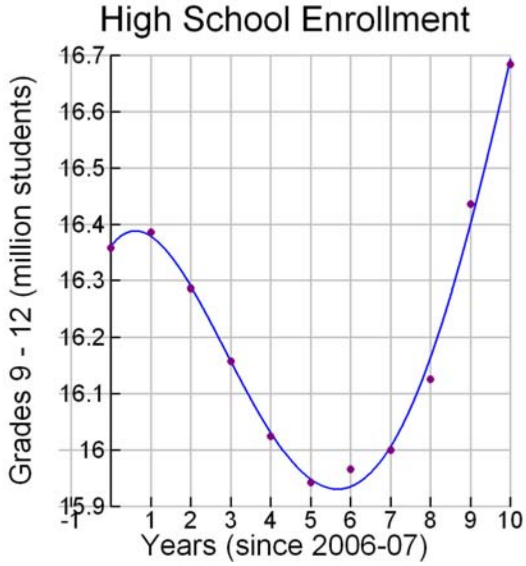
$$s(t) = -0.4936t^4 + 13.27t^3 - 89.50t^2 + 95.10t + 16{,}360 \text{ thousand students}$$

where t is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

2. Use technology and the derivative of s to find the absolute extrema of s on the interval $[0,10]$. Then interpret the real world meaning of each extreme value.

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of s on the interval $[0,10]$ and interpret what it represents in terms of the rate of change of s. (Note: The units on the graph are million students and the units of the equation of s are thousand students.)



4. According to the model, what was the predicted average rate of change in the number of high school students between 2006 – 07 and 2016 – 17?

High School Students

Working with Rates of Change

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Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

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where t is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

$$s'(t) = -1.9744t^3 + 39.81t^2 - 179.0t + 95.10$$

$$s'(3) = -1.9744(3)^3 + 39.81(3)^2 - 179.0(3) + 95.10$$
$$\approx -136.9 \textit{ thousand students per year}$$

$$s'(8) = -1.9744(8)^3 + 39.81(8)^2 - 179.0(8) + 95.10$$
$$\approx 200.0 \textit{ thousand students per year}$$

According to the model, the number of students will be decreasing at a rate of 136.9 thousand students per year in 2009 – 10 and increasing at a rate of 200.0 thousand students per year in 2014 – 15. The number of students is changing more rapidly in 2014 – 15.

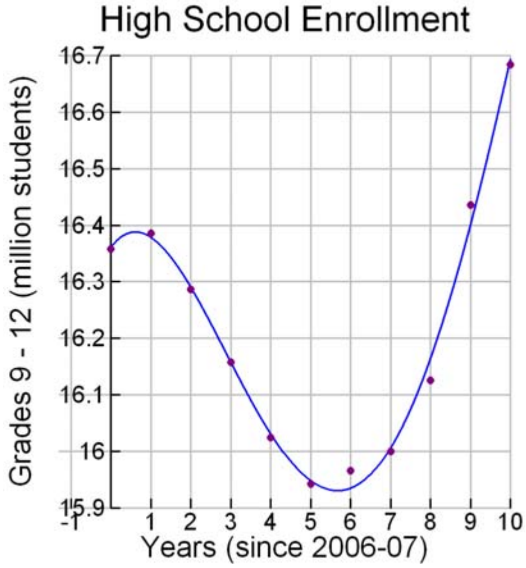
2. Use technology and the derivative of s to find the absolute extrema of s on the interval $[0,10]$. Then interpret the real world meaning of each extreme value.

Relative extrema of s occur wherever s' changes sign. We graph $s'(t)$ and $y = 0$ simultaneously and find the points of intersection to determine the critical values of s. The critical values of s are $s \approx 0.61$ and $s = 5.67$.

t	*s*
0	16,360
0.61	16,390
5.67	15,930
10	16,700

Rounded to the nearest multiple of 10 thousand, the absolute maximum number of students enrolled will be 16,700 thousand and the absolute minimum number of students enrolled will be 15,930 thousand.

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of s on the interval $[0,10]$ and interpret what it represents in terms of the rate of change of s. (Note: The units on the graph are million students and the units of the equation of s are thousand students.)



We are looking for where the graph changes concavity. It appears that the graph changes from concave down to concave up at around $(3,16.15)$. *So the inflection point of* s *is approximately* $(3,16150)$. *That is, in 2009 – 2010 the number of students is about 16,150,000. In terms of the rate of change of* s, *this is the year when the number of students is decreasing most rapidly.*

4. According to the model, what was the predicted average rate of change in the number of high school students between 2006 – 07 and 2016 – 17?

$$\begin{aligned} s(0) &= -0.4936(0)^4 + 13.27(0)^3 - 89.50(0)^2 + 95.10(0) + 16,360 \\ &= 16,360 \end{aligned}$$

$$\begin{aligned} s(10) &= -0.4936(10)^4 + 13.27(10)^3 - 89.50(10)^2 + 95.10(10) + 16,360 \\ &= 16,695 \end{aligned}$$

$$\begin{aligned} \frac{s(10) - s(0)}{10 - 0} &= \frac{16,695 - 16,360}{10 - 0} \frac{\textit{thousand students}}{\textit{year}} \\ &= 33.5\,\textit{thousand students per year} \end{aligned}$$

Between 2006 – 07 and 2016 – 17, the number of students in grades 9 – 12 was expected to increase by an average of 33,500 students per year. That is, although the number of students increased and decreased over the ten year period, the average of the rates of change was 33,500 students per year.

Infant Growth Rates

Using Derivatives

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to forecast the growth rate in boys from birth through 36 months.

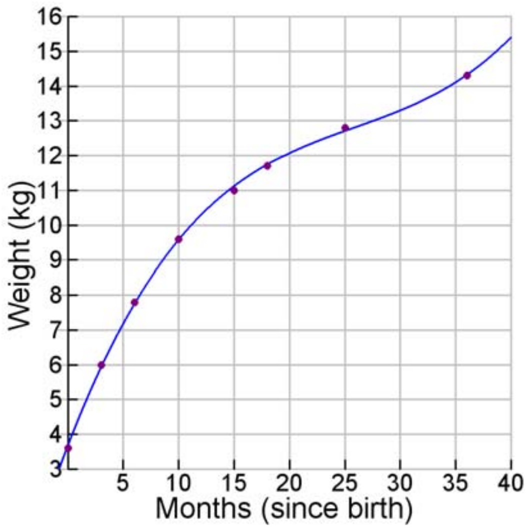
The Centers for Disease Control publishes growth charts to help parents and medical professionals monitor the growth of infants. If a child's growth dramatically exceeds or falls below normal ranges of growth, the child may have health issues that need to be addressed. The equation $L(m) = -8.660 + 27.52\ln(m+9)$ models the average length L of a male infant (in cm) as a function of the number of months since birth, m. (Source: Modeled from CDC Growth Chart)

1. Find $L'(m)$ including units.

2. Evaluate $L'(12)$ and $L'(24)$. Then explain the practical meaning of the results.

3. Referring to the equation of the derivative in Exercise 1, explain what happens to the value of $L'(m)$ as m increases in value.

4. The graph shows the average weight of an infant boy together with a cubic function model. According to the graph, does the weight of a 10-month old boy or a 25-month old boy increase faster? Explain.



5. The equation of the graph given in Exercise 3 is $w(m) = 0.000357m^3 - 0.0277m^2 + 0.832m + 3.67$. What is the derivative of this function and what does it represent in terms of the growth of the child?

6. Evaluate $w'(m)$ at $m = 3$, $m = 27$, and $m = 36$. Then interpret the meaning of each value.

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1. Find $L'(m)$ including units.

$$L'(m) = \frac{27.52}{m+9}\frac{cm}{month}$$

2. Evaluate $L'(12)$ and $L'(24)$. Then explain the practical meaning of the results.

$$L'(12) = \frac{27.52}{12+9}\frac{cm}{month} \approx 1.31\,cm\ per\ month \qquad L'(24) = \frac{27.52}{24+9}\frac{cm}{month} \approx 0.83\,cm\ per\ month$$

According to the model, a 12 month old boy grows at a rate of 1.31 cm per month whereas a 24 month old boy grows at 0.83 cm per month.

3. Referring to the equation of the derivative in Exercise 1, explain what happens to the value of $L'(m)$ as m increases in value.

Recall that $L'(m) = \frac{27.52}{m+9}$. *Observe that the value of the numerator is constant but the value of the denominator varies. In fact, as the value of m increases, the value of the denominator increases. As a result, the value of the derivative decreases as the value of m increases.*

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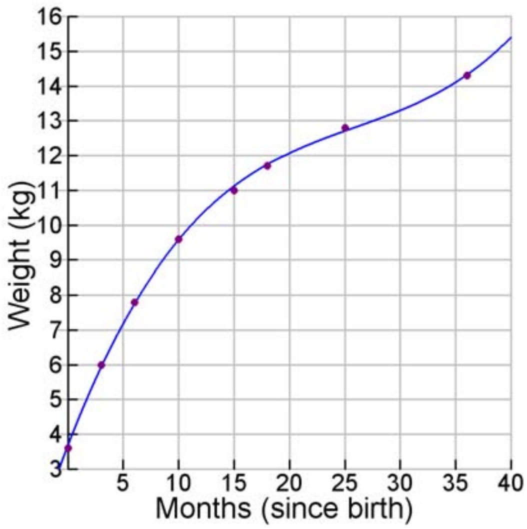
4. The graph shows the average weight of an infant boy together with a cubic function model. According to the graph, does the weight of a 10-month old boy or a 25-month old boy increase faster? Explain.



The slope of the weight graph shows how rapidly the weight is changing at any age. Since the slope is positive, we know the boy's weight continually increases. Since the graph is steeper at $m = 10$ than at $m = 25$, we know that a 10 month old boy grows faster in weight than a 25 month old boy.

5. The equation of the graph given in Exercise 3 is $w(m) = 0.000357m^3 - 0.0277m^2 + 0.832m + 3.67$. What is the derivative of this function and what does it represent in terms of the growth of the child?

$$w'(m) = 0.001071m^2 - 0.0554m + 0.832 \, kg \text{ } per \text{ } month$$

The derivative represents the instantaneous rate of change in the weight of the boy in kilograms per month.

6. Evaluate $w'(m)$ at $m = 3$, $m = 27$, and $m = 36$. Then interpret the meaning of each value.

$$\begin{aligned} w'(3) &= 0.001071(3)^2 - 0.0554(3) + 0.832 \, kg \text{ } per \text{ } month \\ &\approx 0.675 \, kg \text{ } per \text{ } month \end{aligned}$$

$$\begin{aligned} w'(27) &= 0.001071(27)^2 - 0.0554(27) + 0.832 \, kg \text{ } per \text{ } month \\ &\approx 0.117 \, kg \text{ } per \text{ } month \end{aligned}$$

$$\begin{aligned} w'(36) &= 0.001071(36)^2 - 0.0554(36) + 0.832 \, kg \text{ } per \text{ } month \\ &\approx 0.226 \, kg \text{ } per \text{ } month \end{aligned}$$

A 3-month old boy increases his weight by 0.675 kg per month. In contrast, a 24-month old boy gains weight at a rate of 0.117 kg per month and a 36-month old boy gains weight at 0.226 kg per month.

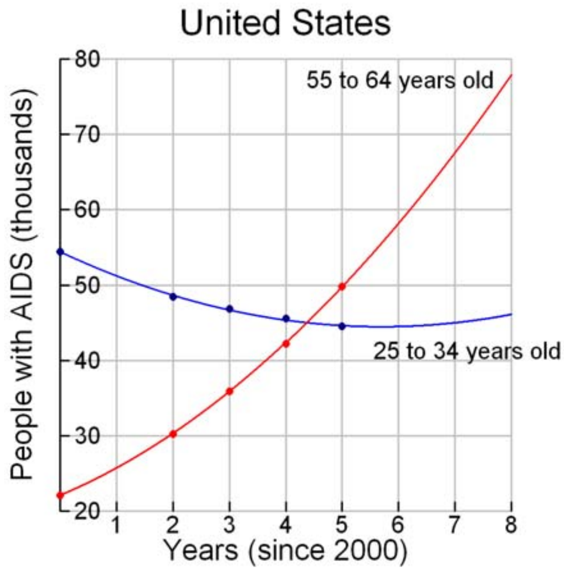
Living with AIDS

Working with Derivatives

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to estimate how rapidly something is changing from a mathematical model.

The figure below shows the estimated number of people living with AIDS in the United States in two different age groups together with a mathematical model for each group (Source: Modeled from Statistical Abstract of the United States 2008, Table 176).



1. According to the model was the number of 55 to 64 year old people with AIDS increasing more rapidly in 2000 or in 2005? Refer to the graph as a part of your explanation.

2. Let T represent the number of 25 to 34 year old people with AIDS (in thousands) in year x. Referring to the graph, determine the sign of $T'(3)$ and $T''(3)$. Justify your conclusion.

3. The function T has equation $T(x) = 0.3067x^2 - 3.493x + 54.41$. Calculate $T'(5)$ and interpret the real world meaning of the result.

4. Let F represent the number of 55 to 64 year old people with AIDS (in thousands) in year x. The function F has equation $F(x) = 0.4791x^2 + 3.141x + 22.09$. Calculate $F''(x)$ and interpret the real world meaning of the result.

6. The table shows the number of 15 to 24 year old people living with AIDS in selected years. Estimate the rate at which the number of 15 to 24 year old people living with AIDS was changing in 2001 and in 2004.

Year	15 to 24 year olds
2000	4,944
2002	5,419
2003	6,056
2004	6,729
2005	7,562

Source: Statistical Abstract of the United States 2008, Table 176

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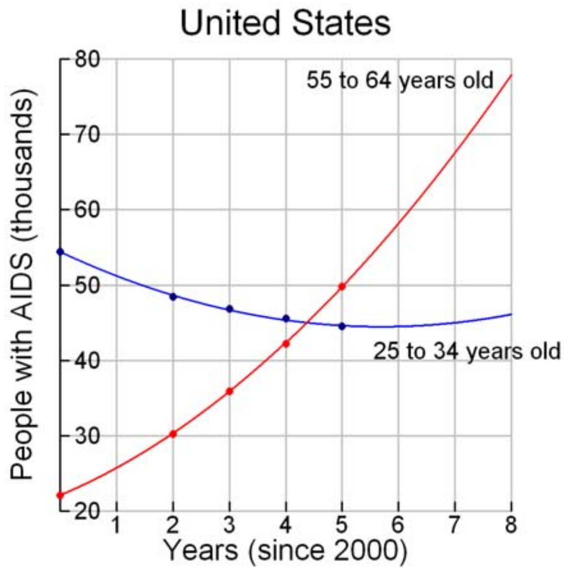


1. According to the model was the number of 55 to 64 year old people with AIDS increasing more rapidly in 2000 or in 2005? Refer to the graph as a part of your explanation.

Because the graph is concave up, the rate of change (in thousand people per year) is itself increasing. That is, as time increases, the rate of change increases. Therefore, the number of 55 to 64 year old people with AIDS was increasing more rapidly in 2005 than in 2000.

2. Let T represent the number of 25 to 34 year old people with AIDS (in thousands) in year x. Referring to the graph, determine the sign of $T'(3)$ and $T''(3)$. Justify your conclusion.

Since the graph of T is decreasing at $x = 3$, $T'(3)$ *is negative.*

Since the graph of T is concave up at $x = 3$, $T''(3)$ *is positive.*

3. The function T has equation $T(x) = 0.3067x^2 - 3.493x + 54.41$. Calculate $T'(5)$ and interpret the real world meaning of the result.

$$T'(x) = 0.6134x - 3.493$$
$$T'(5) = 0.6134(5) - 3.493$$
$$= -0.426 \textit{ thousand people per year}$$

In 2005, the number of 24 – 35 year old people living with AIDS was decreasing at a rate of 426 people per year.

4. Let F represent the number of 55 to 64 year old people with AIDS (in thousands) in year x. The function F has equation $F(x) = 0.4791x^2 + 3.141x + 22.09$. Calculate $F''(x)$ and interpret the real world meaning of the result.

$$F'(x) = 0.9582x + 3.141$$
$$F''(x) = 0.9582 \frac{\textit{thousand people per year}}{\textit{year}}$$

The rate at which the number of 55 to 64 year old people with AIDS is increasing is increasing by 958.2 people per year each year. For example, in year 0 the number of people with AIDS was increasing at a rate of 3141 people per year. This rate will increase by 958.2 to 4099.2 people per year in year 1. This rate will continue to increase by 958.2 people per year each subsequent year.

6. The table shows the number of 15 to 24 year old people living with AIDS in selected years. Estimate the rate at which the number of 15 to 24 year old people living with AIDS was changing in 2001 and in 2004.

Year	15 to 24 year olds
2000	4,944
2002	5,419
2003	6,056
2004	6,729
2005	7,562

Source: Statistical Abstract of the United States 2008, Table 176

We use the average rate of change between 2000 and 2002 to estimate the instantaneous rate of change in 2001. Similarly, we use the average rate of change between 2003 and 2005 to estimate the instantaneous rate of change in 2004.

$$\frac{5419 - 4944}{2002 - 2000} \frac{\textit{people}}{\textit{year}} = 237.5 \textit{ people per year} \qquad \frac{7562 - 6056}{2005 - 2003} \frac{\textit{people}}{\textit{year}} = 753 \textit{ people per year}$$

We estimate that in 2001 the number of 15 to 24 year old people living with AIDS was increasing at a rate of 237.5 people per year. We estimate that in 2004 that rate had increased to 753 people per year.

Movie Ticket Prices

Using Calculus to Analyze Change

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to use calculus concepts to analyze changes in price.

Based on data from 2000 – 2008, the average price of a movie ticket may be modeled by

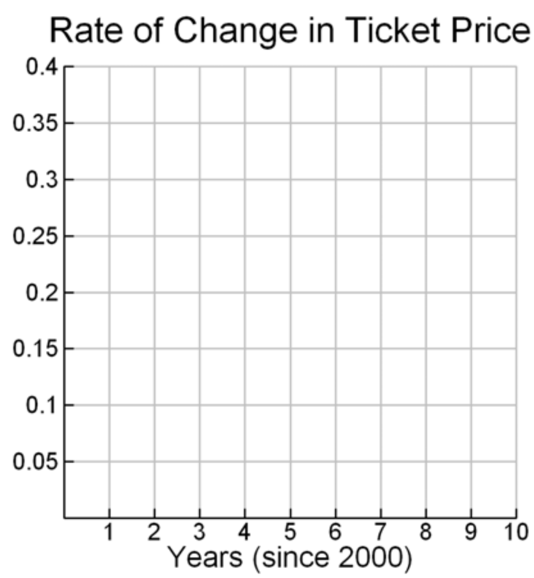
$$p(t) = 0.2055t + 5.405 \text{ dollars}$$

where t is the number of years since 2000 (Source: Modeled from www.boxofficemojo.com).

1. Find $p'(t)$ including units.

2. What is the real world meaning of the result of (1)?

3. Graph $p'(t)$ on the axes below. Then shade the region represented by $\int_3^7 p'(t)\,dt$.



4. Use geometry to calculate the area of the shaded region in (3).

5. Calculate $\int_{3}^{7} p'(t)\,dt$ and interpret what it means in terms of movie ticket prices.

6. According to the model, by how much will ticket prices change between 2007 and 2010?

7. Use calculus to determine the average price of a ticket between 2003 and 2008.

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where t is the number of years since 2000 (Source: Modeled from www.boxofficemojo.com).

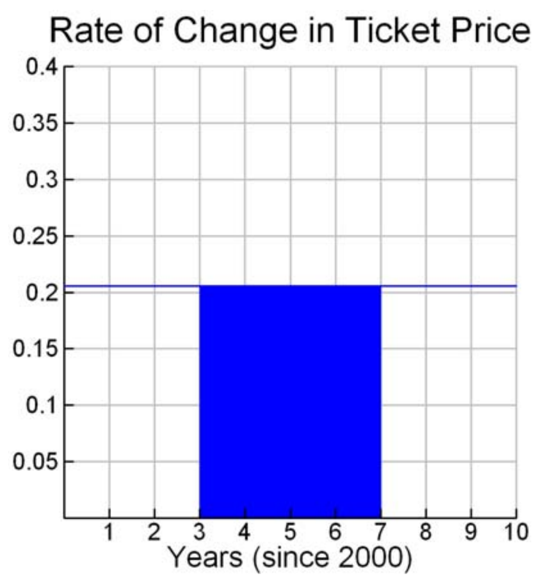
1. Find $p'(t)$ including units.

$$p'(t) = 0.2055 \frac{\textit{dollars}}{\textit{year}}$$

2. What is the real world meaning of the result of (1)?

The price of a movie ticket is changing at a rate of \$0.2055 per year. In other words, ticket prices are going up by about \$0.21 each year.

3. Graph $p'(t)$ on the axes below. Then shade the region represented by $\int_3^7 p'(t)\,dt$.



4. Use geometry to calculate the area of the shaded region in (3).

The shaded region is a rectangle with length 4 and width 0.2055.

$$\begin{aligned} A &= lw \\ &= 4(0.2055) \\ &= 0.822 \end{aligned}$$

5. Calculate $\int_3^7 p'(t)\,dt$ and interpret what it means in terms of movie ticket prices.

$$\begin{aligned} \int_3^7 p'(t)\,dt &= p(7) - p(3) \\ &= 6.8435 - 6.0215 \\ &= 0.822\,\textit{dollars} \end{aligned}$$

Between 2003 and 2007, movie ticket prices increased by about $0.82. That is, 2007 ticket prices were roughly 82 cents higher than 2003 ticket prices.

6. According to the model, by how much will ticket prices change between 2007 and 2010?

We must find $\int_7^{10} p'(t)\,dt$.

$$\begin{aligned} \int_7^{10} p'(t)\,dt &= p(10) - p(7) \\ &= 7.46 - 6.8435 \\ &= 0.6165 \end{aligned}$$

Between 2007 and 2010, we predict that ticket prices will increase by about $0.62.

7. Use calculus to determine the average price of a ticket between 2003 and 2008.

$$\begin{aligned} \textit{average price} &= \frac{\int_3^8 p(t)\,dt}{8-3} \\ &= \frac{0.5\left(0.2055t^2\right) + 5.405t + C\ \Big|_3^8}{5} \\ &= \frac{49.816 - 17.13975}{5} \\ &\approx 6.54 \end{aligned}$$

The average price of a movie ticket between 2003 and 2008 was $6.54.

PreK – Grade 8 Students

Working with Rates of Change

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

Based on projections from 2006–07 to 2016–17, the number of students in pre-K through grade 8 in the United States may be modeled by

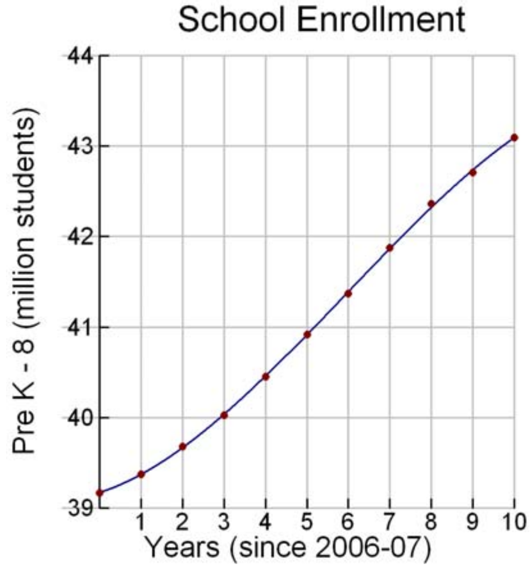
$$s(t) = -3.180t^3 + 56.37t^2 + 146.4t + 39{,}170 \text{ thousand students}$$

where t is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

2. Use calculus to show that s is increasing on the interval $[0,10]$.

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of s on the interval $[0,10]$ and interpret what it represents in terms of the rate of change of s. (Note: The units on the graph are million students and the units of the equation of s are thousand students.)



4. Use calculus to determine the most rapid rate of increase in s.

PreK – Grade 8 Students

Working with Rates of Change

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

Based on projections from 2006–07 to 2016–17, the number of students in pre-K through grade 8 in the United States may be modeled by

$$s(t) = -3.180t^3 + 56.37t^2 + 146.4t + 39{,}170 \text{ thousand students}$$

where t is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

$$\begin{aligned} s'(t) &= -9.540t^2 + 112.74t + 146.4 \\ s'(3) &= -9.540(3)^2 + 112.74(3) + 146.4 \\ &\approx 398.8 \textit{ thousand students per year} \end{aligned}$$

$$\begin{aligned} s'(8) &= -9.540(8)^2 + 112.74(8) + 146.4 \\ &\approx 437.8 \textit{ thousand students per year} \end{aligned}$$

According to the model, the number of students will be increasing at a rate of 398.8 thousand students per year in 2009 – 10 and increasing at a rate of 437.8 thousand students per year in 2014 – 15. The number of students is changing more rapidly in 2014 – 15.

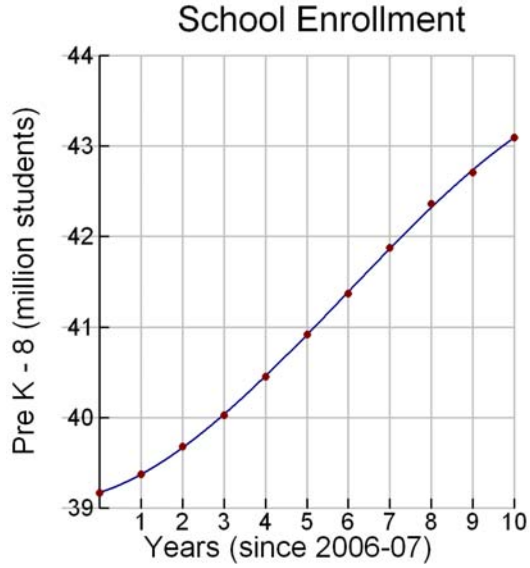
2. Use calculus to show that s is increasing on the interval $[0,10]$.

$$\begin{aligned} s'(t) &= -9.540t^2 + 112.74t + 146.4 \\ 0 &= -9.540t^2 + 112.74t + 146.4 \\ t &= \frac{-112.74 \pm \sqrt{(112.74)^2 - 4(-9.540)(146.4)}}{2(-9.540)} \\ &\approx -1.18,\ 13.00 \end{aligned}$$

$$s'(0) = 146.4 > 0$$

The critical values are −1.18 and 13.00. Since $-1.18 < 0 < 13.00$ *and* $s'(0) > 0$, s' *is positive on* $[-1.18, 13.00]$. *Therefore,* s' *is positive on* $[0,10]$ *and* s *is increasing.*

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of s on the interval $[0,10]$ and interpret what it represents in terms of the rate of change of s. (Note: The units on the graph are million students and the units of the equation of s are thousand students.)



We are looking for where the graph changes concavity. It appears that the graph changes from concave up to concave down at around $(6, 41.3)$. *So the inflection point of s is approximately* $(6, 41300)$. *That is, in 2012 – 2013 the number of students is about 41,300,000. In terms of the rate of change of* s, *this is the year when the number of students is increasing most rapidly.*

4. Use calculus to determine the most rapid rate of increase in s.

$$\begin{aligned} s'(t) &= -9.540t^2 + 112.74t + 146.4 \\ s''(t) &= -19.08t + 112.74 \\ 0 &= -19.08t + 112.74 \\ 19.08t &= 112.74 \\ t &\approx 5.9 \end{aligned}$$

The inflection point occurs at $t \approx 5.9$. *This is where the graph is increasing most rapidly.*

$$\begin{aligned} s'(t) &= -9.540t^2 + 112.74t + 146.4 \\ s'(5.9) &= -9.540(5.9)^2 + 112.74(5.9) + 146.4 \\ &\approx 479.5 \textit{ thousand students per year} \end{aligned}$$

The most rapid rate of increase was 479.5 thousand students per year.

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Shipping Packages

Optimizing Functions with Calculus

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to determine the maximum volume of a package that meets postal service size guidelines.

The United States Postal Service processes thousands of packages each year. The postal service classifies a package as a *machinable parcel* if it meets the following criteria (Source: www.usps.com).

- Not less than 6 inches long, 3 inches high, 1/4 inch thick, and 6 ounces in weight.
- Not more than 34 inches long, or 17 inches high, or 17 inches thick, or 35 pounds in weight. For books, or other printed matter, the maximum weight is 25 pounds.

1. Draw a diagram of a rectangular box with equal height and width. Label each dimension of the box (length, width, height) with an appropriate variable.

2. Referring to the size restrictions in the postal service guidelines, identify the domain of each of the variables identified in Exercise 1.

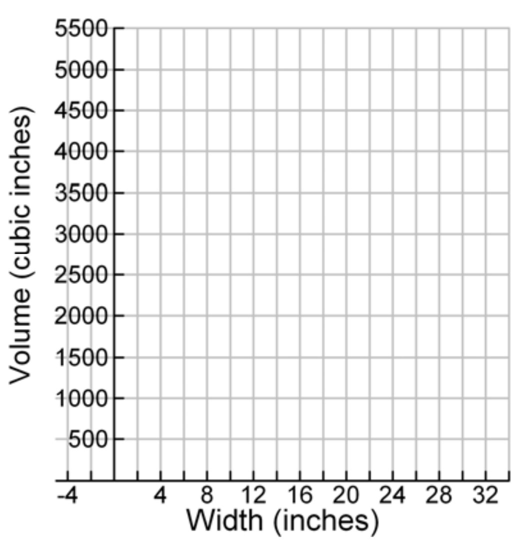
3. Create an equation for the volume of the package using the variables identified in Exercise 1.

4. The girth of the package is the distance around its thickest part. Create an equation for the girth of the package using the variables identified in Exercise 1. Then determine the girth of the package with maximum volume.

5. Suppose that the length plus girth of a particular rectangular package with equal height and width is 84 inches. Write a function for the volume of the package as a function of its width.

6. Use calculus to find the dimensions of the box described in Exercise 5 that has the maximum volume.

7. Verify the solution in Exercise 6 by drawing the graph of the volume function found in Exercise 5 on the axes below.



Shipping Packages

Optimizing Functions with Calculus

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1. Draw a diagram of a rectangular box with equal height and width. Label each dimension of the box (length, width, height) with an appropriate variable.

2. Referring to the size restrictions in the postal service guidelines, identify the domain of each of the variables identified in Exercise 1.

The length of the package must be between 6 inches and 34 inches. Therefore, $6 \le L \le 34$.
The height of the package must be between 3 inches and 17 inches. Since the height and width of the box are equal, $3 \le W \le 17$.

3. Create an equation for the volume of the package using the variables identified in Exercise 1.

Volume is the product of length, width, and height. Therefore, $V = LW^2$.

4. The girth of the package is the distance around its thickest part. Create an equation for the girth of the package using the variables identified in Exercise 1. Then determine the girth of the package with maximum volume.

The girth is given by $G = 4W$ *. The package with maximum volume will have maximum length, width, and height. We calculate the girth:* $G = 4(17) = 68$*. The girth of the package with maximum volume is 68 inches.*

MIRL

5. Suppose that the length plus girth of a particular rectangular package with equal height and width is 84 inches. Write a function for the volume of the package as a function of its width.

We know $G = 4W$. *Since the length plus the girth of this package is 84 inches, we have*

$$84 = L + 4W$$
$$L = 84 - 4W$$

The volume of the package is $V = LW^2$. *So*

$$\begin{aligned} V &= (84 - 4W)W^2 \\ &= -4W^3 + 84W^2 \end{aligned}$$

6. Use calculus to find the dimensions of the box described in Exercise 5 that has the maximum volume.

We first find the derivative of the volume function and set it equal to zero.

$$V' = -12W^2 + 168W$$
$$0 = -12W(W - 14)$$

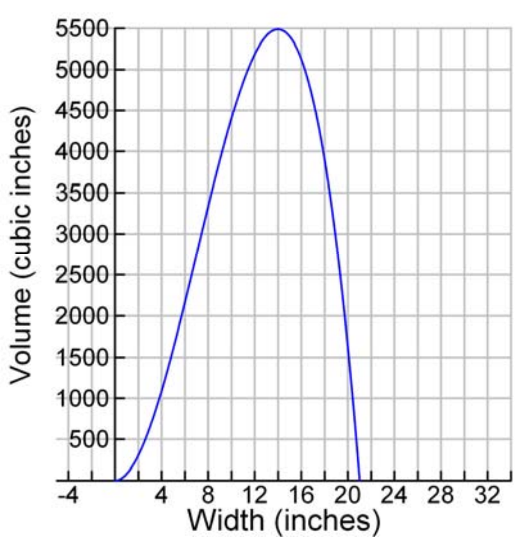
The critical values of the volume function are $W = 0$ *and* $W = 14$. *Clearly, a width of 0 inches will not maximize the volume. We observe that* $V'(13) > 0$ *and* $V'(15) < 0$. *So at* $W = 14$, V *changes from increasing to decreasing. Thus a maximum occurs at* $W = 14$.

In Exercise 5, we showed that $L = 84 - 4W$. *We substitute* $W = 14$ *into this equation and solve for* L.

$$\begin{aligned} L &= 84 - 4(14) \\ &= 28 \end{aligned}$$

According to our calculations, a box with length 28 inches and height and width 14 inches has the maximum volume. However, we must verify that these values are within the domain of the function. From Exercise 2 we have $6 \le L \le 34$ *and* $3 \le W \le 17$. *Since* $6 \le 28 \le 34$ *and* $3 \le 14 \le 17$, *our calculated results satisfy the domain restrictions of the function.*

7. Verify the solution in Exercise 6 by drawing the graph of the volume function found in Exercise 5 on the axes below.



Shipping Packages #2

Optimizing Functions with Calculus

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to determine the minimum volume of a package that meets postal service size guidelines.

The United States Postal Service processes thousands of packages each year. The postal service classifies a package as a *machinable parcel* if it meets the following criteria (Source: www.usps.com).

- Not less than 6 inches long, 3 inches high, 1/4 inch thick, and 6 ounces in weight.
- Not more than 34 inches long, or 17 inches high, or 17 inches thick, or 35 pounds in weight. For books, or other printed matter, the maximum weight is 25 pounds.

1. Draw a diagram of a rectangular box with equal height and width. Label each dimension of the box (length, width, height) with an appropriate variable.

2. Referring to the size restrictions in the postal service guidelines, identify the domain of each of the variables identified in Exercise 1.

3. Create an equation for the volume of the package using the variables identified in Exercise 1.

4. The girth of the package is the distance around its thickest part. Create an equation for the girth of the package using the variables identified in Exercise 1. Then determine the girth of the package with minimum volume.

MIRL

5. Suppose that the length plus girth of a particular rectangular package with equal height and width is 24 inches. Write a function for the volume of the package as a function of its width.

6. Use calculus to find the dimensions of the box described in Exercise 5 that has the *minimum* volume.

Shipping Packages #2

Optimizing Functions with Calculus

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Using the concepts in this worksheet, you will be able to determine the minimum volume of a package that meets postal service size guidelines.

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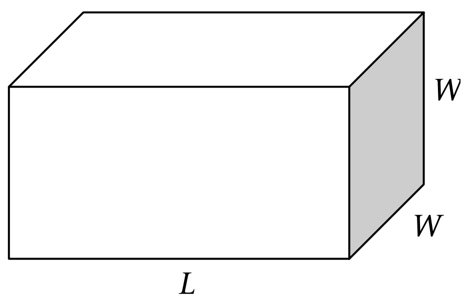
- Not more than 34 inches long, or 17 inches high, or 17 inches thick, or 35 pounds in weight. For books, or other printed matter, the maximum weight is 25 pounds.

1. Draw a diagram of a rectangular box with equal height and width. Label each dimension of the box (length, width, height) with an appropriate variable.



2. Referring to the size restrictions in the postal service guidelines, identify the domain of each of the variables identified in Exercise 1.

The length of the package must be between 6 inches and 34 inches. Therefore, $6 \le L \le 34$.
The height of the package must be between 3 inches and 17 inches. Since the height and width of the box are equal, $3 \le W \le 17$.

3. Create an equation for the volume of the package using the variables identified in Exercise 1.

Volume is the product of length, width, and height. Therefore, $V = LW^2$.

4. The girth of the package is the distance around its thickest part. Create an equation for the girth of the package using the variables identified in Exercise 1. Then determine the girth of the package with minimum volume.

The girth is given by $G = 4W$. *The package with minimum volume will have minimum length, width, and height. We calculate the girth:* $G = 4(3) = 12$. *The girth of the package with minimum volume is 12 inches.*

5. Suppose that the length plus girth of a particular rectangular package with equal height and width is 24 inches. Write a function for the volume of the package as a function of its width.

We know $G = 4W$ *. Since the length plus girth of this package is 24 inches, we have*

$$24 = L + 4W$$
$$L = 24 - 4W$$

The volume of the package is $V = LW^2$ *. So*

$$\begin{aligned} V &= (24 - 4W)W^2 \\ &= -4W^3 + 24W^2 \end{aligned}$$

6. Use calculus to find the dimensions of the box described in Exercise 5 that has the *minimum* volume.

We first find the derivative of the volume function and set it equal to zero.

$$V' = -12W^2 + 48W$$
$$0 = -12W(W - 4)$$

The critical values of the volume function are $W = 0$ *and* $W = 4$ *. However, the domain of the volume function is* $3 \le W \le 17$ *so we must throw out the value* $W = 0$ *. We observe that* $V'(3) > 0$ *and* $V'(5) < 0$ *. So at* $W = 4$ *,* V *changes from increasing to decreasing. Thus a maximum occurs at* $W = 4$ *. But we are looking for a minimum. Since we eliminated both critical values of the volume function, the minimum volume must occur at an end point. We've already identified* $W = 3$ *as the smallest value of W in the domain. What is the largest value of W in the domain? From Exercise 5, we have* $L = 24 - 4W$ *. According to the domain restrictions in Exercise 2,* $6 \le L \le 34$ *. We combine these results.*

$$6 \le L \le 24 - 4W$$
$$6 \le 24 - 4W$$
$$-18 \le -4W$$
$$W \le 4.5$$

We evaluate the volume function at $W = 3$ *and* $W = 4.5$ *.*

$$\begin{aligned} V(3) &= (24 - 4(3))(3)^2 \\ &= 108 \end{aligned} \qquad \begin{aligned} V(4.5) &= (24 - 4(4.5))(4.5)^2 \\ &= 121.5 \end{aligned}$$

The minimum volume of 108 cubic inches occurs when the width of the box is 3 inches. We calculate the corresponding length.

$$\begin{aligned} L(3) &= 24 - 4(3) \\ &= 12 \end{aligned}$$

When the length of the box is 12 inches and the width and height of the box are each 3 inches, the volume of the box is minimized.

Shipping Packages #3

Optimizing Functions with Calculus

"When am I ever going to use this?"

Using the concepts in this worksheet, you will be able to determine the maximum volume of a package that meets postal service size guidelines.

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- Not more than 34 inches long, or 17 inches high, or 17 inches thick, or 35 pounds in weight. For books, or other printed matter, the maximum weight is 25 pounds.

1. Draw a diagram of an enclosed cylindrical tube. Label each dimension of the tube (length, width, height) with an appropriate variable.

2. Referring to the size restrictions in the postal service guidelines, identify the domain of each of the variables identified in Exercise 1.

3. Create an equation for the volume of the tube using the variables identified in Exercise 1.

4. The girth of the tube is the distance around its thickest part. Create an equation for the girth of the tube using the variables identified in Exercise 1.

5. Suppose that the length plus the girth of a particular cylindrical tube is 60 inches. Write a function for the volume of the tube as a function of its width.

6. Use calculus to find the dimensions of the tube described in Exercise 5 that has the *maximum* volume.

Shipping Packages #3

Optimizing Functions with Calculus

"When am I ever going to use this?"

Using the concepts in this worksheet, you will be able to determine the maximum volume of a package that meets postal service size guidelines.

The United States Postal Service processes thousands of packages each year. The postal service classifies a package as a *machinable parcel* if it meets the following criteria (Source: www.usps.com).

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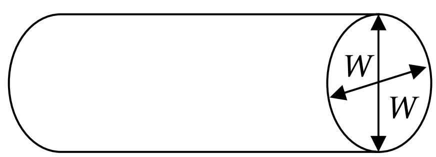
- Not more than 34 inches long, or 17 inches high, or 17 inches thick, or 35 pounds in weight. For books, or other printed matter, the maximum weight is 25 pounds.

1. Draw a diagram of an enclosed cylindrical tube. Label each dimension of the tube (length, width, height) with an appropriate variable.



The diameter W is both the height and width of the tube.

2. Referring to the size restrictions in the postal service guidelines, identify the domain of each of the variables identified in Exercise 1.

The length of the tube must be between 6 inches and 34 inches. Therefore, $6 \le L \le 34$.
The height of the tube must be between 3 inches and 17 inches. Since the height and width of the tube are equal, $3 \le W \le 17$.

3. Create an equation for the volume of the tube using the variables identified in Exercise 1.

Volume of a cylinder is the product of the area of the circular end of the tube and its length. The area of a circle is $A = \pi r^2$. The radius of the tube is $r = \frac{W}{2}$. Therefore,

$$V = \pi\left(\frac{W}{2}\right)^2 L$$
$$= \frac{\pi}{4} W^2 L$$

4. The girth of the tube is the distance around its thickest part. Create an equation for the girth of the tube using the variables identified in Exercise 1.

The circumference of a circle is $C = 2\pi r$ *. Therefore, the distance around the tube is*

$$\begin{aligned} C &= 2\pi\left(\frac{W}{2}\right) \\ &= \pi W \end{aligned}$$

The girth is given by

$$G = \pi W .$$

5. Suppose that the length plus the girth of a particular cylindrical tube is 60 inches. Write a function for the volume of the tube as a function of its width.

We know $G = \pi W$ *. Since the length plus girth of this tube is 60 inches, we have*

$$\begin{aligned} 60 &= L + \pi W \\ L &= 60 - \pi W \end{aligned}$$

The volume of the package is $V = \frac{\pi}{4} W^2 L$ *. So*

$$\begin{aligned} V &= \frac{\pi}{4} W^2 (60 - \pi W) \\ &= -\frac{\pi^2}{4} W^3 + 15\pi W^2 \end{aligned}$$

6. Use calculus to find the dimensions of the tube described in Exercise 5 that has the *maximum* volume.

We first find the derivative of the volume function and set it equal to zero.

$$\begin{aligned} V' &= -\frac{3\pi^2}{4} W^2 + 30\pi W \\ 0 &= -3\pi W\left(\frac{\pi}{4} W - 10\right) \end{aligned}$$

We set each factor equal to zero and solve.

$$\begin{aligned} -3\pi W &= 0 \\ W &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\pi}{4} W - 10 &= 0 \\ \frac{\pi}{4} W &= 10 \\ W &= \frac{40}{\pi} \\ &\approx 12.7 \end{aligned}$$

We observe that V' *changes from positive to negative at* $W \approx 12.7$*. So a maximum occurs at* $W \approx 12.7$*.*

We next calculate the value of L.

$$\begin{aligned} L &= 60 - \pi W \\ L &= 60 - \pi\left(\frac{40}{\pi}\right) \\ &= 20 \end{aligned}$$

So a length of 20 inches and a width and height of 12.7 inches will maximize volume.

Wind Chill Temperature

Using Calculus with Multivariable Functions

When am I ever going to use this?

Using the concepts in this worksheet, you will be able to determine how changes in wind speed and temperature affect the apparent temperature.

When it is cold outside, the wind makes it feel even colder. The wind chill temperature is the "feels like" temperature. For temperatures of 40 degrees Fahrenheit or lower, the following formula is used to calculate the wind chill temperature.

$$C(T,W) = 35.74 + (0.6215T) - \left(35.75W^{0.16}\right) + \left(0.4275TW^{0.16}\right)$$

In the formula, W is the wind speed (in miles per hour) and T is the temperature (in degrees Fahrenheit), and C is the wind chill temperature.

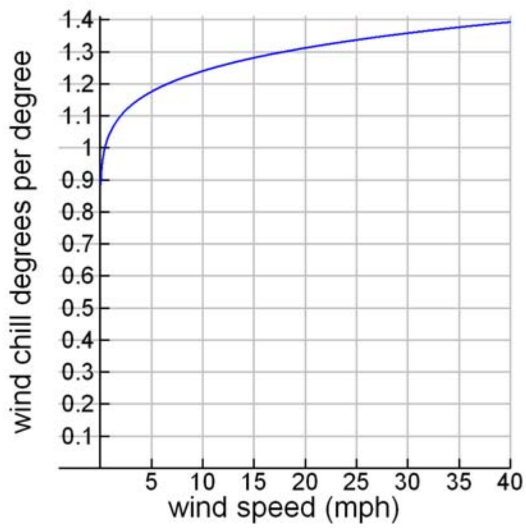
1. Calculate $C(32,16)$ and interpret the real world meaning of the result.

2. Determine the first partial derivatives C_W and C_T, including appropriate units.

3. Calculate $C_W(32,16)$ and interpret the practical meaning of the result.

4. Calculate $C_T(32,16)$ and interpret the practical meaning of the result.

5. The graph of C_T is given on the axes below. Explain what the graph tells us about the change in wind chill temperature.



6. Assuming that the temperature remains constant, does the rate of change in the wind chill temperature depend on both the wind speed and temperature? Explain.

Wind Chill Temperature

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In the formula, W is the wind speed (in miles per hour) and T is the temperature (in degrees Fahrenheit), and C is the wind chill temperature.

1. Calculate $C(32,16)$ and interpret the real world meaning of the result.

$$\begin{aligned} C(32,16) &= 35.74 + (0.6215(32)) - \left(35.75(16)^{0.16}\right) + \left(0.4275(32)(16)^{0.16}\right) \\ &\approx 21.2 \end{aligned}$$

When the temperature is 32 degrees Fahrenheit and the wind is blowing at 16 miles per hour, it feels like 21.2 degrees Fahrenheit.

2. Determine the first partial derivatives C_W and C_T, including appropriate units.

$$\begin{aligned} C_W &= -(0.16)\left(35.75W^{-0.84}\right) + (0.16)(0.4272T)W^{-0.84} \\ &= (-5.72 + 0.068352T)W^{-0.84}\ \frac{\textit{degrees of windchill}}{\textit{mph}} \end{aligned}$$

$$C_T = 0.6215 + 0.4275W^{0.16}\ \frac{\textit{degrees of windchill}}{\textit{degrees of temperature}}$$

3. Calculate $C_W(32,16)$ and interpret the practical meaning of the result.

$$\begin{aligned} C_W(32,16) &= (-5.72 + 0.068352(32))(16)^{-0.84} \\ &\approx -0.34\ \textit{degrees per mile per hour} \end{aligned}$$

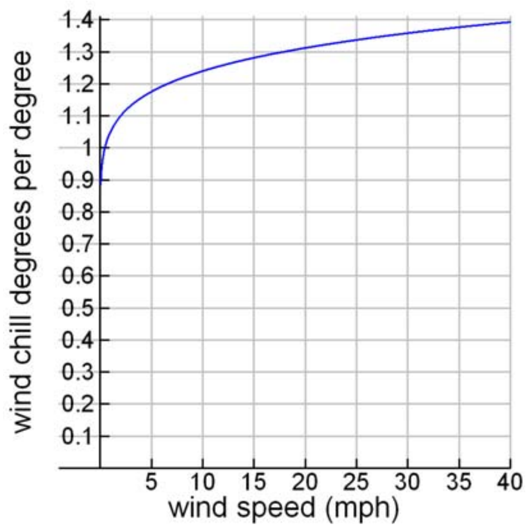
This means that when the temperature is 32 degrees and the wind speed is 16 mph a 1 mile per hour increase in wind speed will decrease the wind chill temperature by about 0.34 degrees.

4. Calculate $C_T(32,16)$ and interpret the practical meaning of the result.

$$C_T(32,16) = 0.6215 + 0.4275(16)^{0.16} \frac{\textit{degrees of windchill}}{\textit{degrees of temperature}}$$
$$\approx 1.3 \textit{ degrees of windchill per degree of temperature}$$

This means that when the temperature is 32 degrees and the wind speed is 16 mph a 1 degree increase in temperature will increase the wind chill temperature by about 1.3 degrees.

5. The graph of C_T is given on the axes below. Explain what the graph tells us about the change in wind chill temperature.



C_T assumes that wind speed remains constant and temperature is changing. The units on C_T are degrees of wind chill temperature per degree of temperature. Since the graph of C_T is increasing, the greater the wind speed the more substantial the effect of a one degree change in temperature on the wind chill temperature. For example, when the wind speed is about 7 mph, a one degree increase in temperature will increase the wind chill temperature by about 1.2 degrees. In contrast, when the wind speed is 40 mph, a one degree increase in temperature will increase the wind chill temperature by about 1.4 degrees.

6. Assuming that the temperature remains constant, does the rate of change in the wind chill temperature depend on both the wind speed and temperature? Explain.

Yes. Since the temperature remains constant, the rate of change in wind chill temperature is given by C_W. Recall from (2) that $C_W = (-5.72 + 0.068352T)W^{-0.84} \frac{\textit{degrees of windchill}}{\textit{mph}}$. Since the equation contains both T and W, the rate of change in wind chill temperature depends on both the wind speed and the temperature.

About the Author

Frank Wilson earned his B.S. and M.S. degrees in mathematics from Brigham Young University. He spent six years serving as an officer in the United States Air Force before returning to civilian life. He has taught students math at the United States Air Force Academy, Park College, Green River Community College, and Chandler-Gilbert Community College. In addition to teaching, Frank is a popular author and workshop presenter. His college mathematics textbooks (Finite Mathematics, Finite Mathematics and Applied Calculus, Brief Applied Calculus, and Applied Calculus) are used at colleges and universities across the United States. Finite Mathematics and Applied Calculus was selected by the Textbook and Academic Author's Association as the winner of the 2007 TEXTY Textbook Excellence award for mathematics. Frank's picture book on measurement, Measure Up! A Bug Contest, is popular among teachers and children alike.

Frank lives with his wife and five living children in Queen Creek, Arizona.

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