Welcome to Carnegie Learning’s Cognitive Tutor® Teacher’s Guide

Welcome to our community of teachers and students. We are aware that as you begin to prepare for your classes, working with a new curriculum can be exciting and a bit overwhelming. With this Teacher’s Guide, the support of our Educational Service Managers and our K-12 Teacher Community, we hope to facilitate and ease that process for you.

Before you begin planning your lessons, it is important to know that Carnegie Learning was founded on the beliefs that:

- Learning is the ultimate criterion of success.
- Optimal opportunities for success will not be achieved unless there is equity for all learners.
- Every learner must grow to be an active, independent agent in the learning process.

We believe that our ability to meet these tenets is ensured by our rich tradition in developing results-based products rooted in:

- Understanding learning and teaching processes through the use of theory, research, and practice.
- Applying theory, research, and experience to the design of high-quality curricular materials for students of all academic levels and background.
- Conducting ongoing research and development to optimize effectiveness.

And we know that it is the teachers and students that bring our curricula to life! We look forward to working with you and will greatly appreciate your contributions.

To contact our Educational Service Managers or Technical Support, please call 1.888.851.7094.
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Collaborative Classrooms

As you begin the process of planning for the school year, you will want to give serious consideration to how your classroom is structured. Early research on teaching and learning revealed that what happens in the classroom in the first three days determines the environment for the entire year. This finding can be important as you begin to think about your classroom and the Cognitive Tutor® curriculum. An effective implementation of the Carnegie Learning Cognitive Tutor® mathematics curricula is most likely to occur in the collaborative classroom, that is a classroom in which knowledge is shared.

Teachers in the collaborative classroom bring their extensive knowledge about teaching and learning, content, and skills, which build on the informal and formal knowledge, strategies and individual experiences that the students bring to the setting. This classroom differs from the traditional classroom, in which the teacher is seen as an information giver (Tinzmann, M.B.; Jones, B.F.; Fennimore, T.F.; Bakker, J; Fine, C.; and Pierce, J., 1990 www.ncrel.org/sdra/areas/rpl_esys/collab.htm.)

Carnegie Learning’s philosophy - Learning by Doing® - captures the belief that students develop understanding and skill by taking an active role in their environment. Furthermore it is Carnegie Learning’s belief that effective communication and collaboration are essential skills for the successful learner. It is through dialogue and discussion of different strategies and perspectives that students become knowledgeable independent learners. These beliefs can be realized in the collaborative classroom.

Defining a collaborative classroom

A collaborative classroom is an environment in which there is shared knowledge between teachers and students as well as shared authority among teachers and students. It is a classroom in which teachers are facilitators and students are active participants. Lastly it is an environment in which all students, not segregated by ability level, interest, and achievement can benefit.

Characteristics of the collaborative classroom

The collaborative classroom is identified by discussion, with in-depth accountable talk and two-way interactions, whether among members of the whole class or small groups. It is an environment that is well structured, where questioning and dialogue are valued, and appropriate parameters are set so that active learning can occur. Careful planning by the teacher ensures that students can work together to attain individual and collective goals, while capitalizing on developing their own strategies.

In the collaborative classroom, students are encouraged to take responsibility for their learning through monitoring and reflective self-evaluation. The collaborative classroom is one in which teachers spend more time in true academic interactions, as they guide students to search for information and help students to share what they know. As facilitators, teachers have the opportunity to provide the right amount of help to an individual student in a timely manner by providing appropriate hints, probing questions, feedback or help in clarifying thinking or the use of a particular strategy (Tinzmann, M.B.;
Classifying collaborative versus cooperative classrooms

In the collaborative classroom, two types of learning are ongoing: collaborative, which focuses on interaction, and cooperative, which is a structure of interaction that helps students to accomplish the goal or end product (Ten Panitz 1996 www.lgu.ac.uk/deliberations/collab.learning/panitz2.html). While these two forms of learning are often described and used interchangeably, differences do exist. The significant difference between a collaborative learning environment and a cooperative one is the amount of teacher control in setting goals and providing choice. For example, in a collaborative classroom students are positioned to set their own goals and choose activities, whereas in the cooperative learning environment, the teacher directs these activities.

Learning in the collaborative classroom

Critical to teaching and learning in the collaborative/cooperative environment is being able to define the responsibilities of the teacher and students and to understand what are best practices for your classroom. Ultimately, the goals of collaborative classrooms and cooperative classrooms are non-competing. For effective collaboration and cooperative teamwork, teachers and students must take on responsibilities to support the process. The table below reflects the parallel responsibilities of teachers and students.
Effective Collaboration and Cooperative Teamwork

<table>
<thead>
<tr>
<th>Teacher Responsibilities</th>
<th>Student Responsibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor student behavior.</td>
<td>Develop the skills to work cooperatively.</td>
</tr>
<tr>
<td>Provide assistance when needed.</td>
<td>Learn to talk and discuss problems with each other in order to accomplish the group goal.</td>
</tr>
<tr>
<td>Answer questions only when they are team questions.</td>
<td>Ask for help only after each person in the group has considered the questions and the group has a question for the teacher.</td>
</tr>
<tr>
<td>Interrupt the process to reinforce cooperative skill or to provide direct instructions to all students.</td>
<td>Believe they are part of a team and that all members of the team work together toward a common goal</td>
</tr>
<tr>
<td></td>
<td>Understand that the success or failure of the group is to be shared by all members</td>
</tr>
<tr>
<td></td>
<td>Understand the group dynamic and learn mathematics by working in groups.</td>
</tr>
<tr>
<td>Provide closure for the lesson.</td>
<td>Reflect back on the work of the group.</td>
</tr>
<tr>
<td>Evaluate the group process by discussing the actions of the team member.</td>
<td>Appreciate that working together is a process that capitalizes on the presence of student peers, encourages the interaction among students, and establishes relationships among team members.</td>
</tr>
<tr>
<td>Help students to become individually accountable for learning and reinforce this understanding regularly.</td>
<td>Realize that each member must contribute as much as he or she can to the group goal.</td>
</tr>
<tr>
<td></td>
<td>Understand that the success of the group is dependent on, and a direct effect of, the individual work of each member of the group.</td>
</tr>
<tr>
<td></td>
<td>Understand that group members are individually accountable for their own learning.</td>
</tr>
</tbody>
</table>
Finally it must be agreed that collaborative/cooperative learning is not one in which students:

- Work in small groups on a problem or group of problems without direction or individual responsibility.
- Work individually while sitting in a group working on problems.
- Work without conversation or interaction regarding the method or process being used for solving the problem.
- Allow one member of the group to do all the work while others sit passively.

**Shaping the collaborative classroom**

To ensure that the spirit and purpose of the collaborative classroom is clear from the onset of school, you will want to engage your students in a collaborative/cooperative activity the first day. In doing so, you can accomplish two important goals. First, students immediately understand the importance and value of working together, and second, students quickly move into their role as active participants.

Facts in Five, as you may have experienced in training, is an activity designed to meet these goals. Another popular activity with students is the one known “Broken Squares” (Spencer Kagan: Cooperative Learning©). In this activity, members of the team are each given several pieces of a broken square. The pieces belong to different squares. Students must create the whole square by taking turns giving each other one piece. No one may speak during the activity, i.e. no one can ask for what he/she needs. This activity is perfect for teaching students sensitivity and the importance of communication.

In addition to establishing the need and benefits of collaboration and cooperation, it is extremely important that expectations and “rules of the game” be defined. The best approach is to have the students work together in small groups to generate the guidelines for teamwork (See the Lesson Framework for Rule-Making).

As a guidepost for identifying the elements for successful group interactions, we suggest reviewing the “Ten guidelines for students doing group work in mathematics” written by Anne E. Brown for the CLUME Project [http://www.usplatt.edu/~clume/tenguid.htm](http://www.usplatt.edu/~clume/tenguid.htm). She developed these after viewing the video and audio tapes of more than a dozen group sessions of her students. This list reflects the actions that appeared to be critical to the success or failure of the group. In summary, the guidelines state the following:

1. Groups should be formed quickly and members of the group should sit close together, facing each other and get to work quickly. Members should call each other by first name. Members should not engage in “off-task” discussion. Everyone should be encouraged to participate.
2. All instructions should be read aloud so that everyone is aware of the expectations of the assignment.
3. Members of the group should listen to each other and not interrupt. Comments or questions should be acknowledged and/or responded to by other group members.
4. Members of the group should not accept being confused. If a member of the group does not understand the information that is presented, this person should ask someone to paraphrase it or re-phrase what was said.
5. Members of the groups should ask for clarification if a word is used in a way that is confusing.
6. The members of the group should work together on the same problem and check for agreement frequently.
7. Members of the group should explain their reasoning by “thinking out loud” and ask others to do the same. This helps everyone relate the information being presented to what they already know.
8. Members of the group should monitor the group’s progress and be aware of time constraints so that all members of the group meet the goals of the assignment.
9. If the group gets stuck, the members of the group should review and summarize what they have done so far and allow for questions, which can then find errors or missing connections to help the group’s work proceed.
10. Members of the group should engage in questioning, as it is the engine that drives mathematical investigation.

**Group work in the collaborative classroom**

If we expect students to work well cooperatively they will need to understand what it means to learn collaboratively and how it will benefit them. A good description of collaborative learning used by many of our teachers is:

> Collaborative Learning is a process where each individual contributes personal knowledge and skill with the intent of improving his or her learning accomplishments along with those of others.

Students should be aware that one of the most important goals of collaborative learning is to create a “community of learners”. They should understand that the community will grow and thrive only if all members of the group are active participants.

Finally, students need to understand that their role in the classroom will be different than what they may have experienced in other classes and so will yours!

Next you will want to introduce the features of a collaborative classroom to your students. Important characteristics of the classroom include:

- Shared responsibility
- Choice
- Discussion about how we learn from what is right as well as what is wrong
- Work is done by the group, whether it is one group or many groups.

And finally, you will want students to understand the goals and expectations:

- Students learn collaboratively to gain greater individual proficiency.
- Groups “sink or swim” together.
- EVERYONE suggests, questions, and encourages.
- Group members are responsible for each other’s learning.
- All group members bring valued talents and information to the task at hand.
Getting started in the collaborative classroom

Where activities lend themselves to students working in groups, you will want to structure the groups. It is possible to maintain a collaborative classroom even when students work individually, such as when they work in a computer lab, but are free to communicate with each other and share information.

To form groups, initially, you may want to set arbitrary groups and make changes as you observe students. One suggestion for structuring groups is to think about having two types of groups: long-term and short-term groups. The long-term groups, or home groups, stay together for the entire school year and sit together in class. Long-term groups enable students to build trust and confidence and to learn how to negotiate with each other to derive success. On the other hand, the short-term groups are randomly assigned for specific tasks. Short-term groupings develop the ability to work with many different people. Clearly, how you arrange the groups will depend on how to best meet your students needs.

Most importantly, you want to make sure that students are respectful of one another at all times. The success of the group rides on cooperation, which can be achieved only if students accept one another and value the contributions of others.

If you have students who do not want to work in groups, do not force the issue. Let those students work alone. It is important that the student who is working alone understands that you are not a member of his/her group. After the students find they have only themselves to talk to, and that those who are sharing information are progressing more easily, they will naturally gravitate to a group.

You want to structure the success of the experience, and so it is important to use guidelines and timelines. You will want the students to come up with the operating rules (with your help – remember you still have veto power if you feel some suggestion may not work toward creating a successful environment). Timelines are probably better left to you to determine. (See the Lesson Framework for facilitating rule-making).

Once the groups are formed, you might want to have one person from each group be designated as a facilitator. Some responsibilities of the facilitator include:

- Getting and returning all materials
- Communicating to the groups information from the teacher
- Handing in the completed assignment for the group

In the collaborative group, success depends upon everyone working on every part of the problem, so you may find that you do not want to assign roles such as recorder or reader to group members.

Students working together should generate noise and movement in the room. Some have defined this attribute as “controlled chaos”. To ensure that control remains, you will want to monitor group interactions and check for understanding of the task at hand. You may want to ask students to complete parts of the task, stop and discuss the work done,
summarize the main points of the task, and then continue. This works well when the activity is long.

As groups will work at different paces, you might want to have some additional tasks or ways of extending the task at hand for those who finish quickly.

**Facilitating groups in the collaborative classroom**

Facilitating the group process is critical. As noted earlier, you will only want to answer a question posed by the group. Students with individual questions should pose them to the group. You may also want to restrict the number of questions a group can ask per task (being generous the first couple of times they work in groups). When a group asks questions, think about helping them answer their questions by redirecting with questions such as:

- What does your group think?
- How did you arrive at that answer?
- How does this relate to past activities?
- What work have you done so far?
- What do you know about the problem?
- What do you need to figure out?
- What materials might help you figure this out?
- Are there other parts of the problem you can do first?

Other tips to consider as you manage your collaborative classroom include:

- Provide additional instruction to those struggling with a task.
- Listen carefully and value diversity of thought practices that often provide instructional opportunities.
- Balance learning mathematics and working effectively. Remember, no one is on task 100% of the time.
- Deal with conflict constructively.
- Ask students to sign-off on other group members’ papers to acknowledge that everyone understands.

Holding the groups accountable for an end product, such as a presentation, will add further value to the learning activity. As you have surely discovered, when you truly understand a concept or idea yourself, then you are able to explain that concept or idea to someone else.

**Presentations and discussions in the collaborative classroom**

To successfully close or wrap-up an activity with a presentation and discussion, students must know exactly what you expect from them. You should also make sure that the students know that you will hold the entire group accountable for the presentation. (This means they need to make sure they know how to hold each other accountable.)
Some suggestions to facilitate the presentation process include:

- Choosing presenters in a group to ensure that all students have the opportunity to present.
- Requiring students to defend and talk about their solutions as communication reinforces understanding.
- Holding all students accountable by asking questions of group members who are not presenting.
- Asking presenters to make connections and generalizations and extending concepts developed in the activity.
- Allowing groups time to process feedback and to celebrate their achievements. (See Lesson Frameworks for detailed examples.)

To bring closure to the group work and presentation process, you may want to work with the whole class and engage students in discussion or have them keep learning journals. Some suggestions for summary wrap-up questions include:

- What was something you learned from this problem?
- What were the mathematical concepts you applied in solving this problem?
- What do you still have questions about?
- What are three things your group did well?
- What is at least one thing your group could do even better the next time?

Checklist

To help you know where you are in the transition process from a teacher centered to a learner centered classroom use the criteria on the following page to evaluate your classroom (Courtesy of Jacquelyn Snyder, Jan Sinopoli and Vince Vernachhio, Pittsburgh Public Schools). Use your initial evaluation as a baseline measure and check yourself at regular intervals through the term.
### Teacher-Directed Classroom
- The teacher directs all classroom activity.
- Each activity is dependent on the teacher.
- The teacher is in the front of the room instructing the entire class using the blackboard or overhead the majority of the time.
- The teacher models examples of the lesson objective and directs students to practice clone problems found in the text or on handouts designed by the teacher.
- Students are seated in rows, working as a class with the teacher at the helm or working independently.
- The teacher presents the material while students watch and take notes.
- The students work independently as the teacher tries to help each one individually.
- The teacher does the problem for the student when he/she is having difficulty.
- The teacher does the thinking and the work.
- The teacher asks low level or fill-in-the-blank type questions that can be answered with a single number or in a word to two.
- The majority of classroom discourse is teacher to student.
- The teacher encourages students to memorize rules, procedures and formulas.

### Learner-Centered Classroom
- The teacher facilitates the classroom activities.
- Most activities require only guidance by the teacher.
- The teacher walks around the classroom during all activities watching and listening to student-to-student discourse.
- The teacher monitors the students to keep them on task, while the students actively work together on an activity.
- The students are typically paired or grouped to work together while the teacher facilitates the process.
- The teacher systematically brings the class together on several occasions, assuring that the mathematics intended is understood.
- Students are required to make presentations, explaining their progress within the activity.
- If a student is having difficulty understanding something, even after consulting with his/her group members, the teacher may ask the group leading questions to guide them to the desired outcome.
- The students do the thinking and the work.
- The teacher asks thought-provoking questions that required students to explain their thinking and processes.
- The majority of discourse is student to student.
- The teacher encourages students to construct knowledge. Prior knowledge is assessed and new concepts emerge.
Lesson Framework: Rule-Making

<table>
<thead>
<tr>
<th>The Activity</th>
<th></th>
</tr>
</thead>
</table>
| **Preparing for the Lesson** | o Arrange the class seats in groups of three or four such that students face each other. Position the desks in such a way that students will only have to do a half turn of their heads if you call their attention to the front of the room.  
 o Give poster boards to each group.  
 o Give colorful markers to each group. |
| **Expected Student Growth** | o Students should gain experience in working cooperatively, listening and respecting the ideas of others, and coming to consensus regarding the final product.  
 o Students will learn how to share power with the teacher. |
| **Initiating the Activity** | o Ask students if they have ever worked in groups before.  
 o Ask students to think about the good and not so good group experiences.  
 o Have students make lists of things happening in groups or things they think should happen in groups to have the group work productively and get the task done.  
 o Develop social guidelines for group work in class. The guidelines should be phrased positively and refer to observable behavior. Lists should not be too long. |
| **Facilitating the Activity** | o Monitor student behavior.  
 o Offer assistance only if necessary (e.g. student lists are getting too long).  
 o Interrupt the process to reinforce cooperative skills or to provide direct. |
### Lesson Framework: Rule-Making

<table>
<thead>
<tr>
<th>Student Presentations</th>
</tr>
</thead>
<tbody>
<tr>
<td>o Have all groups present their rules.</td>
</tr>
<tr>
<td>o Have students determine which rules are similar and record those.</td>
</tr>
<tr>
<td>o Have students look at the remainder of the rules and determine which should be included in the list of rules. Students should be able to justify their choices. As all students will be using these rules, there should be consensus on the final list.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>o Indicate that the list will be generated and every member in the class must sign off on it. Students have agreed to honor the list of rules and will be held accountable.</td>
</tr>
<tr>
<td>o Indicate that groups not adhering to the rules may have their group grades reduced.</td>
</tr>
</tbody>
</table>
## Presentation Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5</strong></td>
<td>You earn a five for your presentation if your presentation is nearly perfect. Your mathematics must be correct with only a very minor flaw (not having to do with the main idea of the problem). Your public speaking skills must also be perfect or quite close to perfect. You must look at your audience. Your must present yourself well and not make distracting gestures or hand motions during the presentation. Your rate of speech must be neither too fast nor too slow.</td>
</tr>
</tbody>
</table>
| **4** | You earn a four for your presentation if you miss one thing within the mathematical content of your presentation.  
OR  
You earn a four for your presentation if there is one thing you do not do very well within the public speaking part of the presentation. |
| **3** | You earn a three on your presentation if you are willing to get up and present, but have little content knowledge. This score means you are unable to complete the problem.  
OR  
You earn a three on your presentation if you can complete the problem, but your public speaking skills are very low. This score means you do not make eye contact; you speak inaudibly, your mumble your words, etc.  
OR  
You earn a three on your presentation if you have some content knowledge and make one major error as well as omit one of the important aspects of good public speaking. |
| **2** | You earn a two for your presentation if you get up for your presentation but really have very little content knowledge. This score means you are unable to complete the problem and your speaking skills are poor. |
| **1** | You earn a one for your presentation for being willing to get up and try a presentation. |
| **0** | You earn a zero for your presentation if you refuse to get up and try to present. |
The Algebra I Format

This book, in keeping with the presentation of content in our software, presents the lessons in each unit utilizing a window format. The significance of each window is detailed below.

<table>
<thead>
<tr>
<th><strong>Preview Window</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>preview</td>
</tr>
<tr>
<td>Windows that contain the word <em>preview</em> in the sidebar present a problem for which students may not yet possess all the necessary mathematical skills to solve, but they do have the required knowledge to approach the problem situation with ample understanding.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Input Window</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
</tr>
<tr>
<td>Windows that contain the word <em>input</em> in the sidebar present anticipatory information that is intended to get students interested in the problem. It often accomplishes this purpose by relating the problem to real world situations and events.</td>
</tr>
<tr>
<td>The input category presents a problem using one of three methods:</td>
</tr>
<tr>
<td>- <strong>scenario</strong> – presents information as a realistic problem situation.</td>
</tr>
<tr>
<td>- <strong>information</strong> – presents factual information about the math concept.</td>
</tr>
<tr>
<td>- <strong>problem</strong> – presents a mathematical activity not necessarily set in a context.</td>
</tr>
</tbody>
</table>
Windows that contain the word **process** in the sidebar enable the student to become more involved in solving the problem using multiple process modes.

The process category has the following four sub-categories:

- **model** – presents students with multiple representations of the problem and asks students to translate situations into specified representations.
- **evaluate** – students generate values, express reasoning, and solve equations.
- **practice** – presents students with a number of problems in the same skill set.
- **summary** – briefly summarizes the lesson and emphasizes important aspects.

Windows that contain the word **analysis** in the sidebar take the lesson concepts above the level of rote practice. Students “kick it up a notch” by engaging in reflection and justification.
The beginning of a new activity in a lesson is designated by the icon in the top left corner of the window.

<table>
<thead>
<tr>
<th>Extension Window</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windows that contain the word <strong>extension</strong> in the sidebar represent more challenging problems or a higher level of abstraction and generalization.</td>
</tr>
</tbody>
</table>
Unit 1: Patterns and Linear Functions

Unit Objectives and Skills

At the completion of this unit, students will attain general competency in reasoning inductively and deductively. Students will be able to identify a pattern and represent it symbolically. In addition, they will be able to write and evaluate an algebraic expression, to model a situation symbolically by naming variables and units, to understand the relationship between dependent and independent variables, and to define a function as a rule through which, given an input, a unique output can be generated.

Unit Overview

The focus of this unit is to introduce students to the role of mathematics as a tool to model physical or real world phenomena for the purpose of answering specific questions relating to the situation.

The materials are designed to introduce students to the language of mathematics and show them how to model situations symbolically. Research on learning has shown that students are more able to develop an understanding and retain information if they construct formal understanding from their own informal knowledge. Based on this principle, the unit extends the concept of algebra as generalized arithmetic through the use of everyday arithmetic situations. Students learn that, through the application of rules or operations such as addition, subtraction, multiplication and division, and procedures for solving for unknown values, they can systematically arrive at a solution.

Specifically, in this unit students learn how to represent a pattern symbolically. The software and print components provide students with situations that have clearly delineated patterns. This enables students to define the pattern and generalize using an equation.

Each problem represents a starting point as students move from arithmetic to algebraic representations. In the software and the next unit in the text, students will be introduced to everyday situations. Students will be asked a number of questions, which individually can be viewed as arithmetic and solved accordingly. In generating the solutions and capturing the initial information and the result, students are asked to look for the pattern and to generalize the solution process by writing an algebraic expression. Through this process, students begin to see algebra as generalized arithmetic. Each of these concepts is reinforced and enhanced in the software units entitled The Worksheet and Graphing Worksheet Points.
As stated above, the use of multiple representations begins in this unit. Multiple representations are used as general problem-solving strategies to help students organize information and solve novel problems. The representations can also be used as specific content heuristics for representing mathematical situations. As students respond to questions in the activities, the relationship between verbal and numerical representations, numerical and graphical representations, and graphical and algebraic representations are systematically developed.

At the completion of the unit, students will formally pass from an arithmetic, pre-algebraic understanding to an algebraic one. The emphasis of the subsequent exercises is on solidifying the transition and extending the concept of linear functions and equations, especially with respect to evaluating and solving equations. Students have multiple opportunities to practice solving linear equations and literal equations in the many *Equation Solving* units in the software.

**Unit Activities**

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

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**Suggestions for Classroom Implementation**

The process of describing a pattern or model in general terms is the main emphasis of the activities in this unit, beginning with the *$8 an Hour Problem*. The goal of this problem is to generate numerical data patterns by answering a series of arithmetically based questions. Patterns such as this
set the stage for generalization and modeling situations algebraically. To further students’ transition from arithmetic to algebraic representation, a variety of relevant situations common to their experiences are presented. The problem closes with a formal introduction to the definition of variables and constants, and algebraic expressions and equations. In the “process: practice” problem, students specifically define quantities symbolically, identify dependency relationships, and informally solve equations by working backwards arithmetically.

The **$8 an Hour Problem** introduces many key concepts integral to the development of the strands in the course, so you may find yourself taking a couple of class meetings to work through it completely. Through the modeling and evaluating activities, the groups construct relationships between numbers, operations, and statistics while building their transition from arithmetic to algebraic representations. Together, this provides the foundation for classroom discourse that highlights the connection between prior and new knowledge as well as informal and formal knowledge.

To guide students toward finding an expression that enables them to find any term and write the expression symbolically, number patterns and sequences of geometric objects are presented in **Number Patterns** and in **Finding the 10th Term**. Group discussions should focus on the importance of being able to describe relationships in general, whether between data inputs and outputs, repeating forms, or other sequences of numbers. As another activity, have each group generate a pattern to be solved by one of the other groups. Closing this iteration of activities is **Soaps and Bathbeads**. This problem represents a full integration of the concepts presented in the last three problems and serves as a transition back to problems in situational contexts designed to extend students’ understanding of linear functions.

**The Consultant Problem** is quite similar to the **$8 an Hour Problem** and therefore allows concepts to be reinforced. At the same time, the problem formalizes the concept of independent (input) and dependent (output) variables. This formalization sets the stage for defining functions as a dependency relationship.

At this point, you may want to compare the **$8 an Hour Problem** with **The Consultant Problem**. It may be interesting to discuss the relationship of the algebraic representation to the problem scenario. The problems share a similar model, but have different story lines. Ask students why they think this commonality is possible. As the constant rate of change is the key element here, you will want to draw out as much as you can about the ratio of the change in dependent variable with respect to the independent variable. Ask them if they think any problem with a constant rate of change will be modeled the same way. At this stage in the discussion, it is not necessary to formally introduce the term slope or its definition.

You might want to ask your students if they can think of another context that might yield the same mathematical model and graph. Moreover, you
may want to offer an example in which the rate of change is negative to see if students can extend understanding to this case.

Following this exercise, students see *U.S. Shirts* and *Hot Shirts*. These problems introduce them to initial or start values other than zero. Discussions should focus on how the graphs of these functions look different from the earlier problems. Have students work in groups on these problems, with half the groups doing *U.S. Shirts* and the other half working on *Hot Shirts*. Have the groups present their work and then have the entire class contribute to the problem *Comparing U.S. Shirts and Hot Shirts*. For this problem, have the graphs of the two functions drawn on the same grid and displayed using an overhead projector as you begin the comparative analysis.

As students work through the activities in this unit, you will want to stress the importance of always showing their work and writing answers in full sentences. If students question how you want to see their work, suggest that it should look similar to entries in their graphing calculator. One final note: students will often write $y = b + mx$ at this point in the course - accept it. It is reasonable, given the interpretation of the information in many of the problems.

The remainder of the problems in the unit focus on finding the $N^{th}$ term in a pattern. For *Points and Lines, Telephone Network Problem,* and the *Handshake Problem,* split the students into small groups, assigning one problem to each group so that all tasks are assigned to at least one group in the class. Student presentations provide for interesting discussions and analyses and help them to understand that a model describes an inherent characteristic rather than a context, as similar models are used in all three problems. One aspect to bring out during class discussions is that all the problems, despite differences in their scenarios, are represented by similar models.

The capstone activity brings mathematical history alive as students work on *Gauss’ Solution.* You may want to use this exercise as an opportunity to have students look up information on Gauss and other prominent mathematicians.

**Unit Assessments**

- Number Patterns Test
- Toothpick Test
- Earning Money
- Vacation Flights
Unit 1: Patterns and Linear Functions

Contents

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Patterns are used to visualize data and help us predict outcomes. Patterns also represent an order that allows us to generalize. In the following problem, you will use patterns to help you describe situations algebraically.

Your parents tell you that it is time for you to start paying for some of your own expenses, so you decide to look for a part-time job. Pat-E-Oh Furniture is hiring furniture assemblers. The ad says that the job will be to remove furniture from its shipping containers and assemble it. The job pays $8 an hour for the first six months.

1. How much will you make in a summer week, working 8 hours a day for a five-day week? In a complete sentence, describe how you found this answer.
   
   **Sample Response:**
   
   *I will make $320 that week. I multiplied the number of hours I work per day times the amount I earn per hour.*

2. During the school year you can only work 4 hours a day. How much will you make each day? Write a complete sentence, describing how you found this answer.
   
   **Sample Response:**
   
   *I will make $32 each day. I multiplied the number of hours I work by the amount I earn per hour.*
At this point, there will be differences in how students go about solving this type of problem. Some will use arithmetic, others algebra and still others a mix of approaches. Let students do what is comfortable for them. Share the alternative approaches and emphasize how the final results are the same.

### $8 an Hour Problem

3. One of the reasons you took this job is to earn enough money to buy a bicycle that costs $375. If you save every cent you earn, how many hours must you work to make enough to buy the bike? Write a complete sentence, describing how you found this answer.

**Sample Response:**

*I would have to work 46.875 hours to earn enough to buy the bike, but if I can be paid for only a full hour, then I would have to work 47 hours.*

*I found this answer by dividing the total amount I need to buy the bike by the $8 hour per hour to get the number of hours I need to work.*

4. If you save only half of the money you earn, how many hours must you work to have enough money for the bike?

*It would require working 93.75 hours. I would have to work twice as long if I wanted to save half of my money.*

How many days would it require working 8 hours a day?

*It would take 11.72 days.*

Write complete sentences, describing how you found these answers.

*To find how many hours it would take to buy the bike if only half the earnings are saved, you double the number of hours it would take if you saved all the money.*

*Divide the total number of hours by the hours worked per day.*

5. One way to keep track of how much money you make is to use a table. In the table below, fill in your total earnings based on the number of hours worked for each week.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Weeks Worked</th>
<th>Hours Worked</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>hours</td>
<td>dollars</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Week 1</td>
<td>11.5</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Week 2</td>
<td>20</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Week 3</td>
<td>16</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Week 4</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Week 5</td>
<td>9.5</td>
<td>76</td>
</tr>
</tbody>
</table>
It is assumed that students have a basic understanding of average and are able to compute averages. Nonetheless, you may want to review the definition and computation. Also mention mean as a statistic and note that the average or mean helps to summarize data and to explain the tendency of the data toward a particular value. Measures of central tendency are discussed more formally and in greater detail in Unit 5.

**$8 an Hour Problem**

Using the information from the table, answer the following questions in complete sentences.

6. Which week did you make:
   a. the most money? *The most money was earned in the second week.*
   b. the least money? *The least money was earned in the fifth week.*

7. Explain how you found the total weekly earnings to fill in the table.
   *The total weekly earnings are found by multiplying the number of hours worked by the hourly rate of $8.*

8. How much more did you make the second week than the third week?
   **Sample Response:**
   *I made 32 dollars more the second week. 160 – 128 = 32*

9. How much did you make for all five weeks?
   **Sample Response:**
   *I made a total of 536 dollars. 92 + 160 + 128 + 80 + 76 = 536*

10. What were your average earnings per week?
    *The average earnings per week were $107.20.*

    How did you find this answer?
    *Divide the total earnings in #9 by 5.*
Provide groups with overhead transparencies of this simple grid from the student text to use in their presentations.

### $8 an Hour Problem

11. Use the information from your table to construct a bar graph on the following grid. Clearly label the graph so that anyone looking at it will understand the information displayed.
$8 an Hour Problem

Use your bar graph to answer the following questions. Answer in complete sentences.

12. What information can you see immediately by looking at the bar graph?

   **Sample Response:**
   
   I can easily find the highest and lowest amounts earned in a week.

13. Can you use the graph to find out how much you made if you worked 7 hours?

   **No.**

   Why or why not?

   Seven hours was not given as a time worked during the period for which earnings were calculated.

14. What information does a bar graph illustrate well?

   **Sample Response:**

   The bar graph illustrates highs and lows very well.

   Bar graphs can also show values for specific instances.

15. What are some shortcomings of bar graphs, or things that a bar graph does not display well?

   Bar graphs do not display values between those given in the table.
$8 an Hour Problem

16. Complete this table.

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>hours</td>
<td>dollars</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>11</td>
<td>88</td>
</tr>
<tr>
<td>15</td>
<td>120</td>
</tr>
<tr>
<td>18</td>
<td>144</td>
</tr>
<tr>
<td>20</td>
<td>160</td>
</tr>
</tbody>
</table>

Make the connection between the difficulty in using a bar graph to make predictions (from the previous page) with the relative ease of using a line graph to make predictions.

17. Would a line graph be useful to show the information in this table? Why or why not?

Yes, it would show how our earnings are changing. It would also allow us to find values other than those given.
Make sure that the students are marking every unit value along the horizontal axis and every 10 units along the vertical axis.

The format of the graph and graph setup as shown here is identical to the software. Continue to reinforce this concept throughout the year, so that students develop the habit of labeling axes and establishing bounds and intervals.

Note that graphs in Units 1 and 2 only require plotting the points in the 1st quadrant. However, consider having students place their own axes in the grids so that when they make the transition to 2nd and 4th quadrant graphing, there is no stumbling block to the placement of axes.

### $8 an Hour Problem

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Worked</td>
<td>0</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>Total Earnings</td>
<td>0</td>
<td>250</td>
<td>10</td>
</tr>
</tbody>
</table>

18. Construct a line graph on this grid. Include all of the information from the table on the previous page.
$8 an Hour Problem

Use your graph to answer the following questions. Answer in complete sentences.

19. How much would you make if you worked:
   a. 10 hours?  **Based on the graph at t = 10 hours, 80 dollars would be earned.**
   b. 22 hours?  **I would make 176 dollars since (10,176) is a point on the graph.**
   c. $\frac{3}{2}$ hours?  **For 3.5 hours of work, the earnings would be $28.**

20. What are the advantages of using a line graph?
   **It can show the exact relationship between time and earnings.**

21. Looking back to your table and line graph, determine if there is a number pattern in this problem. Describe the pattern. How do you know the pattern is correct?
   **Yes, there is a pattern that shows the amount earned increases with each hour worked.**

22. Find the total earnings for any number of hours worked, if you use H to represent the number of hours.
   **8H**

23. To determine your exact earnings after working 40 hours per week for three months, would it be best to use the expression you just found or a graph? Explain.
   **It would be best to use the expression because the graph does not show numbers that high (480) along the horizontal (x) axis.**
$8 an Hour Problem

The $8 an Hour Problem and many other problems you will solve in this course involve analyzing the relationship between two variable quantities.

In the $8 an Hour Problem, the variable quantities are hours worked and total earnings. To understand the idea of a variable quantity, consider what you did in completing the table of values for a five-week period. During this time period, the number of hours you worked varied from week to week and so did your total earnings.

The situation also involved another quantity: the hourly wage of $8 an hour. Unlike the variable quantities of hours worked and total earnings, the hourly wage did not vary from week to week, but remained fixed. A quantity that remains fixed in a problem situation is called a constant.

What you have just written to calculate the total earnings based on hours worked is called an algebraic expression. An algebraic expression uses variable quantities represented by letters that take the place of numbers (like H) and it uses constants (like 8). An algebraic expression is used to generalize a pattern.

Two algebraic expressions with an equal (=) sign between them create an algebraic equation. Using an algebraic equation for a situation allows you to answer many questions quickly and accurately. The formula you found is an example of an algebraic equation.

This year, you will spend a great deal of time and energy finding, using, and trying to understand many different algebraic equations.
This is the first instance where students are asked to define variables with units and to write an algebraic expression. This same procedure should be followed when completing any problem situation, both in the text and in the software.

If students are having difficulty deriving the expression, urge them to create a small table of values to assist them in generalizing.

Ask students to define the word profit. Verify their understanding of differences in income or earnings and profit. Be sure to include money spent as well as money earned in defining profit.

### $8 an Hour Problem

A grocery store owner wants to keep track of the profit he makes from the sale of bread each day. He knows that his profit on each loaf of bread is $0.50.

24. Using complete sentences, state the two variable quantities and the single constant quantity that are involved in the problem situation.

   - **The first variable quantity is amount of bread sold.**
   - **The second variable quantity is profit.**
   - **The constant quantity is $0.50, which is the profit on each loaf of bread.**

25. How would you measure each quantity of the problem situation? That is, what are the units of each quantity?

   - **The unit for bread is loaves.**
   - **The unit for profit is dollars.**

26. Write an algebraic expression to calculate the profit based on the number of loaves of bread sold in the store.

   \[0.50L\]
Number Patterns

When you look at certain number patterns, it is fairly obvious which two numbers are next in the list:

Example 1: 1, 3, 5, 7, 9, 11, 13, ...

In the $8 an Hour Problem, you saw another number pattern. Which two numbers are next in that pattern?

Example 2: 8, 16, 24, 32, 40, 48, ...

Other patterns may not be so obvious:

Example 3: 0, 1, 5, 14, 30, 55, 91, 140, ...

By discovering the pattern in the first several terms, it becomes easier to find other terms. In this list of numbers, use the pattern in the first few numbers to find the tenth term.

Example 4: 4, 8, 12, 16, ..., 40, ...

However, sometimes it may be extremely difficult to find the tenth term:

Example 5: 1, 1, 2, 3, 5, 8, 13, ..., 55, ...

In mathematics, a goal of modeling is to find an algebraic expression that enables you to find any term in a pattern.

In the following table, for example, the bottom row shows the number pattern and the top row shows the number of each term. (The 1st term in the pattern is 6, the 2nd term is 12, and so on.) Can you find an expression that provides the value of any term, which is called the Nth term? Fill in the numbers for the 4th and 5th terms. Then fill in the algebraic expression for the Nth term.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>6N</td>
</tr>
</tbody>
</table>
The famous mathematician and scientist Galileo Galilei spent many years trying to find a formula to fit the number pattern produced when an object is dropped from a building. The following table shows the number pattern produced by dropping a ball from a high building. The top row shows the elapsed time in seconds from the moment the ball was dropped, and the bottom row shows the number of feet the ball has fallen.

Fill in the number of feet the ball has fallen after 6 seconds and after 7 seconds. Can you find a formula for the number of feet the ball has fallen after $T$ seconds? The answer is not obvious at all.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>16</td>
<td>64</td>
<td>144</td>
<td>256</td>
<td>400</td>
<td>576</td>
<td>784</td>
<td>$16T^2$</td>
</tr>
</tbody>
</table>

Number patterns such as these are also called sequences. Using sequences, you may be asked to find:
- The next several terms
- A specific term (the 10th term or the 20th term, for example)
- An expression for a general term (the $N$th term, for example).

Patterns in mathematics are not always number patterns. The patterns you will see may involve shapes or other objects.

**Example 1:**

```
 o
 o  o
 o  o  o
```

**Example 2:**

```
```

Pages 1-14 through 1-16 focus on finding the next several terms. Pages 1-17 through 1-44 focus on finding a specific term. Pages 1-45 through 1-56 focus on finding an expression for a general term.

Problems given throughout the rest of the unit will integrate both algebraic and geometric examples. Require that students attempt all problems. Assure them that if they cannot find a solution right away, they will be able to return to the problems later. The key to this group of problems is that students develop a process for approaching novel decontextualized problems.
Continue to stress that students write their responses in full sentences. Writing helps them clarify their thinking and enhances their ability to communicate mathematical terminology and concepts.

Number Patterns

Here are some examples of pattern problems. Find the next two terms in each sequence. Write a sentence describing how you found them.

1. 1, 12, 123, 1234, __12345__, __123456__ ...  
   Write the next whole number as the last digit.

2. 6, 11, 16, 21, __26__, __31__ ...  
   Add 5 on to the previous term to get the next term.

3. 12, 10, 8, __6__, __4__ ...  
   Decrease the previous term by 2 to get to the next term.

4. 1, -3, 9, -27, 81, -243, __729__, __-2187__ ...  
   Multiply the previous term by -3 to get the next term.

5. 2, 3, 5, 8, 12, 17, __23__, __30__ ...  
   Add the next integer, so add 6 to 17 and 7 to 23.

6. 15, 14, 16, 13, 17, 12, __18__, __11__ ...  
   Subtract consecutive odd integers from the odd terms and add consecutive even integers to the even terms.

7. 1, 2, 4, 8, 16, __32__, __64__ ...  
   Multiply each term by two.

8.  
   Increase the length and width by 1 to form the next square.

9.  
   Increase the number of dots in each row and column by 1 to the next rectangular set of dots.
Patterns and Linear Functions

Number Patterns

10. Create one more triangle on the end of the shape.

11. Rotate the flag by 60° in a clockwise direction.

12. Divide the smallest shape into two smaller ones with a vertical or horizontal segment.

Since the groups may have very different strategies for this challenge problem, leave time for presentations.

Number Patterns

Discuss your answers with the rest of your group. Make sure that everyone in your group understands how you found your answers.

Challenge Problem

0, 11, 34, 69, 116, 175, 246, 329, ...

Describe some of the strategies you used to get the next term in this sequence.

- Guessed and checked
- Subtracted the numbers
- Multiplied by different numbers
Finding the 10th Term

The second type of number-pattern problem is to find a specific term in a sequence. Here are two solved examples.

Example 1:
Find the 10th term in this sequence:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>10</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>...</td>
<td>90</td>
<td>...</td>
</tr>
</tbody>
</table>

Each term in this sequence is 9 times its term number, so the 10th term is 90.

Example 2:
Find the 10th term in this sequence:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>10</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>125</td>
<td>...</td>
<td>1000</td>
<td>...</td>
</tr>
</tbody>
</table>

Each term in this sequence is used as a factor three times and multiplied. So the 10th term is $10 \cdot 10 \cdot 10$ or 1000.
## Finding the 10th Term

Here are some problems for you to solve. Find the 10th term in each sequence. Write a complete sentence describing how you found the 10th term.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

| 3 | 5 | 7 | 9 | ... | 21 | ... |

### Add 2 on to the previous term until the 10th term.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

| 2 | 5 | 8 | 11 | ... | 29 | ... |

### Multiply the term number by 3 and subtract 1.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

| 1 | 4 | 9 | 16 | ... | 100 | ... |

### Multiply the term number by itself.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

| 1 | 6 | 11 | 16 | ... | 46 | ... |

### Add 5 on to the terms until the 10th term is reached.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

| 16 | 26 | 36 | 46 | ... | 106 | ... |

### Multiply the term number by 10 and add 6.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

| 5 | 13 | 21 | 29 | ... | 77 | ... |

### Multiply the term number by 8 and subtract 3.

<table>
<thead>
<tr>
<th>Term Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Sequence**

All the even figures are tilted to the left, so the 10th term is too.
### Finding the 10th Term

8. **Term Number** | 1 | 2 | 3 | 4 | ... | 10 | ...
--- | --- | --- | --- | --- | --- | --- | ---
**Sequence** | 2 | 6 | 12 | 20 | ... | **110** | ...

*Multiply the term number by the next integer so that the 10th term is 10*11.*

9. **Term Number** | 1 | 2 | 3 | 4 | ... | 10 | ...
--- | --- | --- | --- | --- | --- | --- | ---
**Sequence** | | | | | | | ...

*Make a triangular array of dots that has a length and width equal to the term number.*

10. **Term Number** | 1 | 2 | 3 | 4 | ... | 10 | ...
--- | --- | --- | --- | --- | --- | --- | ---
**Sequence** | 1 | 0.1 | 0.01 | 0.001 | ... | **0.000000001** | ...

*Keep moving the decimal point to the left one place for each term until the 10th term.*

11. **Term Number** | 1 | 2 | 3 | 4 | ... | 10 | ...
--- | --- | --- | --- | --- | --- | --- | ---
**Sequence** | | | | | | | ...

*Keep turning the flag by 60° until reaching the 10th term.*

---

12. **Term Number** | 1 | 2 | 3 | 4 | ... | 10 | ...
--- | --- | --- | --- | --- | --- | --- | ---
**Sequence** | | | | | | | ...

*Answers will vary, but students should be describing how they deduced the pattern and any process they went through to make sure that they could find any term.*

---

**Finding the 10th Term**

---

**process: summary**

12. Discuss your answers with the rest of your group. Make sure that everyone in your group understands how you found your answers. Summarize your group discussion below.

*Answers will vary, but students should be describing how they deduced the pattern and any process they went through to make sure that they could find any term.*
### Finding the 10th Term

#### Question 13

Examine questions 1, 2, and 4 – 6. What do all of these have in common? Is there an easy way to find the 10th term without finding the first 9 terms?

They all involve increasing by the same amount.

Yes, form an expression in which the common difference between the terms is multiplied by the term number.

#### Extension

14. Create a sequence of numbers or shapes. Then switch papers with a partner and find the 10th term of your partner’s sequence.

15. Can you find the 10th term of the following sequence?

| Term Number | 1 | 2 | 3 | 4 | 5 | 6 | ... | 10 | ... |
|-------------|---|---|---|---|---|---|      |    |     |
| Sequence    | 1 | 1 | 1 | 3 | 5 | 9 | ...   | 96 | ... |

Have students refer to the 5th example on page 1-13 as a hint to complete the extension problem.
This problem should be used to bring out the tie between geometric patterns, spatial relations, arithmetic generalizations and the use of ratios to show how multiple areas of mathematics are tied together in a single situation.

Soaps and Bathbeads

Appletree & Elizabeth, a company that manufactures bath products, has created a line of new gift packages just in time for Valentine’s Day. The packages contain scented soaps and bath beads arranged so that bath beads surround each of the scented soaps. There are small, medium and large gift boxes. Each of the packages is pictured below.

Small: (2 by 2)  
Medium: (2 by 3)  
Large: (3 by 4)

There are multiple possible solutions for question 1 (3 x 3, 2 x 4, 2 x 5 and even 2 x 6). Stress to students that there are multiple correct solutions, not just one. This is hard for students to grasp, but they need to see that in open-ended problems, this is more the norm than not.

Have students use trial and error if it helps them find the solution. Watch for them to use 3 x 2 and 4 x 3. These are the current medium and large sizes turned 90°.

1. The new line is so successful that the company decides to introduce two new sizes for the spring season (along with a few new scents).
   a. One size will be bigger than the current medium-sized package, but smaller than the current large-sized package. Create two possible designs for the new middle-sized box. Show your designs below and describe how you created each.

   Add 1 column to the medium one.   Subtract 1 column from the large one.

(3 by 3)   (2 by 4)
### Soaps and Bathbeads

<table>
<thead>
<tr>
<th>Process: model</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. The other new size will be called “extra extra large.” Create a new design for this size by adding a row and a column of beads to the current large size. Show your design below.</td>
</tr>
</tbody>
</table>

2. Describe, in words, any patterns you see in the design of the boxes. 

*There is one less column and one less row of soap than there are beads.*

3. How can you predict the number of bath beads in any sized box? Create an algebraic equation to calculate the number of bath beads if you know the dimensions of the box.

*Multiply the number in a row by the number in a column.*

\[ B = R \times C \]

4. How can you predict the number of soaps in any sized box? Create an algebraic equation to calculate the number of soaps if you know the dimensions of the box.

*We can subtract 1 from the bead column and 1 from the bead row and multiply those.*

\[ S = (R - 1)(C - 1) \]

5. How can you predict the total number of items in the gift box? Create an algebraic equation to calculate the total number of items if you know the dimensions of the box.

*We can add the total number of beads and the total number of soaps.*

\[ T = R \times C + (R - 1)(C - 1) \]
The number of soaps received by each of the four daughters is not a whole number.

Call attention to the use of ratios and/or proportional reasoning when solving this problem.

---

### Soaps and Bathbeads

6. A mother of four daughters buys them one large-sized box to split evenly, while a father of two daughters buys them one medium-sized box to split evenly. Do the girls in the family of four daughters get the same number of soaps and bath beads as the girls in the family of two daughters? How many soaps and bath beads does each girl receive? Answer in complete sentences.

<table>
<thead>
<tr>
<th></th>
<th>Four Daughters</th>
<th>Two Daughters</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>$4 \times 3 = 12$</td>
<td>$2 \times 3 = 6$</td>
</tr>
<tr>
<td>S</td>
<td>$3 \times 2 = 6$</td>
<td>$2 \times 1 = 2$</td>
</tr>
<tr>
<td>$\frac{12}{4}$</td>
<td>$3$ beads per girl</td>
<td>$\frac{6}{2}$ = $3$ beads per girl</td>
</tr>
<tr>
<td>$\frac{6}{4}$</td>
<td>$1.5$ soaps per girl</td>
<td>$\frac{2}{1}$ = $1$ soap per girl</td>
</tr>
</tbody>
</table>

Each girl receives 3 beads and 1.5 soaps.

Each girl in both families receives 3 beads, but in the family with four daughters each receives $\frac{1}{2}$ more soaps each.

---

7. What methods did you use to complete the problems in the Soaps and Bathbeads problem?

**Sample Responses:**

I considered the rows separately from the columns.

To find the total items, I used the pattern found for the bath beads and simply added the soaps.

I separated the two families, found the total number of beads and soaps in each box, and then divided that number by the number of girls in each family.
The Consultant Problem

Your older sister works as an architectural consultant. She recently opened her own firm. As is typical with any consulting firm, jobs are billed at an hourly rate to the client. During the last two months, your sister worked on four projects. Her billable rate was $112.75 per hour.

Some students may point out that just because the consultant bills $112.75, it does not mean that she makes $112.75 per hour. Respond to this observation with a brief discussion of income vs. profit. Many problems in the text and software will revisit this concept.

1. How much did she make on the first project for USY if she worked 31 hours? Write a sentence describing how you found this answer.

   She will make $3495.25.

   This was calculated by multiplying $112.75 by 31.

2. How much did she make on the second project for TTG if she worked 23 hours? Write a sentence describing how you found this answer.

   She will make $2593.25.

   The answer was calculated by multiplying 23 by the hourly billing rate, $112.75.
The Consultant Problem

3. How much did she make on the third project for the Barbara Hanna Company if she worked 39 hours? Write a sentence describing how you found this answer.

She will make $4397.25.
The answer was found by multiplying the hourly rate by the number of hours worked.

4. How much did she make on the last project for ALCOM if she worked 19 hours? Write a sentence describing how you found this answer.

She will make $2142.25.
The answer was found by multiplying $112.75 and 19.

5. Your sister needs $33,000 in cash to pay for a new car. Has she made enough money from the four projects? Write a sentence describing how you found this answer.

She has made $12,628. She has not made enough money for the car.
The answer was found by adding the earnings from all the consulting projects.

The Consultant Problem

6. One way to keep track of how much money your sister has made on each project is to use a table. Fill in this table showing the hours worked, and her earnings for each project.

<table>
<thead>
<tr>
<th>Project</th>
<th>Hours Worked</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>USY</td>
<td>31</td>
<td>3495.25</td>
</tr>
<tr>
<td>TTG</td>
<td>23</td>
<td>2593.25</td>
</tr>
<tr>
<td>Hanna Co.</td>
<td>39</td>
<td>4397.25</td>
</tr>
<tr>
<td>ALCOM</td>
<td>19</td>
<td>2142.25</td>
</tr>
</tbody>
</table>

7. Explain how you found the total earnings for each project to fill in the third column of the table.

The hours worked were multiplied by 112.75.
### The Consultant Problem

Answer the following questions in complete sentences.

8. On which project did she make:
   - a. the most money?  *She made the most with Barbara Hanna Company.*
   - b. the least money?  *She made the least amount with ALCOM.*

9. How much more did she make on the third project than on the first project?  *She made $902 more on the third project.*

10. What are her total earnings for all four projects?  *Her total earnings are $12,628.*

11. What were her average earnings per project?  How did you find this number?
    *Her average is $3157 for each project.*
    *Divide $12,628 by 4 to find the average earned per project.*
Provide groups with overhead transparencies of the blank grid to use in their presentations.

**The Consultant Problem**

12. Use the information from your table to construct a bar graph on this grid. Clearly label the graph so that anyone looking at it will understand the information it displays.
Patterns and Linear Functions

The Consultant Problem

Use your bar graph to answer the following questions. Answer in complete sentences.

13. What information can be seen immediately by looking at the bar graph?
   
   You can tell when the consultant made the most/least money.

14. Can you use the graph to find out how much your sister made if she worked 7 hours?
    Why or why not?
    
    No, that information is not given in the graph.

15. What information does a bar graph illustrate well?
    
    It illustrates highest and lowest values as well as differences between values.

16. What are some shortcomings of bar graphs?
    
    Bar graphs do not give values for points between the information given.
The Consultant Problem

17. Complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Hours Worked</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>hours</td>
<td>dollars</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>112.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>225.50</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>563.75</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>1240.25</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1691.25</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>2029.50</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2255</td>
</tr>
</tbody>
</table>

18. Would a line graph be useful to show the information in this table? Why or why not?

Yes, it will show the values given as well as the values that are not provided in the table.
The Consultant Problem

19. Construct a line graph on this grid. Include all the information from the table. Label the graph clearly.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Worked</td>
<td>0</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>Earnings</td>
<td>0</td>
<td>2500</td>
<td>100</td>
</tr>
</tbody>
</table>

---

Make sure that the students are marking each unit of 1 along the horizontal axis and every 100 units along the vertical axis.

Have students use round numbers for their bounds. They will need to include the smallest and largest numbers with the bounds, so they should not choose just the smallest and largest numbers in the table.

The format of the graph and graph setup as shown here is identical to the software. Continue to reinforce the students’ labeling of axes and establishing bounds and intervals.

Note that graphs in Units 1 and 2 only require plotting points in the 1st quadrant. However, consider having students place their own axes in the grids so that when they make the transition to 2nd and 4th quadrant graphing, there is no stumbling block to the placement of axes.
The Consultant Problem

Use your graph to answer the following questions. Answer in complete sentences.

20. How much would your sister make if she worked:
   a. 10 hours?  She would make $1127.50.
   b. 22 hours?  She would make $2480.50.
   c. $\frac{3}{2}$ hours?  She would make $394.53.
   d. 5 hours 15 minutes?  She would make $591.94.
   e. 6 hours 20 minutes?  She would make $714.08.

21. What advantages are there in using a line graph?
   
   You can find values that are not given in the table.

22. Is there a number pattern in your table?
   
   Yes, each value is a multiple of 112.75.

23. What is the algebraic expression for finding your sister’s total earnings if she worked $H$ hours?
   
   $112.75H$
The ability to generate an algebraic expression or equation is the next step in being able to determine the $N^{th}$ term in a sequence. Use examples from the software and have students identify the independent and dependent variables.

### The Consultant Problem

At some point during the process of analyzing the last few problem situations, you wrote algebraic equations to express the relationship between the variable quantities. In these algebraic equations, one quantity depends on the other.

The following algebraic equation shows how the two variable quantities in the $8$ an Hour Problem are related:

$$E = 8H$$

In this algebraic equation, the variable $E$ represents total earnings and the variable $H$ represents the number of hours worked.

In the $8$ an Hour Problem, did the hours worked determine the person’s earnings, or did the earnings determine the hours worked? Which quantity depended upon the other?

*Earnings depended on the Time Worked.*

Because the number of hours worked determines the amount earned, the number of hours worked is the independent variable and the amount earned is the dependent variable. The amount earned depends on the number of hours worked.
To solidify the ability to recognize the variable quantities with units and independent and dependent variables, students should identify these values in every problem they encounter throughout the curriculum even if this information is not specifically requested.

If students struggle with these questions, refer them to the table on page 1-29. Point out that the variable quantities are the labels in each column in the table.

Ask the students to define the word constant.

### The Consultant Problem

Use The Consultant Problem to answer the following questions.

24. What are the variable quantities in the problem situation? Include the units used to measure these quantities.

*The variable quantities are Time Worked in hours and Earnings in dollars.*

25. Which of these two variable quantities depends on the other? Explain.

*Earnings depends on Time Worked.*

26. Which variable quantity is the independent variable (input) and which is the dependent variable (output)?

*The independent variable is Time Worked. The dependent variable is Earnings.*

27. What is the constant quantity in the problem situation? Include the unit used to measure this fixed quantity.

*The constant quantity is 112.75 measured in dollars per hour.*

### The Consultant Problem

Use the grocery store problem from within the $8 an Hour Problem to answer the following questions. A grocery store owner wants to keep track of the profit he makes from the sale of bread each day. He knows his profit on each loaf is $0.50.

28. Of the two variable quantities in the problem situation, which depends on the other? Explain.

*The profit depends on the number of loaves sold.*

29. Which variable quantity is the independent variable (input), and which is the dependent variable (output)?

*The number of loaves sold is independent. The profit is dependent.*
U.S. Shirts varies from the previous problem. It includes a y-intercept in addition to the rate of change. The concept of intercepts will be developed through Unit 4, where it is formally defined.

As students move from problems of the form \( y = mx \) to \( y = mx + b \), watch that they do not make the common error of adding the 8 and 15 before multiplying.

---

### Input

**U.S. Shirts**

Pat-E-Oh Furniture, Inc. went out of business. You went to work for your sister at her architectural firm, but that didn’t work out. Now, you have taken a new job at U.S. Shirts, a custom T-shirt shop.

One of your responsibilities is to calculate the price of the orders. For each order, U.S. Shirts charges $8 per shirt, plus a one-time charge of $15 to set up the design.

---

### Model

**U.S. Shirts**

1. In your own words, clearly identify the problem situation.

   *We are purchasing shirts from U.S. Shirts for $8 each, but we must first pay them $15 as a setup fee.*

2. How does the U.S. Shirts problem differ from the $8 an Hour Problem and The Consultant Problem?

   *There is a setup fee in this problem.*
When multiple steps are involved in solving equations using arithmetic rather than algebraic strategies, some difficulties may arise for students. Be sure they understand the intended goal, namely to find the unknown value. Ask them to talk through how they can ‘unwind’ the equation.

Allow students to choose the values included in the table. Most students will use values from questions 3 and 4. However, allow them to choose their own values for the number of shirts ordered.

### U.S. Shirts

3. How much will you charge for an order of:
   - a. 3 shirts? **The charge is $39.** \((3 \times 8 + 15)\)
   - b. 10 shirts? **The charge is $95.**
   - c. 100 shirts? **The charge is $815.**

   In a complete sentence, describe how you found your answers.
   **Multiply the number of shirts by 8 and add 15.**

4. How many shirts can a customer purchase with:
   - a. $50? **They can buy 4 shirts.** \((50 - 15)/8\)
   - b. $100? **They can buy 10 shirts.**
   - c. $1000? **They can buy 123 shirts.**
   - d. $52.50? **They can buy 4 shirts.**

   In a complete sentence, describe how you found your answers.
   **Subtract the given amount by 15 and then divide the result by 8.**

   **If there is a decimal amount, drop it, since a portion of a shirt cannot be purchased.**

5. Make a table of values based on the problem situation.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Number of Shirts Ordered</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>shirts</td>
<td>dollars</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>815</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>215</td>
</tr>
</tbody>
</table>

**Answers will vary depending on the number of shirts the student chooses.**
When describing the constant rate of change, be sure to use the full unit of dollars per shirt. This enables you to correctly interpret the rate of change for linear functions as the change in the dependent variable for every one unit increase of the independent variable.

---

### U.S. Shirts

Reflect on the work you have just completed for this problem situation to answer the following questions. Answer in complete sentences.

6. What are the variable quantities in the problem situation? Include the units used to measure these quantities.

   **The first quantity is number of shirts measured in shirts.**
   **The second quantity is total cost measured in dollars.**

7. What are the two constant quantities in the problem situation? Include the units used to measure these fixed quantities.

   **The two constants are 8 measured in dollars per shirt and 15 measured in dollars.**

8. Of the two variable quantities, which quantity depends on the other quantity?

   **The total cost depends on the number of shirts ordered.**

9. What is the independent variable quantity, and what is the dependent variable quantity in the problem situation?

   **The independent quantity is the number of shirts.**
   **The dependent quantity is total cost.**
Since the students selected which values to include in their table, not all students will use the same bounds or intervals. Reinforce the idea that there are multiple correct ways to set the bounds. It is important that all points from the table are visible in the graph. This concept is reinforced in the software.

Note that in previous problems, students were asked to write an algebraic expression, but now they are asked to write an algebraic equation. This switch should help students to think about the relationship between one variable quantity and the other.

Start to refer to the x-axis as the independent variable and the y-axis as the dependent variable.

Tie the representations together by stressing that data points in the table are coordinate points on the graph and lie on the line represented by the equation. Also stress that the points on the line are solutions to the algebraic equations.

### U.S. Shirts

10. Plot the points for the data from the U.S. Shirts problem and draw a line graph on the grid below.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Shirts</td>
<td>0</td>
<td>125</td>
<td>5</td>
</tr>
<tr>
<td>Total Cost</td>
<td>0</td>
<td>1250</td>
<td>50</td>
</tr>
</tbody>
</table>

11. Write an algebraic equation for the problem situation.

\[ c = 8s + 15 \]
Discuss the advantages of all four representations and when one might be better than another and why. Discuss general disadvantages of the representations. Mention that disadvantages depend on what you are trying to find.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>U.S. Shirts</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. You have just represented this problem situation in four different ways, using sentences, a table, a graph, and an algebraic equation. Discuss and record the advantages and disadvantages of each of these representations with your group members. Be prepared to contribute to a general class discussion of these problem representations.</td>
<td></td>
</tr>
<tr>
<td>Samples Responses might include:</td>
<td></td>
</tr>
<tr>
<td>The advantage of sentences is to understand how the result is used.</td>
<td></td>
</tr>
<tr>
<td>The disadvantage is that it might be difficult to visualize the problem.</td>
<td></td>
</tr>
<tr>
<td>The advantage of a table is that it helps you graph the situation and gives specific instances about the situation.</td>
<td></td>
</tr>
<tr>
<td>The disadvantage of a table is that it does not show values between those specific instances.</td>
<td></td>
</tr>
<tr>
<td>The advantage of a graph is that it allows you to find values between those on the table and shows the tendency in the data.</td>
<td></td>
</tr>
<tr>
<td>The disadvantage of a graph is that it is more difficult to find values.</td>
<td></td>
</tr>
<tr>
<td>The advantage of an equation is that it allows you to find any value and to generalize the solution.</td>
<td></td>
</tr>
<tr>
<td>The disadvantage of an equation is that you cannot “see” the situation as you can with a visual display such as a graph.</td>
<td></td>
</tr>
</tbody>
</table>
Identifying the problem situation should include the definition of variables with units and identification of constants with units. Reinforce this idea throughout the year.

Have students think about the similarities between this problem and U.S. Shirts. Students should identify that each has an initial value and constant cost per shirt.

**Hot Shirts**

An advertisement for Hot Shirts, a competitor of U.S. Shirts, states that they will make custom T-shirts for $5.50 a shirt, with a one-time fee of $49.95 to set up the design. When your boss sees this ad, he asks you to figure out how this might affect his business.

**Hot Shirts**

1. In your own words, clearly identify the problem situation.

   **Sample Response:**

   *We must find out the cost of shirts in dollars from the competitors based on the number of shirts ordered, knowing that they charge $5.50 per shirt with a one-time start-up fee of $49.95.*
At this point in the curriculum, students must complete their own tables. Ask which two variable quantities they will use in setting up their tables. Be sure that students understand how units differ from labels for the variable quantity.

Hot Shirts

2. How much will Hot Shirts charge for an order of:
   a. 3 shirts?  The cost will be $66.45.  (3*5.50) + 49.95
   b. 10 shirts? The cost will be $104.95.
   c. 100 shirts? The cost will be $599.95.
   In a complete sentence, describe how you found your answers.
   Multiply the number of shirts by $5.50 and then add $49.95.

3. How many shirts can a customer purchase from your competitor with:
   a. $50?  They can get 0 shirts.  (50 – 49.95)/50
   b. $100? They can get 9 shirts.
   c. $1000? They can get 172 shirts.
   d. $92.50? They can get 7 shirts.
   In a complete sentence, describe how you found your answers.
   Subtract $49.95 from the given amount and divide the result by 5.50. If there is a decimal amount, drop it because a portion of a shirt cannot be purchased.

4. Make a table of values based on the problem situation.

<table>
<thead>
<tr>
<th>Number of Shirts Ordered</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>shirts</td>
<td>dollars</td>
</tr>
<tr>
<td>3</td>
<td>66.45</td>
</tr>
<tr>
<td>10</td>
<td>104.95</td>
</tr>
<tr>
<td>100</td>
<td>599.95</td>
</tr>
<tr>
<td>9</td>
<td>99.45</td>
</tr>
<tr>
<td>7</td>
<td>88.45</td>
</tr>
</tbody>
</table>

Answers will vary depending on the number of shirts the student chooses.
To make it easier to graph the system of equations on page 1-43, you should urge students to use the same bounds as those in U.S. Shirts.

Continue to reinforce the x and y-axes as the independent and dependent variables.

Problems in Unit 1 have been in context. In Unit 4, students begin to work more out of context. You want them to become comfortable with more formal abstract representations, but you want it to be a slow process so that meaning is applied and deeper understanding gained.

**Hot Shirts**

5. Plot the points for the data from the Hot Shirts problem and draw a line graph on the grid below.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Shirts</td>
<td>0</td>
<td>125</td>
<td>5</td>
</tr>
<tr>
<td>Total Cost</td>
<td>0</td>
<td>1250</td>
<td>50</td>
</tr>
</tbody>
</table>

6. Write an algebraic equation for the problem situation.

\[ c = 5.50s + 49.95 \]
Comparing U.S. Shirts and Hot Shirts

Use the information from U.S. Shirts and Hot Shirts to compare the cost of shirts when ordering from these two companies.

1. For an order of fewer than 5 shirts, would you (U.S. Shirts) or your competitor (Hot Shirts) give customers the best price? What would each charge for exactly five shirts?

   **For less than 5 shirts, U.S. Shirts will be cheaper.**
   
   **U.S. Shirts would charge $55 for 5 shirts.**
   
   **Hot Shirts would charge $77.45 for 5 shirts.**

   In a complete sentence, describe how you found your answer.
   
   **Multiply the number of shirts by the cost of each shirt, and add the setup fee.**

2. For a larger order of shirts, say 18, which company's price would be the least? By how much?

   **For an order of 18 shirts, Hot Shirts would be cheaper.**
   
   **U.S. Shirts would charge $159 for 18 shirts.**
   
   **Hot Shirts would charge $148.95 for 18 shirts.**

   **Hot Shirts are cheaper by $10.05.**

   In a complete sentence, describe how you found your answer.
   
   **Multiply the number of shirts by 18 and add the setup fees. Then subtract the costs to find the difference between them.**

3. For a much larger order of shirts, say 80, which company would be less expensive? By how much?

   **For an order of 80 shirts, Hot Shirts would still be cheaper. They would charge $165.05 less than U.S. Shirts.**

   In a complete sentence, describe how you found your answer.
   
   **Multiply each cost by 80 and add the setup fees. Subtract the final costs to find the difference.**
Students need to set their bounds to be able to see the intersection point and portions of the graph to either side of the intersection point.

Make sure students use the same values as when they graphed the lines separately. If students did not use the same values, discuss the problems this might cause in generating an appropriate graph.

### Comparing U.S. Shirts and Hot Shirts

4. Graph each of the lines from the U.S. Shirts and Hot Shirts problems on the following grid.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Shirts</td>
<td>0</td>
<td>125</td>
<td>5</td>
</tr>
<tr>
<td>Total Cost</td>
<td>0</td>
<td>1250</td>
<td>50</td>
</tr>
</tbody>
</table>

![Graph of U.S. Shirts and Hot Shirts](image)
Comparing U.S. Shirts and Hot Shirts

5. When are the costs the same?
   *The costs are the same at 13.98 shirts.*

6. When is U.S. Shirts more expensive?
   *U.S. Shirts will be more expensive if you order 14 shirts or more.*

7. When is Hot Shirts more expensive?
   *Hot Shirts will be more expensive if you order less than 14 shirts.*

8. Write a one-paragraph report for your boss comparing the price structure of the two companies. Try to give him an answer to his original question: “Will Hot Shirts affect our business at U.S. Shirts?”

   **Sample Response:**

   *Hot Shirts will affect our business for those customers who will order a lot of shirts. Our main business will come from customers who will order a small number of shirts. We are going to lose big order customers to Hot Shirts if we do not lower our fee per shirt.*
Now that the students have put together concepts from $8 an Hour, Number Pattern, Consultant, U.S. Shirts, and Hot Shirts problems, they will apply them to patterns where they are looking for an unknown term. This activity is intended to make students more comfortable with out-of-context modeling and generalizations before proceeding with linear equations.

Encourage students to find a pattern between the term number and the numbers in the sequence. The goal is determining the $N^{th}$ term, with a method that allows them to find any term in the sequence.

The last type of number-pattern problem is to find an expression for any term in a sequence. Here is a solved example.

Example:
Find an expression for the $N^{th}$ term in this sequence:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>N</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>...</td>
<td>9N</td>
<td>...</td>
</tr>
</tbody>
</table>

Each term in this sequence is 9 times its term number, so the $N^{th}$ term is 9 times $N$.

Notice that you can use this expression to find any specific term in this sequence. For instance, the 25th term is 9(25), or 225. The 101st term is 9(101), or 909.

For some sequences, it is more difficult to find an expression for the $N^{th}$ term. For example:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>N</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>...</td>
<td>$N^3$</td>
<td>$(N^3+N^2)$</td>
</tr>
</tbody>
</table>

Each term in this sequence is the term number used as a factor 3 times, so the $N^{th}$ term is $N^3$.

In much of this course, you will be working with number patterns for which you will find expressions to represent the $N^{th}$ term.
Students will see a pattern between the successive sequence numbers in questions 1 through 6. Students should make the connection between constant change and multiplying that constant by \( N \) in order to create the algebraic expression for the \( N \)th term.

### Finding the \( N \)th Term

Here are some problems for you to solve. Find the \( N \)th term in each sequence. Write a complete sentence describing how you found the \( N \)th term.

1. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 3 | 5 | 7 | 9 | \( \ldots \) | 2\( N \) + 1 | \( \ldots \)  

   **Multiply the term number by 2 and add 1 to find the \( N \)th term.**

2. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 2 | 5 | 8 | 11 | \( \ldots \) | 3\( N \) – 1 | \( \ldots \)  

   **Multiply the term number by 3 and subtract 1 to find the \( N \)th term.**

3. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 2 | 6 | 10 | 14 | \( \ldots \) | 4\( N \) – 2 | \( \ldots \)  

   **To find the \( N \)th term, multiply the term number by 4 and subtract 2.**

4. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 1 | 6 | 11 | 16 | \( \ldots \) | 5\( N \) – 4 | \( \ldots \)  

   **Multiply the term number by 5 and subtract 4 to find the \( N \)th term.**

5. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 16 | 26 | 36 | 46 | \( \ldots \) | 10\( N \) + 6 | \( \ldots \)  

   **Multiply the term number by 10 and add 6 to find the \( N \)th term.**

6. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 5 | 13 | 21 | 29 | \( \ldots \) | 8\( N \) – 3 | \( \ldots \)  

   **To find the \( N \)th term, multiply the term number by 8 and subtract 3.**

7. **Term Number** | 1 | 2 | 3 | 4 | \( \ldots \) | \( N \) | \( \ldots \)  
   **Sequence** | 1 | 4 | 9 | 16 | \( \ldots \) | \( N \)² | \( \ldots \)  

   **To find the \( N \)th term, square the term number.**
Finding the $N^{th}$ Term

8. Term Number | 1 | 2 | 3 | 4 | ... | $N$ | ...  
Sequence   | 2 | 8 | 14 | 20 | ... | $6N - 4$ | ...  

*Multiply the term number by 6 and subtract 4.*

9. Term Number | 1 | 2 | 3 | 4 | ... | $N$ | ...  
Sequence   | 1 | 0.1 | 0.01 | 0.001 | ... | $\frac{1}{10^{N-1}}$ | ...  

*To find the $N^{th}$ term, divide 1 by 10 raised to the term number minus 1.*

10. Term Number | 1 | 2 | 3 | 4 | ... | $N$ | ...  
Sequence | 2 | 5 | 10 | 17 | ... | $N^2 + 1$ | ...  

*To find the $N^{th}$ term, add one to the square of the term number.*

Finding the $N^{th}$ Term

11. Discuss your answers with the rest of your group. Make sure that everyone in your group understands how you found your answers. Summarize the group’s observations here.

**Sample Response:**

In some patterns, we saw that the difference between successive terms was the number we should multiply by $N$. Then, we added or subtracted a certain number every time to get the sequence.

Finding the $N^{th}$ Term

12. Make up a sequence similar to #1 – 6. In complete sentences, describe what these patterns all have in common. How does your pattern share this commonality?

| Term Number | 1 | 2 | 3 | 4 | ... | $N$ | ...  
Sequence   | 3 | 8 | 13 | 18 | ... | $5N - 2$ | ...  

All of the patterns increase by the same amount each time. This sequence of numbers increases by 5 each time.
The next three activities all deal with the same underlying pattern. Assign one of these activities to each group. Have each group present its results and then discuss the similarities and differences in the problems.

**Points and Lines Problem**

How many lines are required to connect a set of points so that every point is connected to every other point by a separate line?

*Students will be drawing and counting. They are not expected to know the answer. The question is designed to generate the sequence, which can be generalized.*

**preview**

1. These diagrams illustrate the solution for 3 points and 4 points. Count the lines in these diagrams.

   Total Lines = 3  
   Total Lines = 6

2. Draw diagrams for 5 points, 6 points, and 7 points.

   Total Lines = 10  
   Total Lines = 15  
   Total Lines = 21
Points and Lines Problem

3. Try to complete this table.

<table>
<thead>
<tr>
<th>Number of Points</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Lines</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>45</td>
<td>190</td>
<td>1225</td>
<td>4950</td>
<td>(\frac{N(N-1)}{2})</td>
</tr>
</tbody>
</table>

Answer the following questions using complete sentences.

4. How many lines does it take to connect:
   a. 7 points?
      It takes 21 lines to connect 7 points.
   b. 20 points
      It takes 190 lines to connect 20 points.

5. How many points are connected if there are 190 lines drawn?
   Twenty points are connected if there are 190 lines drawn.

6. Is it still difficult to find the answer for large numbers and for any number of points, N? Why or why not?
   Sample Response:
   It is difficult. You have to connect each point with a line, and you may lose track of how many lines you draw or miss several lines. It is also hard to draw a 100-point figure. If you know the expression, then it is easy to find the number of lines given any number of points.
Telephone Network Problem

You have been hired by the Central Intelligence Agency (CIA) to design a Safe-Telephone Network for its headquarters in Langley, Virginia.

The network must be set up so that each person is connected to every other person by a direct line. This way, any two people can talk to each other without someone else being able to gain access to their conversation.

Your task is to determine how many different lines will be needed for 10 employees.

This problem is similar to the previous problem and to the next problem. If you are having all students work each problem, they will gain familiarity as they progress. If you are using groups for these problems as suggested, then the parallel construction of the problems will help when students do presentations.

<table>
<thead>
<tr>
<th>Number of People</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Lines</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>28</td>
<td>45</td>
<td>190</td>
<td>1225</td>
<td>4950</td>
<td>N(N-1)</td>
<td></td>
</tr>
</tbody>
</table>
For question 3, remind students that, since we are working in terms of telephone lines and people, there is no in-between value. If there are not enough lines to connect that next person, then they must stop at 10.

### Telephone Network Problem

<table>
<thead>
<tr>
<th>process: evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer the following questions using complete sentences.</td>
</tr>
<tr>
<td>2. How many direct telephone lines does it take to connect:</td>
</tr>
<tr>
<td>a. 5 people? <em>It would take 10 lines.</em></td>
</tr>
<tr>
<td>b. 10 people? <em>It would take 45 lines.</em></td>
</tr>
<tr>
<td>3. How many people are connected if there are 50 direct telephone lines?</td>
</tr>
<tr>
<td><em>There would be 10 people connected with 50 lines.</em></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Is it difficult to find the number of lines for a large number of people and for any number of people, ( N )? Why or why not?</td>
</tr>
<tr>
<td><em>It is difficult to find the number of lines for a large number of people because it gets hard to draw. If we knew the expression, then it would be easy to plug in the numbers.</em></td>
</tr>
</tbody>
</table>
Handshake Problem

How many handshakes does it take for every person in a group to shake the hand of every other person in the group?

Students may be able to use a calculator to find the answer for 20, 50 and 100 people.

1. One approach that is often useful for solving problems is to find the answer for small numbers and work up to larger numbers. Try to complete the following table.

<table>
<thead>
<tr>
<th>Number of People</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Handshakes</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>45</td>
<td>190</td>
<td>1225</td>
<td>4950</td>
<td>( \frac{n(n-1)}{2} )</td>
</tr>
</tbody>
</table>

2. How many handshakes does it take for a group of:
   a. 100 people? It takes 4950 handshakes.
   b. 1000 people? It takes 499,500 handshakes.

3. How many people are in the group if there were 1275 handshakes?
   There would be 51 people in the group.
Patterns and Linear Functions

Handshake Problem

4. Is it difficult to find the number of handshakes for any number of people, \( N \)? Why or why not?

*It was difficult because the pattern was not obvious. If we had the expression, we could use it to find the number of handshakes given any number of people in a group.*

Handshake Problem

Let’s re-examine the handshake problem and try to find the formula.

5. If there are 10 people, how many handshakes were done with:

   a. the first person? **9**
   b. the second person? **8** (Don’t include the handshakes you’ve already counted.)
   c. the third person? **7**
   d. the fourth person? **6**
   e. the fifth person? **5**
   f. the sixth person? **4**
   g. the seventh person? **3**
   h. the eighth person? **2**
   i. the ninth person? **1**
   j. the tenth person? **0**

6. Write this information as a mathematical equation.

   **Handshakes for 10 = 9 +8+7+6+5+4+3+2+1+0 or**

   \[(N - 1) + (N - 2) + (N - 3) + (N - 4) + (N - 5) + ... (N - N) = \text{Handshakes for } N \text{ people.}\]

Now the problem has changed from finding an expression for \( N \) people shaking hands to a mathematical problem of finding the sum of consecutive numbers.
Gauss’ Solution

Carl Friedrich Gauss was perhaps the world’s greatest mathematician. When Carl was in third grade, his teacher was annoyed with him because he finished his lessons too quickly. His teacher gave him the following problem, thinking it would take Carl a long time to solve: “Add the numbers from 1 to 100.” As Carl walked back to his desk, he immediately found the answer. How?

### Gauss’ Solution

#### analysis

Let’s see how Gauss was able to do this.

1. Start by adding the numbers from 1 to 9. Let’s write the sum twice: once with the numbers running from highest to lowest and again from lowest to highest. You should see some patterns emerging.

   \[
   \begin{align*}
   9 & + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 \\
   1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
   \end{align*}
   \]

   a. What do you notice?

   **Each column of numbers will add up to 10**

   b. What is the relationship between the total sum you calculated above and the sum of the numbers 1 through 9?

   **The total sum is twice that of the sum of the numbers 1 through 9.**

   c. What should you do with the answer to the problem above to find the sum of the numbers 1 through 9?

   **Take the total sum and divide it by 2.**
For question 2, allow students to write out the 20-number sequence and evaluate the solution by hand. Then see if they can use the pattern for 50 and 100 integers.

### Gauss’ Solution

### process: evaluate

2. What is the sum of the numbers 1 through 20? Show how you can find this sum using Gauss’ method.

\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 \]
\[ 20 + 19 + 18 + 17 + 16 + 15 + 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 \]
\[ \frac{20(21)}{2} = \frac{420}{2} = 210 \]

3. What is the sum of the first 50 integers? Show how you can find this sum using Gauss’ method.

\[ 50(51)/2 = 2550/2 = 1275 \]

4. What was Gauss’ solution for the sum of the first 100 integers?

\[ 100(101)/2 = 10100/2 = 5050 \]

### analysis

Help students connect the sums here to the results of the last three problems. Gauss’ Solution for the sum from 1 to \( n \) is the same as the result for \( n + 1 \) points in Points and Lines, \( n + 1 \) people in Handshake, or \( n + 1 \) lines in Telephone Network.

5. Is this number the same as the answer for drawing lines for 100 points, or the number of handshakes for 100 people, or the number of telephone lines for 100 people? Why or why not?

*No, the answer is not the same. However, the sum from 1 to 100 is the number of handshakes for 101 people, while the number of handshakes for 100 people is the sum of the numbers from 1 to 99. This is because a person cannot shake his or her own hand.*

6. Based on what you learned here, represent the equation, for \( N \) people doing handshakes, or connecting \( N \) points with lines, or \( N \) people connected with telephone lines.

\[ \frac{N(N-1)}{2} \]

*Note that for Gauss, the solution is* \[ \frac{N(N+1)}{2} \]
Finding patterns and generalizing the results illustrates an important problem-solving strategy that mathematicians use. Even though the problem situations we examined seemed totally different, we found the same mathematical problem underlying all of them. In finding the patterns, we first reduced the problem to a similar problem with smaller numbers. Then we looked for patterns to help us predict the answer for larger numbers. We next tried to find a mathematical equation to model the problem. Finally, by looking at a mathematical equation, we eventually found the result for any number, $N$, of people, telephone lines, or points. The ability to model a variety of situations will continue to be an important part of this course.
Unit 2: Proportional Reasoning and Linear Functions

Unit Objectives and Skills

In addition to the objectives and skills stated in the prior unit, at the completion of this unit, students will understand a ratio such as a fraction, decimal, or percent as a constant rate of change, and be able to solve equations.

Unit Overview

This unit offers students an opportunity to enhance their understanding of basic linear functions, while further developing the concept of constant rate of change. These problems introduce another array of familiar situations that provide students with a clear perspective on the role of mathematics in daily life.

Problems in this unit focus on representative examples of ratios as a rate of change, i.e. problems addressing a part of a population, tax rates, commissions, and tipping. In particular, activities in this unit help students to see a part of a whole as a ratio and ratio as a rate of change. This set of problems complements the problems the students are completing in the software. By the very nature of the problems, students will be reviewing the basic arithmetic associated with fractions, decimals, and percents, especially when they are required to solve for the input given the output. The equation-solving units in the software provide students with extensive practice in solving linear equations.

Completing this unit broadens students’ perspective on the extensive range of situations that can be modeled and solved using linear equations. Students’ understanding is further enhanced in Unit 4 where they work with alternative symbolic representations of linear relationships, i.e. situations that may be modeled in the form \( y = m(x + x_0) \).
Unit Activities

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

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<tr>
<td>Tipping in a Restaurant</td>
<td>pg. 41</td>
</tr>
<tr>
<td>Earning Sales Commission</td>
<td>pgs. 43, 45</td>
</tr>
</tbody>
</table>

Suggestions for Classroom Implementation

The key new concepts are using ratios as a rate of change and solving for the input given the output. The introduction of both concepts is rooted in students’ existing knowledge. Students have solved numerous problems of this type in arithmetic. Tying their arithmetic knowledge to the algebraic symbolic representation is helpful, especially when introducing the concept of solving an algebraic equation.

*Left-Handed Learners, Shadows and Proportions,* and *Making Punch* all involve ratios that can be represented symbolically. After students complete *Making Punch,* open a discussion that compares the three problems. It will support a review while furthering students’ ability to engage in comparative analyses. The discussion should also be used to help clarify students’ understanding of ratios and ratios as rates.

*TV News Ratings, Truck Life* and *Women at a University* involve studying a part of the population and using part of a whole as the rate to model and make inferences about the population. These problems, along with *Left-Handed Learners,* introduce the notion of sampling from a population and therefore have a more statistical flavor, setting the stage for material in Unit 5. The remaining three problems require finding a constant percentage of a defined value. Solving equations appears in *Taxes Deducted from your Paycheck, Tipping in a Restaurant,* and *Earning Sales Commission,* but is treated arithmetically.
**TV News Ratings** and **Truck Life** are similar with respect to their slopes and solution methodology, so it is useful to form small groups where half the groups work one problem each. At the completion of the work, have each group present its results and engage students in a discussion about the similarities (in terms of being able to model them using a linear equation) between these problems and the problems in the previous unit.

**Women at a University** is slightly different from **TV News Ratings** and **Truck Life**. In this problem, students are asked to look at the ratio of women to total students and solve for the ratio of the number of women to the number of men. Given this variation, you will want to have everyone work on this problem.

**Plastic Containers** articulates all aspects of the problems so far and helps students summarize the main themes of this unit. Have groups present the memo or note required in the problem. So that the presentations have meaning, students should bring supporting materials and reference them as they present.

For the final three problems, **Taxes Deducted from your Paycheck**, **Tipping in a Restaurant** and **Earning Sales Commission**, break the students up into groups, having a third of the groups work on each of the problems. Ask students to compare the rates of change in this group of problems. Also, have students compare these problems to the problems in the previous unit. Have students clearly explain how they went about finding the input as a means of reviewing basic arithmetic procedures.

**Unit Assessments**

**Students Who Don’t Take Algebra**

**Paying Taxes**
Unit 2: Proportional Reasoning and Linear Functions

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Collect class data by a simple show of hands.

Assign one of the additional population questions to each group a day prior to performing this activity, or provide students with data about the populations of the locations in which they live. Research the subject so you can provide accurate numbers.

Many qualities that people possess are learned. For example, some people are very organized, while others may be great cooks. Other traits are inherited, like your natural hair color or the shape of your face. Some people debate whether being right-handed or left-handed is a learned or inherited characteristic.

You recently overheard a conversation about this topic. One person claimed that her child was going to be ambidextrous because she was encouraging him to use either hand to eat or write or catch a ball. Another said he hoped his child would be left-handed like he was. A third person believed that she would have been left-handed, except that in grade school she had been forced to use her right hand to write in order to be like everybody else. Listening to this made you wonder if there has been an increase in the number of left-handed youngsters, since the practice of altering handedness has stopped.

You would like to find out if this is true nationally, but you decide to start your investigation by looking at the students in your school. You know you can’t ask every single student. Instead, you use the students in your math class as a sample.

<table>
<thead>
<tr>
<th>Total number of students in class</th>
<th>Number of right-handed students</th>
<th>Number of left-handed students</th>
<th>Number of ambidextrous students</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>29</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Before you examine the data any further, you also need to know the number of students in your school. You might also consider finding the number of people who live in your borough or township, your county, the nearest city, and the country. Record any data you or your teacher find in the table below.

Number of students in your school | Number of people...
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>864</td>
<td></td>
</tr>
</tbody>
</table>
Students may not have written a ratio or proportion to solve this problem, but they will have used proportional reasoning in one way or another. For example, students may have used repeated addition or multiplication to find the results for the school. If students did make use of ratios to find the school’s results, they may not have used a formal procedure for solving proportions. Do not give students a technique for finding the number of left-handed people in each group. Allow them to work together and devise strategies within their small groups.

Presentations and class discussions should focus on the method used rather than the final solution. Question student groups about strategies they used. Solicit several different methods and discuss the advantages and disadvantages of each.

A quick review of fractions and simplification may be necessary for students to focus on the relationship formed by the ratio.

<table>
<thead>
<tr>
<th>Class: Left-handed</th>
<th>School: Left-handed</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{4}{36} = \frac{1}{9} ]</td>
<td>[ \frac{96}{864} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Right-handed</th>
<th>Right-handed</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{29}{36} ]</td>
<td>[ \frac{696}{864} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ambidextrous</th>
<th>Ambidextrous</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{2}{36} = \frac{1}{18} ]</td>
<td>[ \frac{48}{864} ]</td>
</tr>
</tbody>
</table>

Multiply the numerator and denominator by 24.

2. Use the other data you have gathered about the population of your city or state to predict how many left-handed people, right-handed people, and ambidextrous people live in the different areas. Show all of your work.

These questions will be answered using ratios that are created based on the populations entered in the table at the bottom of the previous page.
Students should see that a relationship exists between the sample and the ability to generalize to larger populations. During discussions of linear functions, students will once again be asked to make predictions based on a model.

In question 3, students may have valid reasons for why the class is not a representative sample of the school and therefore should not be used to make predictions about the entire school population.

In question 4, students may have difficulty grasping the feasibility of extrapolating to large numbers. Help them by taking a smaller group, for example 500 or 1,000, and asking the question. Then increase to 3,000 through 10,000 and finally to very large numbers. Examine whether their views change about the reasonableness of using their class to make predictions as the numbers increase.

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Left-Handed Learners</strong></td>
</tr>
<tr>
<td>3. Is it reasonable to use your class data to make the predictions about left-handed, right-handed, and ambidextrous people for your school? Why or why not?</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td>Yes. The number of students in the class is a good sample of the school population. The students in this class are representative of the school population</td>
</tr>
<tr>
<td>4. Is it reasonable to use your class to make predictions about your town, city, or state? What about the entire country? Explain your reasoning.</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td>No. Such a small sample is not a good representation of an extremely large population. Even though the students in the class may be representative of students in the school, you cannot say that they will be representative of the town’s population.</td>
</tr>
<tr>
<td>5. What method did you use to help you find the number of left-handed (or right-handed) people in your school, town, city, state, or country?</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td>I used ratios and proportions. It was easier than asking every person.</td>
</tr>
</tbody>
</table>
After analyzing the two studies, ask students to design their own study to determine the number of left-handed people in the entire country. What elements would they include from the studies discussed? What additional elements would they include?

Of the two surveys above, the Bertley survey has more validity than the Hachkins survey because the sample is random and has a greater likelihood of including all types of people, therefore paralleling the population.

Cite the presidential election of 1948, when the Republican candidate Dewey was predicted as a landslide winner over Democrat Harry Truman based on a telephone poll. The poll was biased since only the wealthy, who tended to vote Republican, had telephones in 1948. This resulted in premature and incorrect headlines announcing Dewey as the winner of the election.

Discuss why surveys can be misleading or biased. Ask students to think of situations where findings for studies have been reported and have them consider how the findings may have been biased. You might even introduce the concept of variability at this point. (Variability is discussed again in Unit 5.)

### Left-Handed Learners

#### extension

6. Even after performing this investigation in class, you are still not satisfied that you have answered your question. That evening, you log onto the Internet and search for a study of left-handedness. You find the results of two separate studies.

- One study was performed at James Hachkins University. Within this study, graduate students in statistics took a sample of people from a list of local Internet users by using every 10th name on the list, and emailing them to ask which hand they used to write.
- The other study was performed at Bertley. Within this study, address lists from towns throughout the country were gathered. Each address was assigned a number. Random numbers were generated to choose the participants. People were contacted by mail asking which was their dominant hand.

Will either of these studies provide better information about left-handed people than the investigation you did? Explain why or why not.

**Student responses might include:**

- **The Bertley experiment is better because the information they gather is based on a bigger and more widespread sample.**
- **Both surveys may be worse because you do not know how many people will respond. You may get too few responses.**
- **The Bertley survey may be better since it is a random sample and is more likely to mirror the population.**
- **The Hachkins survey may be worse because it only used local Internet users, and that doesn’t seem very random.**
One afternoon, you and your friend Kirk were walking along your street. You noticed a very large pine tree, which had obviously been there for quite some time. You wondered aloud about how tall the tree was. Kirk suggested that you measure the shadow cast by the tree. He thought the shadow’s length would be the same as the tree’s height. You weren’t convinced, but went along with his idea.

Upon measuring the length of the shadow, you found that it was approximately 8 meters long, although the tree looked a lot taller. You also noticed that Kirk’s shadow looked a lot smaller than he was, so you decided to measure it as well. Kirk’s shadow was 60 cm long, but Kirk is 150 cm (1.5 m) tall.
Shadows and Proportions

1. What is the ratio between Kirk's actual height and Kirk's shadow's length? Put this ratio in simplest terms.

\[
\frac{150 \text{ cm}}{60 \text{ cm}} = \frac{5}{2}
\]

2. Should the ratio of the tree's height to the tree's shadow length be the same as or different than the ratio between Kirk's height and his shadow's length? Explain the reason for your response.

It should be the same. The angle at which the sun hits both the tree and Kirk is the same.

3. What would the length of the tree's shadow be if the tree's height was 3 meters (300 cm.)? Explain how you know.

The shadow would be 1.2 meters long. Since Kirk's height is 1.5m, double it to get a height of 3m. Kirk's shadow is 0.6m, so double it to get 1.2m.

or

The shadow would be 120cm long. Since Kirk is 150cm tall, double his height to get 300cm. Kirk's shadow is 60cm so double it to get 120cm.

4. If the tree's shadow measured 6 meters (600 cm.), what would the tree's height be?

The height of the tree would be 15 meters. The ratio of Kirk's height to his shadow is 150:60. This can also be expressed as 15:6. If the shadow is 6m, the height has to be 15m.

5. Find the approximate height of the tree. Show all your work.

\[
\frac{5}{2} = \frac{x}{8} \quad \text{or} \quad \frac{2}{5} = \frac{8}{x}
\]

Since these are equivalent fractions, 2 has to be multiplied by 4 to get 8 and 5 has to be multiplied by 4, so \(x = 20\).

The height of the tree is about 20 meters.
Shadows and Proportions

Just as you were finishing your calculations, an elderly woman came out from the house near the tree. She asked what you were doing. You explained that you were curious about the height of her tree, and you were calculating its height.

Upon hearing this, the woman gave you some useful information. She told you that she and her husband planted the tree together on their wedding day in 1927. When they planted the tree, it was about 1 meter tall. She had tracked its growth until it was too tall to measure directly. Up until that point, the tree had grown an average of about 30 cm each year.

Reinforce the use of proper units when describing the rate of change. For example, the growth rate of the tree should be measured in either centimeters per year or meters per year.

6. If the tree continued to grow at the same rate each year, determine its height …
   a. 10 years after it was planted.
      \[
      10 \times 30 = 300 \text{ cm} = 3 \text{ m} \\
      1 \text{ m} = 3 \text{ m} = 4 \text{ m} \\
      \text{The tree is 4 meters tall.}
      \]
   b. 20 years after it was planted.
      \[
      20 \times 30 = 600 \text{ cm} = 6 \text{ m} \\
      1 \text{ m} = 6 \text{ m} = 7 \text{ m} \\
      \text{The tree is 7 meters tall.}
      \]
   c. 50 years after it was planted.
      \[
      50 \times 30 = 1500 \text{ cm} = 15 \text{ m} \\
      1 \text{ m} = 15 \text{ m} = 16 \text{ m} \\
      \text{The tree is 16 meters tall.}
      \]

7. What is the rate of growth of this tree?

   \text{The rate of growth is 0.3 meters per year or 30 cm per year.}
8. Create a table of values for the year, the number of years after the tree was planted, and the height of the tree.

<table>
<thead>
<tr>
<th>Time</th>
<th>Height of Tree</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year</strong></td>
<td><strong>Years since 1927</strong></td>
</tr>
<tr>
<td>1927</td>
<td>0</td>
</tr>
<tr>
<td>1937</td>
<td>10</td>
</tr>
<tr>
<td><strong>1947</strong></td>
<td>20</td>
</tr>
<tr>
<td>1977</td>
<td>50</td>
</tr>
<tr>
<td>1997</td>
<td>70</td>
</tr>
<tr>
<td><strong>2002</strong></td>
<td>75</td>
</tr>
</tbody>
</table>
Choose bounds so that the current height of the tree is visible on the graph. Reinforce the ability to use the graph to find a solution.

### Shadows and Proportions

9. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>0</td>
<td>75</td>
<td>5</td>
</tr>
<tr>
<td>Height</td>
<td>0</td>
<td>24</td>
<td>1</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between time and height]
Students began to solve and evaluate equations like the one generated in question 10 within Unit 5 of the software curriculum.

Shadows and Proportions

10. Using \( h \) to represent the height of the tree and \( t \) to represent the time after 1927, write an equation to model this situation.

\[
h = 0.3t + 1 \quad \text{(for a result in meters)}
\]

or

\[
h = 30t + 100 \quad \text{(for a result in centimeters)}
\]

11. Using the algebraic equation you created above as a model, estimate the current height of the tree. Make sure to show your work.

\[
h = 0.3(75) + 1
\]

\[
h = 23.5\text{m}
\]

Have students make comparisons between ratios to develop the concept of proportions.

Shadows and Proportions

12. Does the result of your calculation using Kirk’s height and his shadow agree with your new calculation using the rate of growth of the tree?  

No, the rate of growth calculation says the tree is 3.5 m higher.

13. What might account for any differences between the two results?  

The growth of the tree may have slowed over the years.

14. Which of the two results do you think is more accurate? Why?

Answers will vary.

Sample Response:

I believe that the calculation using Kirk’s height would be more accurate because that one used actual measurements.
Students can analyze the strength of the punch by calculating the ratio of grapefruit juice to soda pop, or by calculating the ratio of grapefruit juice to total liquid, or even by calculating the ratio of soda pop to total liquid.

Making Punch

Each year, your class presents its mathematics portfolios to parents and community members. This year, your homeroom is in charge of the refreshments for the reception that follows the presentations. You have a great recipe for punch, so you volunteer to let the class use it, but so do three other students in the class. To avoid favoritism, the class decides to analyze the recipes to determine which one will make the fruitiest tasting punch for their guests. The recipes are as follows:

- **Adam’s Recipe:**
  4 parts lemon lime soda pop
  8 parts grapefruit juice

- **Bobbi’s Recipe:**
  3 parts lemon lime soda pop
  5 parts grapefruit juice

- **Carlos’ Recipe:**
  2 parts lemon lime soda pop
  3 parts grapefruit juice

- **Zeb’s Recipe:**
  1 part lemon lime soda pop
  4 parts grapefruit juice


1. Which of these recipes will have the strongest taste of grapefruit? Show your work and explain your reasoning.

   **Sample Response:**

   
   \[
   \frac{4}{8} = 0.5 \quad \frac{3}{5} = 0.6 \quad \frac{2}{3} = 0.666 \quad \frac{1}{4} = 0.25
   \]

   Since Zeb’s recipe has the smallest ratio of lemon lime soda pop to grapefruit juice, his would have the strongest taste of grapefruit.

   Other solution methods are possible.
### Making Punch

2. Which of the recipes will have the strongest taste of lemon lime? Show your work and explain your reasoning.

   Since Carlos’ recipe has the highest ratio of lemon lime soda pop to grapefruit, his would have the strongest lemon lime taste.

---

### Making Punch

3. The plastic cups your class bought to serve the punch are small. Each cup holds 6 ounces (¾ cup). There are a total of 70 students in your grade, and you are expecting about 90 parents and community members to be in attendance. Assume everyone at the reception has a glass of punch. If you use Adam’s recipe, how much soda pop and how much grapefruit juice would you need to make enough punch to serve everyone?

   One possible solution strategy:

   \[
   \begin{align*}
   \frac{70}{90} &= \frac{160}{180} \text{ people} \\
   160 \cdot \frac{3}{4} &= 120 \text{ cups} \\
   4 \cdot 8 &= 12 \text{ total parts} \\
   \frac{120}{12} &= 10 \text{ cups per part} \\
   4 \cdot 10 &= 40 \text{ cups of lemon lime soda pop} \\
   8 \cdot 10 &= 80 \text{ cups of grapefruit juice}
   \end{align*}
   \]

4. For Bobbi’s recipe, how much soda pop and how much grapefruit juice would you need to make enough punch to serve everyone?

   \[
   \begin{align*}
   \frac{5}{3} &= 8 \text{ total parts} \\
   \frac{120}{8} &= 15 \text{ cups per part} \\
   5 \cdot 15 &= 75 \text{ cups of grapefruit juice} \\
   3 \cdot 15 &= 45 \text{ cups of lemon lime soda pop}
   \end{align*}
   \]
Making Punch

5. For Carlos’ recipe, how much soda pop and how much grapefruit juice would you need to make enough punch to serve everyone?

\[ \frac{240}{5} = 24 \text{ cups per part} \]

\[ 2 \cdot 24 = 48 \text{ cups of lemon lime soda pop} \]

\[ 3 \cdot 24 = 72 \text{ cups of grapefruit juice} \]

6. For Zeb’s recipe, how much soda pop and how much grapefruit juice would you need to make enough punch to serve everyone?

\[ \frac{120}{5} = 24 \text{ cups per part} \]

\[ 1 \cdot 24 = 24 \text{ cups of lemon lime soda pop} \]

\[ 4 \cdot 24 = 96 \text{ cups of grapefruit juice} \]

Making Punch

7. Summarize this information in the table below.

<table>
<thead>
<tr>
<th>Amount of soda pop needed</th>
<th>Amount of grapefruit juice needed</th>
<th>Total amount of punch needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adam’s Recipe</td>
<td>40 cups</td>
<td>80 cups</td>
</tr>
<tr>
<td>Bobbi’s Recipe</td>
<td>45 cups</td>
<td>75 cups</td>
</tr>
<tr>
<td>Carlos’ Recipe</td>
<td>48 cups</td>
<td>72 cups</td>
</tr>
<tr>
<td>Zeb’s Recipe</td>
<td>24 cups</td>
<td>96 cups</td>
</tr>
</tbody>
</table>

Remind students that they may use cups or ounces, but should be consistent once the choice is made.
8. If you were using an 8 ounce plastic cup instead of a 6 ounce cup, how much grapefruit juice and how much soda pop would be contained in one cup of punch if you used Zeb’s recipe?

   You would have 1/5 cup soda pop and 4/5 cup grapefruit or 1.6 ounces of soda and 6.4 ounces of grapefruit juice.

9. How much grapefruit juice and how much soda pop would be contained in one cup of punch if you used Carlos’ recipe?

   You would have 2/5 cup soda pop and 3/5 cup grapefruit or 3.2 ounces of soda and 4.8 ounces of grapefruit juice.

10. How much of each ingredient would be contained in 1 cup of punch made using Bobbi’s recipe?

    You would have 3/8 cup soda pop and 5/8 cup grapefruit or 3 ounces of soda and 5 ounces of grapefruit.

11. Lastly, how much of each ingredient would be in 1 cup of Adam’s recipe?

    You would have 1/3 cup soda pop and 2/3 cup grapefruit or about 2.7 ounces of soda and 5.3 ounces of grapefruit juice.
### Making Punch

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Explain how ratios helped you solve the problems in the process section of this activity.</td>
</tr>
<tr>
<td><strong>Answers will vary.</strong></td>
</tr>
<tr>
<td><strong>Sample response:</strong></td>
</tr>
<tr>
<td>The ratios in questions 1 and 2 told me which mix was the weakest or strongest.</td>
</tr>
<tr>
<td>The ratios of lemon lime to grapefruit had to be maintained in the rest of the questions. That helped to determine the number of cups or ounces in each recipe mix.</td>
</tr>
</tbody>
</table>

13. Could you use all of these recipes to make exactly 100 cups of punch? If so, explain your reasoning in detail. If not, will any of the recipes make exactly 100 cups? Explain.

**Answers will vary.**

**Sample response:**

Yes. To do so, you would need to use fractions of cups as in the case of Adam and Bobbi.

For Carlos’ and Zeb’s recipes, there is a total of 5 cups of liquid. Since 100 is 20*5, you can multiply the number of parts of lemon lime soda and grapefruit juice by 20 to get the number of cups of each in 100 cups of punch.

---

Students may not fully understand how ratios helped them, so you may need a guiding question such as, “How did you know that Bobbi’s recipe called for so many cups (ounces) of soda?”

Students’ interpretation of question 13 may vary. Critical to the interpretation is whether it is possible for a ‘part’ to consist of a fractional number of cups. For example, Bobbi’s recipe could be used if each ‘part’ consists of 12 ½ cups.
Many of the remaining problems in this unit consist of the same components: calculation of dependent variable, calculation of independent variable, derivation of algebraic equation, interpretation of algebraic equation, creation of table of values, and plotting of points from table.

TV News Ratings

Advertisers must pay to run their commercials on television. The cost of commercials depends on the popularity of a show. The more popular a show is, the larger the viewing audience, so more people see the commercial advertisements. Therefore, ratings of shows become important for producers. They use the information to determine the cost of a one-minute commercial spot.

One news show may be more popular than another because of the commentators or personal interest stories. The results of a recent survey indicate that two out of every five people who watch the news watch Channel 11 News at Six.

1. Approximately how many people watch Channel 11 if the number of people who watch any news at 6:00 is:
   a. 10 people? 4 people watch Channel 11.
   b. 1,000 people? 400 people watch Channel 11.
   c. 10,000 people? 4,000 people watch Channel 11.

   In a complete sentence, describe how you found these answers.

   **Sample response:**
   Divide the number of people by 5 and then multiply the result by 2.

2. Approximately how many people watch any news at 6:00 if the number of people who watch Channel 11 News is:
   a. 20 people? 50 people
   b. 200 people? 500 people
   c. 20,000 people? 50,000 people

   In a complete sentence, describe how you found these answers.

   **Sample response:**
   Divide the number of people by 2 and then multiply the result by 5, or multiply the number of people who watch Channel 11 by 5/2.
You want students to see the concept of constant rate, namely 2 out of every 5, emerging in a slightly different way than it did in the previous problems. The format of the table in question 5 is identical to the format of the worksheet in the software curriculum.

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3. Let \( P \) represent the total number of people who watch any news at 6:00. Write an equation for \( T \), the number of people who watch the Channel 11 News at Six.

\[
T = \frac{2}{5}P
\]

4. Use the information you have developed so far to clearly redefine the problem situation.

*Given the total number of people who watch the news and given 2 out of every 5 people who watch any news watch Channel 11, we need to find the number of people who watch Channel 11 News.*

5. Create a table of values for this situation and include the expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>All News Watchers</th>
<th>Channel 11 Watchers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>People</td>
<td>People</td>
</tr>
<tr>
<td>Expression</td>
<td>( P )</td>
<td>( \frac{2}{5}P )</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>1,000</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>4,000</td>
</tr>
</tbody>
</table>

Table entries will vary.
The format of the graph and graph setup in question 6 is identical to the format of the Grapher window in the software curriculum.

### TV News Ratings

6. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total News</td>
<td>0</td>
<td>10,000</td>
<td>500</td>
</tr>
<tr>
<td>Channel 11 News</td>
<td>0</td>
<td>4,000</td>
<td>200</td>
</tr>
</tbody>
</table>

**Bounds, intervals, and graphs will vary according to entries from the table.**
### TV News Ratings

7. What are the variable quantities in the problem situation? Include the units used to measure these quantities.

   - Number of people who watch any news measured in people.
   - Number of people who watch Channel 11 News measured in people.

8. What is the constant quantity in the problem situation?

   The constant is 2/5.

9. Of the two variable quantities, which quantity depends on the other quantity?

   The number of people who watch Channel 11 News depends on the total number of people watching any news.

10. What is the independent variable quantity, and what is the dependent variable quantity in the problem situation?

    - People who watch any news is the independent variable quantity.
    - People who watch Channel 11 News is the dependent variable quantity.

### TV News Ratings

11. The producers of a competing news show on Channel 4 tell the commercial sponsors that their six o’clock news has one of every three viewers watching it. Should the advertisers place their commercial with Channel 11 or Channel 4 if they want to reach the largest television viewing audience? Explain your reasoning.

    \[
    \frac{2}{5} = \frac{6}{15} \quad \frac{1}{3} = \frac{5}{15}
    \]

    *Since 6/15 is greater than 5/15, advertisers should use Channel 11 News.*
Note that students are asked to write the algebraic equation after answering specific questions. This is done to give students practice detecting the pattern prior to writing the equation. As students proceed through the problems of this unit, they may be able to write the equation first and then use it to answer the initial questions.

### Truck Life

To sell an expensive vehicle, like a 4 x 4 truck, advertising agencies talk about its power, design features, and durability. In a recent advertising campaign to convince buyers of the value of Motorman trucks, the following claim was made: “Nine out of 10 Motorman trucks sold in the last ten years are still on the road.”

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
<th>Truck Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Approximately how many Motorman trucks are still on the road if the number of trucks sold in the last ten years is:</td>
<td></td>
</tr>
<tr>
<td>a. 100 trucks?</td>
<td><strong>90 trucks</strong></td>
</tr>
<tr>
<td>b. 10,000 trucks?</td>
<td><strong>9,000 trucks</strong></td>
</tr>
<tr>
<td>c. 1,000,000 trucks?</td>
<td><strong>900,000 trucks</strong></td>
</tr>
<tr>
<td>In a sentence or two, explain how you solved these three problems.</td>
<td></td>
</tr>
<tr>
<td><strong>Sample response:</strong></td>
<td>I multiplied the number of trucks by 9 and then divided the result by 10.</td>
</tr>
</tbody>
</table>

2. Approximately how many trucks were sold in the last ten years if the number of Motorman trucks still on the road is:

| a. 90,000 Motorman trucks? | **100,000 trucks** |
| b. 1,800 Motorman trucks? | **2,000 trucks** |
| c. 27,000 Motorman trucks? | **30,000 trucks** |
| In a sentence or two, describe how you found these answers. | |
| **Sample response:** | Multiply the number of Motorman trucks by 10 and then divide the result by 9, or multiply the number of Motorman trucks by 10/9. |
3. Let $T$ represent the total number of Motorman trucks sold in the last ten years. Write an equation for $M$, the number of Motorman trucks still on the road.

$$M = \frac{9}{10}T$$

4. Use the information you have developed so far to clearly redefine the problem situation.

**Sample response:**

*We need to find the quantity of Motorman trucks sold in the last ten years that are still on the road, given the claim that 9 out of 10 of all the Motorman trucks sold in the last ten years are still on the road.*

5. Create a table of values for this situation and include the expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Motorman Trucks</th>
<th>Trucks Still on the Road</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>trucks</td>
<td>trucks</td>
</tr>
<tr>
<td></td>
<td>$T$</td>
<td>$\frac{9}{10}T$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>9,000</td>
</tr>
<tr>
<td></td>
<td>100,000</td>
<td>90,000</td>
</tr>
<tr>
<td></td>
<td>2,000</td>
<td>1,800</td>
</tr>
<tr>
<td></td>
<td>30,000</td>
<td>27,000</td>
</tr>
</tbody>
</table>
Since larger numerical values are presented in this problem, pay special attention to choosing appropriate bounds and intervals.

### Truck Life

6. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Trucks</strong></td>
<td>0</td>
<td>100,000</td>
<td>5,000</td>
</tr>
<tr>
<td><strong>Trucks Still on the Road</strong></td>
<td>0</td>
<td>90,000</td>
<td>5,000</td>
</tr>
</tbody>
</table>
7. What similarities exist between the TV News Rating problem and this Truck Life problem? Make sure to compare the scenario, the tables, the algebraic expressions, and the graphs.

*Answers will vary.*

*Sample response:*

*Each problem scenario involves a ratio. Each table contains small and large values.*

*The expressions contain fractions. The lines move in a positive direction and are at less than a 45° angle in the first quadrant. The lines all pass through the origin.*

8. What differences exist between the TV News Rating problem and this Truck Life problem? Make sure to compare all the different representations of the problem.

*Answers will vary.*

*Sample response:*

*Each problem uses different ratios. The problems both involve different units.*

*The graphs increase at different rates, with the Motorman truck graph increasing more quickly.*

*Each algebraic expression contains a different fraction.*
Women at a University

Government agencies and civil rights groups concerned with diversity in higher education monitor enrollment data for underrepresented groups. One study focused on the enrollment of women at a certain university. The study found that three out of every five students enrolled were women.

1. Approximately how many women are enrolled if the total number of students enrolled is:
   a. 1000 students? 600 women
   b. 10,000 students? 6,000 women
   c. 5,000 students? 3,000 women

   In a few sentences, explain how you solved these three problems.
   **Sample response:**
   
   *First divide the total number of students by 5. Then multiply that number by 3 to find the number of women.*

2. What is the approximate total number of students enrolled if there are:
   a. 3,000 female students? 5,000 students
   b. 1,800 female students? 3,000 students
   c. 27,000 female students? 45,000 students

   In a few sentences, describe how you found these answers.
   **Sample response:**
   
   *First multiply the number of female students by 5. Then divide the result by 3.*
Women at a University

3. Let $T$ represent the total number of students enrolled. Write a formula for $F$, the number of female students enrolled.

$$F = \frac{3}{5}T$$

4. Use the information you have developed so far to clearly redefine the problem situation.

*Given the total number of students at a university and given that $3/5$ of all the students are women, we want to find the number of women enrolled.*

5. Create a table of values for this situation and include the expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Students</th>
<th>Female Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>students</td>
<td>students</td>
</tr>
<tr>
<td>Expression</td>
<td>$T$</td>
<td>$3/5T$</td>
</tr>
<tr>
<td></td>
<td>1,000</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>5,000</td>
<td>3,000</td>
</tr>
<tr>
<td></td>
<td>3,000</td>
<td>1,800</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>6,000</td>
</tr>
<tr>
<td></td>
<td>45,000</td>
<td>27,000</td>
</tr>
</tbody>
</table>

*Table entries will vary.*
6. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Students</td>
<td>0</td>
<td>45,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Female Students</td>
<td>0</td>
<td>27,000</td>
<td>2,000</td>
</tr>
</tbody>
</table>

*Bounds, intervals and graphs will vary according to entries from the table.*
In question 7, watch for students to use $\frac{3}{5}$. Explain that they are no longer comparing to the total population, but instead men to women and women to men. However, students may first need to compute the total population in order to compute the number of male students.

Students will compute fractional solutions in questions 7 and 8. When discussing the results, address the appropriateness of these solutions, as well as the difference between an algebraic solution and a realistic solution.

---

### Women at a University

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7.</strong> Approximately how many men are enrolled if there are:</td>
</tr>
<tr>
<td>a. 1,000 female students?</td>
</tr>
<tr>
<td><strong>There are 666 male students enrolled.</strong></td>
</tr>
<tr>
<td>b. 10,000 female students?</td>
</tr>
<tr>
<td><strong>Six thousand six hundred sixty-six students are male.</strong></td>
</tr>
<tr>
<td>c. 5,000 female students? <strong>There are 3,333 male students</strong></td>
</tr>
<tr>
<td>Explain how you calculated the results for the three questions above.</td>
</tr>
<tr>
<td>Sample response:</td>
</tr>
<tr>
<td><strong>Multiply the number of females by 2 and divide the result by 3.</strong></td>
</tr>
</tbody>
</table>

---

8. Approximately how many women are enrolled if there are:

| a. 3,000 male students? |
| **There are 4,500 female students enrolled.** |
| b. 1,800 male students? |
| **Of the total number of students, 2,700 are female.** |
| c. 27,000 male students? |
| **There are 40,500 female students.** |
| Explain how you found your answers to the three questions above. |
| Sample response: |
| **Multiply the number of males by 3 and divide the result by 2.** |
Women at a University

9. Let $F$ represent the total number of female students enrolled. Write an equation for $M$, the number of male students enrolled.

$$M = \frac{2}{3}F$$

10. Use the information you have developed so far to clearly redefine the problem situation.

*Given the number of female students, we need to find the quantity of male students enrolled based on the fact that the number of male students is $2/3$ the number of female students.*

11. Create a table of values for this situation and include the expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Female Students</th>
<th>Male Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>students</td>
<td>students</td>
</tr>
<tr>
<td>Expression</td>
<td>$F$</td>
<td>$2/3F$</td>
</tr>
<tr>
<td></td>
<td>1,000</td>
<td>666</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>6,666</td>
</tr>
<tr>
<td></td>
<td>5,000</td>
<td>3,333</td>
</tr>
<tr>
<td></td>
<td>4,500</td>
<td>3,000</td>
</tr>
<tr>
<td></td>
<td>2,700</td>
<td>1,800</td>
</tr>
</tbody>
</table>

*Table entries will vary.*
12. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female Students</td>
<td>0</td>
<td>40,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Male Students</td>
<td>0</td>
<td>30,000</td>
<td>1,500</td>
</tr>
</tbody>
</table>

**Bounds, intervals, and graphs will vary according to entries from the table.**
13. Which of the representations of this problem (scenario, table, algebraic equation, or graph) provides the most useful information? Explain why you chose a particular representation.

**Sample responses:**

The scenario helps understand the problem; it gives the formula for the problem.

The table provides values for graphing purposes.

The equation helps construct the table.

The graph provides a visual method of finding points and values that are not given in the chart.

---

14. Use the Internet to gather demographic information about a college or university in your state. What is the ratio of women to men at this college or university? What is the ratio of minority students to the total student population? Find other ratios of interest to you.

**Answers will vary based on individual student research, but you should limit the students' search to less than ten colleges or universities of interest. In this way, you can also gather the information to check against their reported numbers.**
You have been hired by the Nikkerware Company to help create new packaging and shipping containers for its plastic containers. The company makes all different shapes and sizes of plastic containers. To ship the containers, the lids are removed to allow for stacking. The company wants to design boxes for shipping that will hold two dozen containers no matter what the size or shape of the container.

The table below shows the data gathered from measuring the heights of different sized stacks of the various plastic containers.

<table>
<thead>
<tr>
<th>Number of containers</th>
<th>Height of stack (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Round</td>
</tr>
<tr>
<td>1</td>
<td>5.4</td>
</tr>
<tr>
<td>2</td>
<td>5.9</td>
</tr>
<tr>
<td>3</td>
<td>6.4</td>
</tr>
<tr>
<td>4</td>
<td>6.9</td>
</tr>
<tr>
<td>5</td>
<td>7.4</td>
</tr>
</tbody>
</table>

If you have some stackable plastic containers in your classroom, you should add the data about them to this table.
Remind students that the bounds must be large enough to contain all three graphs.

Questions on the next page ask students to make predictions for 24 containers. If you have the students make this visible on the graph, it will reinforce the use of the graph to obtain solutions. The graph is 20 x 20, so if students use an interval of 1 on the horizontal axis, they will need to extend the grid to view 24 containers.

### Plastic Containers

1. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis. You may want to use different markings for each type of plastic container (dots, x’s, open circles, etc.) or you may want to use different colored pencils for each type of container.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Containers</strong></td>
<td>0</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td><strong>Height of Containers</strong></td>
<td>0</td>
<td>20</td>
<td>1</td>
</tr>
</tbody>
</table>

2. What are the variables in this problem situation?

   **The number of containers and the height of the stack measured in centimeters are the variables in this scenario.**

3. What is the relationship between these variables (which variable depends on the other)?

   **The height of the stack depends on the number of containers in the stack.**
Plastic Containers

4. Predict the height of a stack of two dozen round containers? Explain your reasoning in full sentences.

   \[ 5.4 + 23(0.5) = 16.9 \text{ cm} \]

   **Sample response:**
   The height of the stack should be 16.9 cm. In arriving at this answer, I kept adding 0.5 until I reached 24 containers, or I could have multiplied 0.5 by 23 and added 5.4.

5. Predict the height of a stack of two dozen square containers? Explain your reasoning in full sentences.

   \[ 4.5 + 23(1.7) = 43.6 \text{ cm} \]

   **Sample response:**
   The height of the stack should be 43.6 cm. In arriving at this answer, I kept adding 1.7 until I reached 24 containers, or I could have multiplied 1.7 by 23 and added 4.5.

6. Predict the height of a stack of two dozen rectangular containers. Explain your reasoning.

   \[ 14.6 + 23(0.3) = 21.5 \text{ cm} \]

   **Sample response:**
   The height of the stack should be 21.5 cm. In arriving at this answer, I kept adding .3 until I reached 24 containers, or I could have multiplied 0.3 by 23 and added 14.6.

7. If you did not use algebraic equations to make the predictions above, create algebraic equations that will allow you to compute the height of any stack of containers.

   a. **round** \[ H = 5.4 + 0.5(C - 1) \]
   b. **square** \[ H = 4.5 + 1.7(C - 1) \]
   c. **rectangular** \[ H = 14.6 + 0.3(C - 1) \]
Proportional Reasoning and Linear Functions

### Plastic Containers

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>analysis</strong></td>
<td></td>
</tr>
</tbody>
</table>
| **8.** What ratios were important for you to know to predict the height of a stack of 24 containers and to generate algebraic equations? Why were these ratios helpful?  
  
  **Sample response:**  
  
  *We needed to know the rate of growth of each stack of containers, but I did not use the ratio of the number of containers to the height of the stack because it did not stay the same. The rate of growth was helpful because I could use it to tell how much the stack grew.*  
  
  **9.** What similarities do you notice in all of the algebraic equations you wrote? Why do these similarities exist?  
  
  **Sample response:**  
  
  *The algebraic equations are similar in that both utilize multiplication and addition, have the same variables and require two steps of calculation. These similarities exist because we were starting from the height of the first container and adding on an additional amount for each new container in the stack.*  
  
  **10.** What differences do you notice in all of the algebraic equations you wrote? Why do the differences exist?  
  
  **Sample response:**  
  
  *The algebraic equations are different in that each has a different change in height and a different starting height. These dissimilarities exist because different containers were being used and because some could nest further into the stack than others.*  
  
  **11.** Write a memo to your supervisor in which you recommend the sizing of cartons for the different sized containers, in order to ship orders most effectively. Explain why you chose a particular sizing.  
  
  *Students may recommend three different sized shipping cartons for each of the different sized plastic containers, or they may recommend one size large enough to accommodate any of the three types of containers. Students may also choose to stack the containers in one set of 24, two sets of 12, three sets of 8, etc. In any case, the shipping cartons should slightly exceed the size of the stacked containers. If the carton exceeds the size of the containers by too much, then more Styrofoam peanuts must be used, which will raise the cost of shipping.*

---

Recall that shipping cartons need to accommodate 24 plastic containers. Include recommendations for any additional containers used as well.  

You may suggest that students write formal memos on a separate sheet of paper.
Students began to analyze systems of equations in Unit 1 with the comparison between U.S. Shirts and Hot Shirts. Refer back to that activity if needed while analyzing this system of equations. Students will be comparing rental car rates in Unit 3 as another example of systems of equations presented informally.

Students are now developing concepts about growth that will soon lead to discussions about slope.

<table>
<thead>
<tr>
<th>Plastic Containers</th>
</tr>
</thead>
</table>
| 12. The square containers start with the smallest stack, yet this stack ends up being the tallest when two dozen are stacked. Explain how this can be true.  
*The change in height each time was greater than the others.* |
| 13. Up to what point does the square stack stay the shortest stack? Why doesn’t it always stay the shortest?  
*It is only the shortest at the beginning. As soon as a container is added, it is no longer the shortest. It does not stay the shortest because additional containers do not nest as much as with the other types.* |
| 14. When does the stack of round containers become the shortest stack? After that point, will the round containers remain the shortest stack? Why or why not?  
*The stack becomes the shortest stack at 2 containers.*  
*The stack of round containers remains the shortest until it reaches 46 containers. It does not always remain the shortest because the change in height for the rectangular containers is less than the change in height for the round containers.* |
| 15. Will the stack of rectangular containers ever be the shortest stack? If so, when? If not, why not?  
*The rectangular stack becomes the shortest stack at 47 containers.* |
Have students think of famous people and estimate their salaries. Then allow students to compute the amount paid in taxes, assuming the 37\% figure is accurate. Students may be surprised at the dollar amounts paid in taxes by these famous (and frequently wealthy) individuals.

Students should discuss this type of rate – part of the whole – with respect to the previous problems. Does part of the whole mean the same when defining the relationship between the variables?

### Taxes Deducted from Your Paycheck

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{1. Based on the information above, how much is deducted from a person's gross pay if that gross pay is:}</td>
<td>\textbf{2. \begin{align*} \text{\textit{$11.10 is deducted. (30\times0.37)}} \end{align*} \text{\textit{$444 is deducted. (1200\times37%)}}} \text{\textit{$22,200 is deducted. (0.37\times60,000)}} \text{\textit{$444,000 is deducted. (37/100\times1,2000,000)}}}</td>
</tr>
<tr>
<td>\textbf{a. $30}</td>
<td>\textbf{b. $1,200?}}</td>
</tr>
<tr>
<td>\textbf{$11.10 is deducted. (30\times0.37)}</td>
<td>\textbf{$444 is deducted. (1200\times37%)}</td>
</tr>
<tr>
<td>\textbf{$22,200 is deducted. (0.37\times60,000)}</td>
<td>\textbf{$444,000 is deducted. (37/100\times1,2000,000)}</td>
</tr>
<tr>
<td>\textbf{c. $60,000?}}</td>
<td>\textbf{d. $1,200,000?}}</td>
</tr>
<tr>
<td>\textbf{Show all of your work and describe how you found your answers in complete sentences.}}</td>
<td>\textbf{I multiplied the gross pay by 0.37.}}</td>
</tr>
</tbody>
</table>

Those of us who work know that our paychecks are less than the full amount of our wages or salary (gross pay).

The amount you are taxed will depend on your income and what state you reside in. Some people may pay 37\% of their gross pay for federal, state, and local taxes. The portion that is left is called the take-home or net pay.

Interpreting this properly requires an understanding of percentages. The word percent means literally per hundred, so the statement above translates into “for every $100 in gross pay, a person receives about $63 in net pay” or “a person pays $37 in federal, state, and local taxes for every $100 earned.”

You can translate a percentage into either a decimal or a fraction by using the definition of \textit{per hundred}. For example: $37\% = \frac{37}{100} = 0.37$. Both the fraction and the decimal are read “thirty-seven hundredths.”
Students should calculate these solutions using division and/or ratios.

The explanations given are sample student responses only. Other solution methods and explanations are possible.

<table>
<thead>
<tr>
<th>Taxes Deducted from Your Paycheck</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. How much is a person’s gross pay if the amount deducted for federal, state, and local taxes is:</td>
</tr>
<tr>
<td>a. $37?</td>
</tr>
<tr>
<td><em>The gross pay would be $100. This was given in the input window.</em></td>
</tr>
<tr>
<td>b. $185?</td>
</tr>
<tr>
<td><em>Since 37</em>5 = 185, and 100<em>5 = 500, the gross pay is $500.</em></td>
</tr>
<tr>
<td>c. $462.50?</td>
</tr>
<tr>
<td>462.50 divided by 0.37 = 1250</td>
</tr>
<tr>
<td>$1,250 is the gross pay.</td>
</tr>
<tr>
<td>d. $370?</td>
</tr>
<tr>
<td>$370 is 10<em>37 so the gross pay has to be 10</em>100 = 1000.</td>
</tr>
<tr>
<td>$1000 is the gross pay.</td>
</tr>
<tr>
<td>e. $3,700?</td>
</tr>
<tr>
<td>3700/37 = 100 and 100*100 = 10,000</td>
</tr>
<tr>
<td>$10,000 is the gross pay.</td>
</tr>
</tbody>
</table>

Show all your work and describe how you found your answers in complete sentences.

*Divide the amount deducted by 0.37.*
Taxes Deducted from Your Paycheck

3. Create a table of values for this situation, and include the expression.

<table>
<thead>
<tr>
<th>Gross Pay</th>
<th>Amount Deducted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollars</td>
<td>Dollars</td>
</tr>
<tr>
<td>$G$</td>
<td>$0.37G$</td>
</tr>
<tr>
<td>100</td>
<td>37</td>
</tr>
<tr>
<td>300</td>
<td>111</td>
</tr>
<tr>
<td>500</td>
<td>185</td>
</tr>
<tr>
<td>1,000</td>
<td>370</td>
</tr>
<tr>
<td>1,250</td>
<td>462.50</td>
</tr>
</tbody>
</table>

4. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Pay</td>
<td>0</td>
<td>1,300</td>
<td>100</td>
</tr>
<tr>
<td>Amount Deducted</td>
<td>0</td>
<td>500</td>
<td>50</td>
</tr>
</tbody>
</table>
Taxes Deducted from Your Paycheck

5. Let \( G \) represent a person’s gross pay. Write an algebraic equation for \( N \), the person’s net pay.

\[
N = 0.63G = G - 0.37G
\]

a. What is the independent variable quantity in this problem?

Gross pay is independent.

b. What is the dependent variable quantity in this problem?

Net pay is dependent. The amount deducted from the pay has also been a dependent variable in this situation up to this point.

c. Is there a constant quantity in this problem? If so, what is it?

The constant is 0.63 or 0.37.

6. What similarities do you see among the problems you have done so far in this unit? What mathematical differences can you cite among the problems?

Similarities:

The problem situations can be modeled by algebraic equations.

Patterns can be found.

Each of the problems can be represented numerically, graphically, and algebraically.

The problems involve quantities where one depends on the other.

Differences:

Some problems have initial or start values, whereas others do not.

Most of the problems involved data points that formed a straight line, but Making Punch did not.
## Tipping in a Restaurant

The earnings of servers in restaurants are made up of a small hourly wage plus tips from customers. The general rule of thumb when dining in a restaurant is to tip the server 15% of the total bill. If you receive better than average service, a tip amounting to 20% of the bill is a common practice.

It is not specified whether students should use the general rule of thumb, whether they should base their tipping on above average service, or something in between. This ambiguity is in the problem purposefully. When students do their small group presentations, make sure to ask for a justification as to why students chose a certain tip amount.

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
<th>Tipping in a Restaurant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Based on the information above, how much should the tip be if the total bill is:</td>
<td>1. Based on the information above, how much should the tip be if the total bill is:</td>
</tr>
<tr>
<td>a. $10? (10 \times 0.15 = $1.50) tip</td>
<td>a. $10? (10 \times 0.15 = $1.50) tip</td>
</tr>
<tr>
<td>b. $75? (75 \times 0.15 = $11.25) tip</td>
<td>b. $75? (75 \times 0.15 = $11.25) tip</td>
</tr>
<tr>
<td>c. $120? (120 \times 0.15 = $18) tip</td>
<td>c. $120? (120 \times 0.15 = $18) tip</td>
</tr>
<tr>
<td>d. $450? (450 \times 0.15 = $67.50) tip</td>
<td>d. $450? (450 \times 0.15 = $67.50) tip</td>
</tr>
<tr>
<td>Describe how you found your answers in complete sentences.</td>
<td>Describe how you found your answers in complete sentences.</td>
</tr>
<tr>
<td>2. What was the total bill if the tip is:</td>
<td>2. What was the total bill if the tip is:</td>
</tr>
<tr>
<td>a. $3? (\frac{3}{0.15} = $20) is the total bill</td>
<td>a. $3? (\frac{3}{0.15} = $20) is the total bill</td>
</tr>
<tr>
<td>b. $1.75? (\frac{1.75}{0.15} = $11.67) is the total bill</td>
<td>b. $1.75? (\frac{1.75}{0.15} = $11.67) is the total bill</td>
</tr>
<tr>
<td>c. $3.75? (\frac{3.75}{0.15} = $25) is the total bill</td>
<td>c. $3.75? (\frac{3.75}{0.15} = $25) is the total bill</td>
</tr>
<tr>
<td>d. $15? (\frac{15}{0.15} = $100) is the total bill</td>
<td>d. $15? (\frac{15}{0.15} = $100) is the total bill</td>
</tr>
<tr>
<td>e. $22.50? (\frac{22.50}{0.15} = $150) is the total bill</td>
<td>e. $22.50? (\frac{22.50}{0.15} = $150) is the total bill</td>
</tr>
<tr>
<td>Describe how you found your answers in complete sentences.</td>
<td>Describe how you found your answers in complete sentences.</td>
</tr>
</tbody>
</table>

*Multiply the total bill by 0.15.*

*Divide the tip by 0.15 or 15%.*
### Tipping in a Restaurant

3. Create a table of values for this situation and include the expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Bill</td>
<td>Dollars</td>
<td>$B$</td>
</tr>
<tr>
<td>Tip</td>
<td>Dollars</td>
<td>$0.15B$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>0.15B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.50</td>
</tr>
<tr>
<td>75</td>
<td>11.25</td>
</tr>
<tr>
<td>120</td>
<td>18</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>150</td>
<td>22.5</td>
</tr>
</tbody>
</table>

4. Let $B$ represent the total bill. Write an algebraic equation for $T$, the tip.

$$T = 0.15B$$

a. What is the independent variable quantity in this problem?

The total bill is independent.

b. What is the dependent variable quantity in this problem?

The tip is dependent.

c. Is there a constant quantity in this problem? If so, what is it?

The constant is 0.15 or 15%.
5. Use the points from the equation and the data table to create a graph, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Bill</td>
<td>0</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>Tip</td>
<td>0</td>
<td>30</td>
<td>1.5</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between Total Bill and Tip]
### Tipping in a Restaurant

6. You and your fellow waiters and waitresses have been analyzing the tips that you have earned. To do the analysis, Yuri created an algebraic equation to model his earnings in tips. Jane used a table of values to keep track of her tips. Hal used a graph to display his earnings from tips by using the total bills of his customers. Currently, the three are arguing about whose representation is correct. You claim that they are all correct because all of the representations are really showing the same thing. Justify your claim.

*The equation comes from the description of the problem. We use the equation to place values in the table, and you use the values in the table to create the graph, so they are all related to the exact same problem situation. If we wanted, we could create the table directly from the problem situation and then use it to get the equation and graph.*
Earning Sales Commissions

Another situation where percentages are used is to calculate commissions. Often, salespeople are paid, at least in part, on commission. This means they receive a percentage of their total sales, in addition to any salary.

A person selling cars may receive a salary of $200 per week, plus a commission of 2% of her total sales.

Students may be confused about which numbers to multiply or divide. Discuss what is involved with total income.

Show all your computations for the following problems.

1. Based on the information above, how much will the salesperson receive in all if her total sales are:
   a. $10,000? $10,000 • 0.02 + 200 = $400
   b. $25,000? $25,000 • 0.02 + 200 = $700
   c. $100,000? $100,000 • 0.02 + 200 = $2,200
   d. $200,000? $200,000 • 0.02 + 200 = $4,200
   Describe how you found your answers. Use complete sentences.
   Multiply total sales by 0.02 and add 200.

2. How much was the salesperson’s total sales if her total pay is:
   a. $300? $300 – 200 = 100; 100 / 0.02 = $5,000
   b. $200? $200 – 200 = 0; 0 / 0.02 = $0
   c. $220? $220 – 200 = 20; 20 / 0.02 = $1,000
   d. $400? $400 – 200 = 200; 200 / 0.02 = $10,000
   e. $1,000? $1,000 – 200 = 800; 800 / 0.02 = $40,000
   Describe how you found your answers. Use complete sentences.
   Subtract the weekly salary from the pay to get the pay from commission. Divide the result by 0.02.
3. Let $S$ represent the total sales. Write an algebraic equation for $P$, the total pay.

$$P = 0.02S + 200$$

a. What is the independent variable quantity in this problem?

*Total sales is independent.*

b. What is the dependent variable quantity in this problem?

*Total pay is dependent.*

c. Is there a constant quantity in this problem? If so, what is it?

*There are 2 constants: 0.02 and 200.*

4. Create a table of values for this situation and include the expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Sales</th>
<th>Total Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Dollars</td>
<td>Dollars</td>
</tr>
<tr>
<td>Expression</td>
<td>$S$</td>
<td>$0.02S + 200$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>1,000</td>
<td>220</td>
</tr>
<tr>
<td></td>
<td>5,000</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>700</td>
</tr>
</tbody>
</table>
Earning Sales Commissions

5. Use the points from the data table and the equation to create a graph, making sure to label your axes appropriately and using an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Sales</td>
<td>0</td>
<td>30,000</td>
<td>1,500</td>
</tr>
<tr>
<td>Total Pay</td>
<td>0</td>
<td>1,000</td>
<td>50</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between Total Sales and Total Pay. The graph plots Total Sales (dollars) on the y-axis and Total Pay (dollars) on the x-axis. The line graph shows a linear relationship with points plotted at various sales levels.]

Total Sales (dollars)
Have students choose specific base salaries and rates of commission for each of three job options. Have students refer to the Plastic Containers activity for a general procedure to use when making comparisons. Have groups present all work and results.

This is another informal view of systems of linear equations.

### Earning Sales Commissions

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
</table>

6. You are considering a job in sales. One job opportunity gives you a lower base salary and a higher rate of commission on the products you sell. A second job offer entails a higher base salary and a lower rate of commission. A third job opportunity offers only a straight salary. Under which conditions will each job be the best? Which offer would you take? Why?

*Answers will vary, but students’ responses should include such considerations as the product being sold, its cost and the amount of commission earned from the product. For example, the first job offer may be the best if the rate of commission is rather high and the product has good sales potential. Conversely, the second job offer may be preferable in the case of a less-profitable commission rate and perhaps a product of lesser marketability. The third job, however, does not account for such considerations and may be the best when evaluated apart from dependence on product sales and commission.*
Unit 3: Modeling Situations Using Multiple Representations

Unit Objectives and Skills

In addition to the objectives and skills developed in the prior units, at the completion of this unit, students will be able to understand the relationship between input and output variables and to describe a linear relationship as a linear function.

Students will also be able to solve multi-step linear equations and to graph equations in all four quadrants.

Unit Overview

As many of the basic concepts of linear functions have already been introduced, this unit is designed to enhance students’ understanding of the role of multiple representations in setting up and solving functional relationships.

In this unit, students will have the opportunity to view a broad range of situations that can be modeled linearly. The goal, by the end of the unit, is for students to understand that a situation, in which the rate of change is defined as a constant, can always be modeled by a linear relationship and that that relationship can be represented by a linear function.

Students gain the understanding that underlying characteristics of a situation such as rate of change and dependency determine how the function is modeled rather than the context or story representing the situation.

Two important additional concepts are addressed with regard to the validity of solutions. Students are asked to make sense of answers with respect to the context of the problem, not just with respect to the algebra. In conjunction with this concept, students will also examine the relationship between the full domain and range and the domain and range that meets the constraints of the problem situation.

Lastly, in relation to moving between the abstract and concrete, students develop an understanding of the relationship between the y-intercept and the initial value.

Development of all the concepts in this unit are advanced or supported in software units: The Worksheet with Equation Solver Tool, the Equation Solver itself (more advanced) and Four Quadrant Graphing.
Unit Activities

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

<table>
<thead>
<tr>
<th>Printed Classroom Activities</th>
<th>Homework Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent a Car from Go-Go Car Rentals</td>
<td>pg. 47</td>
</tr>
<tr>
<td>Rent a Car from Wreckem Car Rentals</td>
<td>pg. 51</td>
</tr>
<tr>
<td>Rent a Car from Good Rents Car Rentals</td>
<td>pg. 55</td>
</tr>
<tr>
<td>Comparing Car Rental Rates</td>
<td>-</td>
</tr>
<tr>
<td>Move a Sand Pile</td>
<td>pg. 57</td>
</tr>
<tr>
<td>Engineering a Highway</td>
<td>pg. 59</td>
</tr>
<tr>
<td>Constructing Customized Rain Gutters</td>
<td>pg. 61</td>
</tr>
<tr>
<td>Functions</td>
<td>pg. 63</td>
</tr>
<tr>
<td>Solving and Graphing Inequalities</td>
<td>pgs. 65, 67</td>
</tr>
<tr>
<td>Widgets</td>
<td>pg. 73</td>
</tr>
<tr>
<td>Dumbbells</td>
<td>pg. 75</td>
</tr>
<tr>
<td>Computer Time</td>
<td>pg. 77</td>
</tr>
<tr>
<td>Dumpster</td>
<td>pg. 77</td>
</tr>
</tbody>
</table>

Suggested Materials

Sheets of 8 ½ x 11 inch paper

Suggestions for Classroom Implementation

Students will be working in a variety of software units, but many ties can be made since, at a concept development level, a basic set of concepts is covered in every text and software unit. These concepts include modeling through symbolic representation, graphing, equation solving and examining the relationship between verbal, numeric, graphic, and algebraic representations.

The problem scenarios in this unit offer a rich array of situations that can be represented by linear functions. Students work with a variety of numbers in interesting contexts. Furthermore, some of these problems offer opportunities for different interpretations and/or alternate solution strategies. It is at this point in the curriculum that students are introduced more formally to some of the mathematical concepts.

Students begin with the group of “Rent a Car” problems to facilitate the development of the concepts in the unit. These are designed specifically as group activities since students can compare similar scenarios with different input data. To work with these three problems, split the class into groups.
and have each group work one of the three “Rent a Car” problems and present its results. A comparison of the best deal provides a natural closing activity for a full group conversation or as a homework assignment. For the group discussion, graph the three functions together using the same scaling. In these problems, students work with slopes and initial values represented by decimals.

The concepts developed in the “Rent a Car” problems are extended in the Move a Sand Pile and Engineering a Highway problems. In particular, Move a Sand Pile introduces a large initial value with a negative slope, while in Engineering a Highway, students work with an equation involving a fractional slope and positive initial value. Take note that with Engineering a Highway the scenario is open to interpretation and solutions to the problem will vary, allowing for rather rich group discussions. Use the varied solutions to generate a classroom discussion on the role of interpretation and justification. “Who is right?” is an interesting question to get the discussion started.

From this point, you move to Constructing Customized Rain Gutters and Functions. Combined, these two activities help students to conceptualize and formalize the concept of independence and dependence, linear relationships and functions, along with domain and range. While students work in groups, you may want to weave in whole group instruction as the more formal concepts are presented.

Widgets, Dumbbells, Computer Time, and Dumpster present similar algebraic concepts. The emphasis in this set of problems is the use of the multiple representations in the presentation and solution of the problems. Stress that different representations provide a different view of the problem and information that can influence the solution path.

Unit Assessments

Stocking the Shelves
Integers and Solving Equations Form A
Integers and Solving Equations Form B
At the Driving Range
Unit 3: Modeling Situations Using Multiple Representations

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Rent a Car from Good Rents Car Rentals ....................................................... 3-9
Comparing Car Rental Rates ......................................................................... 3-12
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Functions ....................................................................................................... 3-27
Solving and Graphing Inequalities ................................................................. 3-35
Widgets .......................................................................................................... 3-46
Dumbbells ...................................................................................................... 3-50
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Dumpster ....................................................................................................... 3-57
Rent a Car from Go-Go Car Rentals

You plan to rent a car from Go-Go Car Rentals. Companies that rent cars generally charge a fixed amount per day with an additional charge for each mile driven. The model of car that you want to rent has a daily charge of $28 with an additional charge of $0.25 per mile.

You plan to rent a car from Go-Go Car Rentals. The problems in question 1 use decimal values for slope.

**Input**

1. How much will it cost to rent the car for one day if you drive it:
   a. 10 miles?
   
   **For one day and 10 miles, it will cost $30.50. (0.25*10 + 28 = 30.5)**
   
   b. 50 miles?
   
   **For one day and 50 miles, it will cost $40.50. (0.25*50 + 28 = 40.5)**
   
   c. 536 miles?
   
   **For 536 miles, it will cost $162 to rent the car. (0.25*536 + 28 = 162)**
   
   d. 2500 mile?
   
   **If the car is driven 2500 miles in a day, it will cost $653. This distance is unreasonable for a distance driven in one day. At an average speed of 60 miles per hour, it would take between 41 and 42 hours.**

Use complete sentences to describe how you determined the cost of the car rental.

**Process: Evaluate**

To determine the cost of the car rental, I multiplied the number of miles driven in one day by 0.25 and added 28 for the daily fee.
Rent a Car from Go-Go Car Rentals

2. How many miles did you drive the car if the agent charged:
   a. $30? The car was driven 8 miles, if $30 was charged.
      \[(30 - 28) / 25 = 8\]
   b. $45? If $45 was charged, the car was driven 68 miles.
      \[(48 - 28) / 25 = 68\]
   c. $67.50? The car was driven 158 miles.
      \[(67.5 - 28)/0.25 = 158\]
   d. $121.25? The car was driven 373 miles in one day.
      \[(121.25 - 28) / 25 = 373\]

Use complete sentences to describe how you determined the miles driven given the cost of the car rental. First, subtract 28 and divide the result by 0.25.

Rent a Car from Go-Go Car Rentals

3. Find an algebraic expression for this problem situation and complete the table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Expression</th>
<th>Distance</th>
<th>Total Cost of Rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>$</td>
<td>0.25M + 28</td>
<td>10</td>
<td>30.50</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td>50</td>
<td>40.50</td>
</tr>
<tr>
<td>536</td>
<td></td>
<td></td>
<td>536</td>
<td>162</td>
</tr>
<tr>
<td>2500</td>
<td></td>
<td></td>
<td>2500</td>
<td>653</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>68</td>
<td></td>
<td></td>
<td>68</td>
<td>45</td>
</tr>
<tr>
<td>373</td>
<td></td>
<td></td>
<td>373</td>
<td>121.25</td>
</tr>
<tr>
<td>158</td>
<td></td>
<td></td>
<td>158</td>
<td>67.50</td>
</tr>
<tr>
<td>374</td>
<td></td>
<td></td>
<td>374</td>
<td>121.50</td>
</tr>
</tbody>
</table>

Students may create their own values to be placed in the table.
Rent a Car from Go-Go Car Rentals

4. Plot the points from the data table, making sure to label your axes appropriately and using an appropriate scale for each axis. Draw the line.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Driven</td>
<td>0</td>
<td>2,500</td>
<td>125</td>
</tr>
<tr>
<td>Cost</td>
<td>0</td>
<td>1,000</td>
<td>50</td>
</tr>
</tbody>
</table>

If students have difficulty, have them solve for each distance separately, then compare.

Rent a Car from Go-Go Car Rentals

5. Suppose Go-Go offers you a special deal. The cost per day is $35, but you get 500 miles for free. If you are planning to drive 50 miles, is this deal better? How does it compare if you plan to drive 400 miles, 600 miles or 2500 miles? Explain in detail.

The special deal will be better as long as the car is driven at least 28 miles in a day. When the car is driven more than 28 miles, the cost of the regular deal exceeds $35.
Rent a Car from Wreckem Car Rentals

You plan to rent a car from Wreckem Car Rentals. Like many other companies, Wreckem Car Rentals charges a fixed amount per day with an additional charge for each mile driven. Wreckem, another rental company you called, has the same model as Go-Go. Wreckem charges $29.95 per day with an additional charge of $0.23 per mile.

1. How much will it cost to rent the car for one day if you drive it:
   a. 10 miles?  **It will cost $32.25 to rent this car and travel 10 miles in a day.**  \((0.23\times10 + 29.95)\)
   b. 50 miles?  **It will cost $41.45 to travel 50 miles in a day.**  \((0.23\times50 + 29.95)\)
   c. 536 miles?  **It will cost $153.23 to drive 536 miles in one day.**  \((0.23\times536 + 29.95)\)
   d. 2500 miles?  **The cost will be $604.95, but it is unreasonable to drive 2500 miles in a day.**

Use complete sentences to describe how you determined the cost of the car rental.

**The number of miles driven in a day was multiplied by the cost per mile, $0.23, and then the cost per day, $29.95, was added.**
Rent a Car from Wreckem Car Rentals

2. How many miles did you drive the car if the agent charged:
   
   a. $30.64? Three miles were driven if the charge was $30.64.
      \[\frac{(30.64 - 29.95)}{0.23}\]
   
   b. $33.17? The car was driven 14 miles if the charge was $33.17.
      \[\frac{(33.17 - 29.95)}{0.23}\]
   
   c. $46.28? If the charge for the day was $46.28, the car was driven 71 miles.
      \[\frac{(46.28 - 29.95)}{0.23}\]
   
   d. $83.77? The car was driven 234 miles in one day.

   Use complete sentences to describe how you determined the miles driven given the cost of the car rental.

   *First $29.95, the daily fee, was subtracted from the total amount charged. Then the remaining amount was divided by the fee per mile, $0.23, to determine the number of miles driven.*

Rent a Car from Wreckem Car Rentals

3. Find an algebraic expression for this problem situation and complete the table.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Total Cost of Rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>$0.23 M + 29.95</td>
</tr>
<tr>
<td>10</td>
<td>32.25</td>
</tr>
<tr>
<td>50</td>
<td>41.45</td>
</tr>
<tr>
<td>536</td>
<td>153.23</td>
</tr>
<tr>
<td>2500</td>
<td>604.95</td>
</tr>
<tr>
<td>3</td>
<td>30.64</td>
</tr>
<tr>
<td>14</td>
<td>33.17</td>
</tr>
<tr>
<td>100</td>
<td>52.95</td>
</tr>
<tr>
<td>71</td>
<td>46.28</td>
</tr>
<tr>
<td>234</td>
<td>83.77</td>
</tr>
</tbody>
</table>
Rent a Car from Wreckem Car Rentals

4. Generate a graph using the data table.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Driven</td>
<td>0</td>
<td>2,500</td>
<td>125</td>
</tr>
<tr>
<td>Cost</td>
<td>0</td>
<td>1,000</td>
<td>50</td>
</tr>
</tbody>
</table>

Rent a Car from Wreckem Car Rentals

5. Suppose Wreckem offers you a special deal. The cost per day is $32, but you get 250 miles for free. If you are planning to drive 50 miles, is this deal better? How does it compare if you plan to drive 400 miles, 600 miles or 2500 miles? Explain in detail.

The special deal is better in all of these instances at any distance greater than or equal to 9 miles. Even though the cost begins to rise after 250 miles, the rate at which it rises ($0.23 per mile) means that the price of the special deal will always remain below the regular deal.
Rent a Car from Good Rents Car Rentals

You have already looked at two car rental companies and now you are checking the rates at Good Rents Car Rentals. Good Rents also has the model you want and it rents for a daily charge of $21.65 with an additional charge of $0.27 per mile.

1. How much will it cost to rent the car for one day if you drive it:
   a. 10 miles? The cost will be $24.35, if the car is driven 10 miles in a day. (0.27*10 + 21.65)
   b. 50 miles? It will cost $35.15 to rent the car for the day if 50 miles are driven. (0.27*50 + 21.65)
   c. 536 miles? The cost of the rental is $166.37 for the day. (0.27*536 + 21.65)
   d. 2500 miles? The cost will be $696.65, but it is unreasonable to drive a car 2500 miles in one day.

Use complete sentences to describe how you determined the cost of the car rental.

The number of miles was multiplied by $0.27, which is the cost per mile. Then, the daily fee of $21.65 was added.
Rent a Car from Good Rents Car Rentals

2. How many miles did you drive the car if the agent charged:
   a. $29.75? The car was driven 30 miles if the bill was $29.75. 
      \[\frac{(29.75 - 21.65)}{0.27}\]
   b. $33.26? The car was driven 43 miles. 
      \[\frac{(33.26 - 21.65)}{0.27}\]
   c. $39.74? The car was driven 67 miles in a day, if the charge was 
      $39.74. \[\frac{(39.74 - 21.65)}{0.27}\]
   d. $51.62? The car was driven 111 miles in one day.

Use complete sentences to describe how you determined the miles driven given the cost of the car rental.

First the daily fee of $21.65 was subtracted from the total charge. Then, the result was divided by the fee per mile, which is $0.27, to get the number of miles driven.

Rent a Car from Good Rents Car Rentals

3. Find an algebraic expression for this problem situation and complete the table.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Total Cost of Rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>$</td>
</tr>
<tr>
<td>$0.27 M + 21.65</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>24.35</td>
</tr>
<tr>
<td>50</td>
<td>35.15</td>
</tr>
<tr>
<td>536</td>
<td>166.37</td>
</tr>
<tr>
<td>2500</td>
<td>696.65</td>
</tr>
<tr>
<td>30</td>
<td>29.75</td>
</tr>
<tr>
<td>43</td>
<td>33.26</td>
</tr>
<tr>
<td>100</td>
<td>48.65</td>
</tr>
<tr>
<td>67</td>
<td>39.74</td>
</tr>
<tr>
<td>111</td>
<td>51.62</td>
</tr>
</tbody>
</table>
Rent a Car from Good Rents Car Rentals

4. Create a graph using the data table.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles driven</td>
<td>0</td>
<td>2,500</td>
<td>125</td>
</tr>
<tr>
<td>Cost</td>
<td>0</td>
<td>1,000</td>
<td>50</td>
</tr>
</tbody>
</table>

Rent a Car from Good Rents Car Rentals

extension

5. Suppose Good Rents offers you a special deal. The cost per day is $28, but you get 150 miles for free. If you are planning to drive 50 miles, is this deal better? How does it compare if you plan to drive 400 miles, 600 miles or 2500 miles? Explain in detail.

The special deal is less expensive in all of these instances. If you drive 24 miles or more, the special deal is better. Even after 150 miles, the rate at which the total fee increases is the same as the regular deal. Therefore, the price of the special deal always remains lower.
Comparing Car Rental Rates

Compare the prices of the three car rental companies. Prepare a written analysis indicating which company will cost the least and when. Can you tell if the cost of the companies will ever be the same? If that is possible, when will it occur? If so, how did you figure it out?

**Students might mention the following in the comparison:**

- *Good Rents begins as the least expensive.*
- *Wreckem begins as the most expensive.*
- *Wreckem’s per mile fee is the least.*
- *Good Rents’ per mile fee is the most.*
- *Go-Go has the middle daily and per mile fee.*
- *Go-Go gets more expensive than Wreckem once the car is driven 98 miles or more. It is temporarily the most expensive rental company, and Wreckem becomes the company with the middle price.*
- *Good Rents passes Wreckem in price when the car has been driven 208 or more miles. This is when Wreckem becomes the least expensive. At this point, Go-Go is still the most expensive and Good Rents’ price is in the middle.*
- *Good Rents become the most expensive when it passes Go-Go. If the car is driven 318 miles or more, Good Rents is the most expensive, and Go-Go is again the middle-priced company, like it was at the start.*
**Move a Sand Pile**

You are a materials handler for a sand and concrete firm. There is an enormous pile of sand, estimated to be 2500 cubic feet (ft$^3$) that must be loaded onto a barge on the river. You have a bucket loader to transfer the sand to the barge. The bucket loader can pick up 5 cubic feet of sand in its bucket.

**input**

This is the first scenario that involves a negative rate of change.

It is expected that students will be able to use an algebraic method to find solutions. This skill has been developed through the use of the Equation Solver in the computer software.

**Process: Evaluate**

1. How big is the pile of sand after:
   - a. 50 buckets have been transferred to the barge? 
     The pile will have 2250 cubic feet of sand left.
   - b. 200 buckets have been transferred? 
     The pile will have 1500 ft$^3$ of sand left.
   - c. 400 buckets have been transferred? 
     There is 500 ft$^3$ left in the pile.
   - d. 600 buckets have been transferred? There will be -500 cubic feet of sand. (This is not a reasonable answer in terms of the problem situation since the maximum number of buckets that can be transferred is 500.)

**Process: Model**

2. Write a complete sentence describing how you found your answers in the questions above.
   
   *The number of buckets was multiplied by the amount of sand that was transferred per bucket, 5 cubic feet. This was the total transferred and was subtracted from the initial 2500 cubic feet of sand in the pile.*

3. Write an algebraic expression that represents this problem situation.

   \[2500 - 5B\] where \(B\) represents the number of buckets

   or \(S = 2500 - 5B\) (accept the equation even though only the expression was required) where \(S\) is the amount of sand.
Students may have difficulty doing these “backwards” problems because of negative numbers. Facilitate by asking how much sand was hauled away. Then ask students how many buckets it took to move that quantity of sand.

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
<th>Move a Sand Pile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4. How many buckets of sand did you put on the barge if the amount of sand remaining in the pile is:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>a. 2000 cubic feet?</td>
<td></td>
</tr>
<tr>
<td><strong>One hundred buckets were put on the barge.</strong> (2000 = 2500 –5B; -500 = -5B; 100 = B)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 1550 cubic feet?</td>
<td></td>
</tr>
</tbody>
</table>
| **The number of buckets transferred is 190.**  
There were 950 cubic feet of sand transferred, 950/5 = 190. |
|                   |                  |
| c. 100 cubic feet? |
| **Four hundred eighty buckets were transferred to the barge.** |
|                   |                  |
| d. 3000 cubic feet? |
| **The number of buckets put on the barge was -100.**  
(This seems illogical in the problem situation. See next page.) |
|                   |                  |
| 5. How many buckets of sand did you put on the barge if the pile is completely gone? |
|                   |                  |
| **If the pile is completely gone, 500 buckets had to be loaded onto the barge.** |
| 2500/5 = 500      |

<table>
<thead>
<tr>
<th>Process: Summary</th>
<th>Move a Sand Pile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Write a complete sentence describing how you found your answers in the questions above.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>To calculate the number of buckets transferred, subtract the amount of sand remaining from the original amount of sand (2500 cubic feet). This will be the total amount of sand that was transferred. Divide this number by 5, which is the amount transferred in one bucket.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Begin to discuss the difference between the algebraic representation and the problem situation.
In question 4d for example, there is a valid algebraic solution, but it does not make sense in the problem situation. This difference will be reinforced through this unit. A formal discussion of domain and range will take place on pages 3-31 and 3-33.


No, the answer did not make sense in the problem situation since the quantity exceeds the initial amount of sand.

8. Use the information from questions 1-5 above to complete this table:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Sand Moved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
</tr>
<tr>
<td>Expression</td>
<td></td>
</tr>
<tr>
<td><strong>Buckets</strong></td>
<td><strong>Cubic feet</strong></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td><strong>2500 – 2B</strong></td>
</tr>
<tr>
<td>50</td>
<td>2250</td>
</tr>
<tr>
<td>200</td>
<td>1500</td>
</tr>
<tr>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>600</td>
<td>-500</td>
</tr>
<tr>
<td>100</td>
<td>2000</td>
</tr>
<tr>
<td>190</td>
<td>1550</td>
</tr>
<tr>
<td>480</td>
<td>100</td>
</tr>
<tr>
<td>-100</td>
<td>3000</td>
</tr>
<tr>
<td>500</td>
<td>2000</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>2125</td>
</tr>
<tr>
<td>37</td>
<td>2315</td>
</tr>
<tr>
<td>150</td>
<td>1750</td>
</tr>
<tr>
<td>349</td>
<td>755</td>
</tr>
</tbody>
</table>
You may want students to graph all the data points in the table, or you can choose a more restricted range and domain. Thus, for this graph, you may want the students to place the axes on the grid, so the origin is not necessarily at the lower left-hand corner. Depending on the domain and range chosen, they may need to include the 2nd and 4th quadrants.

### Move a Sand Pile

9. Generate a graph using the data table.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Buckets</td>
<td>0</td>
<td>600</td>
<td>30</td>
</tr>
<tr>
<td>Amount of Sand Remaining in Pile</td>
<td>0</td>
<td>3000</td>
<td>150</td>
</tr>
</tbody>
</table>

![Graph of data table](image)
For additional practice, have students use an algebraic method to check their answers in questions 10 and 11.

### Move a Sand Pile

10. Use your graph to answer the following questions. How many buckets of sand did you transfer if the pile is approximately:
   - a. 1000 cubic feet? **The amount of sand transferred is 300 buckets.**
   - b. 500 cubic feet? **The amount of sand moved is 400 buckets.**
   - c. 2200 cubic feet? **Sixty buckets of sand were moved.**

11. Use your graph to answer the following questions. How large will the pile be after you transfer:
   - a. 100 buckets? **The pile will contain 2000 ft³ of sand.**
   - b. 250 buckets? **The pile will contain 1250 cubic feet of sand.**
   - c. 350 buckets? **The pile will have 750 ft³ of sand in it.**

For question 12, prompt the students by asking them how to use the graph to get values to be entered into a table.

Students should already have a good understanding of how to generate the symbolic representation and a graph when presented with numerical data. Throughout this unit, continue to develop the relationship between the three representations and the ability to generate the additional representations when presented with one.

### Move a Sand Pile

12. If you were presented with the graph of a situation, how could you go about generating a table and symbolic representation? Explain in detail.

**Sample student response:**

First, I would set up a table with the number of buckets in the left-hand column and the amount of sand in the right-hand column. To fill in the column with the number of buckets, I would read across the bottom to see if there was a point. Then I would look across to the Amount of Sand axis to see what value matched. I would write the values in the columns in the table. Once the values were in the table, I would look to see if there was a pattern. If I found a pattern, I would try to write it algebraically using an expression.
This is the first activity where the students are asked to generate the algebraic equation directly from the verbal representation. Not all students will be able to make this jump. Do not discourage students from generating a few specific values until they see the pattern, since this method is supported in the software.

### Engineering a Highway

You are a civil engineer who has been hired to oversee a large interstate-highway construction project connecting Pittsburgh with Cincinnati. Construction began several months ago, and approximately 87 miles of highway have been completed. The total length of this highway will be 267 miles. At present, you estimate that new construction can proceed at the rate of one-fifth of a mile per day.

### Students move from decimals to fractions.

Students may multiply by the fraction 1/5, divide by 5, or find the decimal equivalent of 0.2 to create the algebraic equation.

### process: model

1. Write an algebraic equation that models this situation.

   \[ M = \frac{1}{5} D + 87 \]

   where \( M \) represents the total miles completed and \( D \) represents the number of days worked, starting today.

   **Student interpretations of the problem situation will vary. Some students may use**

   - \( M = (1/5) D \) and remember that they only have 267 – 87 = 180 miles to complete.

     or

   - \( M = \frac{1}{5} (435 + D) \) since another interpretation is to account for the days already worked.
One set of possible answers is provided here. Answers to these questions may vary depending on students’ interpretation of the problem and the model used. Therefore, ask for an explanation from each group as they present their solutions.

Students may interpret questions 2 and 3 such that they do not account for the miles already completed. If that is the case, make sure students are consistent with respect to the interpretation used.

Students may have difficulty dividing by 1/5 in questions 3 through 5. You may need to guide them towards the fact that dividing by 1/5 is the same as multiplying by 5.

---

### Engineering a Highway

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.</strong> How many miles of the highway will be completed in:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a. 10 days?</td>
</tr>
<tr>
<td></td>
<td><em>In 10 days, 89 miles of highway will be completed.</em></td>
</tr>
<tr>
<td></td>
<td>b. 50 days?</td>
</tr>
<tr>
<td></td>
<td><em>In 50 days, 97 miles of highway will be completed.</em></td>
</tr>
<tr>
<td></td>
<td>c. 1 year?</td>
</tr>
<tr>
<td></td>
<td><em>In 365 days, 160 miles of highways will be completed.</em></td>
</tr>
<tr>
<td></td>
<td>d. 2 years?</td>
</tr>
<tr>
<td></td>
<td><em>In two years (730 days), 233 miles of highway will be completed.</em></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>In a complete sentence, describe how you found these answers.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>The number of days was multiplied by the amount completed in each day (one-fifth mile). Then, the amount already completed (87 miles) was added.</em></td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.</strong> How many days will it take to complete:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a. 100 miles of highway?</td>
</tr>
<tr>
<td></td>
<td><em>It will take 65 days from now to complete 100 miles.</em></td>
</tr>
<tr>
<td></td>
<td>b. 150 miles of highway?</td>
</tr>
<tr>
<td></td>
<td><em>To complete 150 miles, it will take 315 days.</em></td>
</tr>
<tr>
<td></td>
<td>c. the entire highway?</td>
</tr>
<tr>
<td></td>
<td><em>The entire highway will be completed in 900 days from now.</em></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>In a complete sentence, describe how you found these answers.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Eighty-seven was subtracted from the number of miles to get the number of miles that still needs to be completed. This number was multiplied by 5 (divided by 1/5) to get the number of days needed.</em></td>
</tr>
</tbody>
</table>
This is the first instance where students see "negative" time interpreted as being in the past.

In order to extend the concept, ask how many days from now it will take to finish the project and how many total days were spent working on the road. No matter what the students' interpretation of the problem situation was, the responses to these two questions should be the same, as long as the project is being completed at the same rate as it was begun. It will take 900 days from now to finish the remaining 180 miles of road at the rate of 1/5 mile per day. The total time to complete the entire 267-mile roadway is 1335 days (or 3 years and 8 months), as long as the entire road was built at 1/5 mile per day.

4. How many days will it take to complete 50 miles?

   It will take \(-185\) days to complete 50 miles. Fifty miles have already been completed.
   
   a. Does this answer make sense? Why or why not?
   Students may say the negative number does not make sense. They may say that the negative number should be interpreted as being in the past.

   b. What is the meaning of this answer within the problem situation?
   Within the problem situation, \(-185\) days means that 50 miles of highway was completed 185 days ago.

5. How many days ago was this project started? How do you know?

   As long as the rate of work was the same in the past, then the project was started 435 days ago. This is true because \(87 \times 5 = 435\), and 87 miles are already completed.

6. Use the information from questions 1 – 5 to complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Miles of Highway</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Days</td>
<td>Miles</td>
</tr>
<tr>
<td></td>
<td>(D)</td>
<td>((1/5)D + 87)</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>89</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>97</td>
</tr>
<tr>
<td>365</td>
<td></td>
<td>160</td>
</tr>
<tr>
<td>65</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>(-185)</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>
Engineering a Highway

7. Use the information from the table to construct a graph, with the number of miles of highway on the vertical axis and the number of days on the horizontal axis. Make sure that all information from the table is displayed on the graph.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Days</td>
<td>-200</td>
<td>400</td>
<td>30</td>
</tr>
<tr>
<td>Miles of Highway</td>
<td>0</td>
<td>200</td>
<td>10</td>
</tr>
</tbody>
</table>

8. Use your graph to estimate the answers to the following questions.
   a. When were 30 miles of highway completed? **285 days ago**
   b. When were 60 miles completed? **135 days ago**
   c. When will 250 miles of highway be completed? **815 days from now**
   d. When will 130 miles be completed? **215 days from now**
   e. How many miles of highway were completed 50 days ago? **77 miles**
   f. How many miles were completed 100 days ago? **67 miles**
   g. How many miles of highway will be completed in 100 days? **107 miles**
   h. How many miles will be completed in 200 days? **127 miles**
### Engineering a Highway

<table>
<thead>
<tr>
<th>For questions 9 through 11, you may choose to have each group answer all three questions or you may assign one question per group. Debrief with student presentations so students have the opportunity to see different answers and interpretations. To further extend question 9, have different groups construct different models to describe the project’s status. Focus the discussion on the impact of changes to the model on the project schedule. Students need to be able to put their thought processes into writing. Analysis questions like these will help students understand how the situation is applied outside the classroom.</th>
</tr>
</thead>
</table>

#### 9. Some of the other engineers who are working on this project also responded to the questions and several had different responses than you. Why did that occur? Explain.

Possible reasons for different answers include a different interpretation of the problem, which does not necessarily account for the amount of highway already completed in the same manner. Other reasons for different responses include human errors or perhaps the use of a different estimate on the project’s rate of completion.

#### 10. You need to present a 15-minute overview of the project to date, and plans for completion. The project manager is concerned that this project may not be on schedule and you have to convince him everything is under control.

**Student responses may include:**
- Starting date of the project as well as projected completion date.
- The rate at which the project is being completed and any variations to the rate from the past or expected variations in the future.
- The amount of highway completed and the amount yet to be completed.
- Perhaps the number of workers and a budget summary is also included, but that was not a part of this scenario.

#### 11. There is a new employee whom you are responsible for training. You must prepare some written instructions so that this person understands the project. Given that sometimes people interpret information differently, you want to make sure you are very clear about the start dates and the number of miles completed.

**Students should provide a starting date for the project that is 435 days in the past (from today) and an ending date that is 900 days in the future. Students should state that 87 miles of the 267-mile highway are completed, so that 180 miles are yet to be completed. The rate of completion is one-fifth mile per day, so that one mile is completed every five working days. Students may also chose to provide the new employee with a summary of work responsibilities, but that is outside the scope of the problem.**
Demonstrate the construction of one possible rain gutter to the entire class to ensure proper visualization. It is sufficient to use a piece of paper to construct the gutter. Then give the students an 8 ½ by 11 sheet of paper and have them create their own gutters of different bottom widths and side lengths.

This activity is useful in the section on Functions to introduce domain and range.

Constructing Customized Rain Gutters

A general contractor has asked you for some help. The contractor is making customized rain gutters for a house. To form the gutters, he uses long rectangular sheets of metal and bends the sides up. If you view a gutter from the end, it looks like this:

Using sheet metal that is 8.5 inches wide, help the contractor determine the relationship between the side length and bottom width. In addition, the contractor needs to know all of the possible side-length and bottom-width measurements that can be used to construct these gutters.

1. Using sheets of paper 8.5 inches wide, follow the procedure described above to construct 5 different gutters. Make some gutters short and wide; make others tall and narrow. In the table below, record the side length and bottom width measurements of each gutter you construct.

<table>
<thead>
<tr>
<th>Gutter</th>
<th>Side Length</th>
<th>Bottom Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gutter 1</td>
<td>1</td>
<td>6.5</td>
</tr>
<tr>
<td>Gutter 2</td>
<td>2</td>
<td>4.5</td>
</tr>
<tr>
<td>Gutter 3</td>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>Gutter 4</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>Gutter 5</td>
<td>0.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Students should begin to see a pattern after the first few rows of the table.

### Constructing Customized Rain Gutters

2. Complete the table below. If necessary, construct models of the gutters.

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Bottom Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>inches</td>
</tr>
<tr>
<td>0</td>
<td>8.5</td>
</tr>
<tr>
<td>0.25</td>
<td>8</td>
</tr>
<tr>
<td>0.5</td>
<td>7.5</td>
</tr>
<tr>
<td>0.75</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>6.5</td>
</tr>
<tr>
<td>1.25</td>
<td>6</td>
</tr>
<tr>
<td>1.5</td>
<td>5.5</td>
</tr>
<tr>
<td>1.75</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Bottom Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>inches</td>
</tr>
<tr>
<td>2.25</td>
<td>4</td>
</tr>
<tr>
<td>2.5</td>
<td>3.5</td>
</tr>
<tr>
<td>2.75</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>3.25</td>
<td>2</td>
</tr>
<tr>
<td>3.5</td>
<td>1.5</td>
</tr>
<tr>
<td>3.75</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>4.25</td>
<td>0</td>
</tr>
</tbody>
</table>

You may tend to focus on three representations: numerical, graphical, and symbolic. However, as demonstrated in question 3, it is also important for students to be able to generate a verbal representation. This is sometimes forgotten since the students are often given the verbal representation in the form of a problem scenario.

### Constructing Customized Rain Gutters

3. Based on the data in the table above, describe the relationship between the side length and bottom width. Explain your answer.

*The bottom width is 8.5 inches minus twice the side length because the gutter has two sides.*
### Constructing Customized Rain Gutters

Answer the following questions. As you do so, think about whether these results support your previous analysis.

4. As the side length values increase by a quarter inch, how do the bottom width values change?
   
   \textbf{The bottom width decreases by \( \frac{1}{2} \) inch for each quarter inch that the side length increases.}

5. As the side length values increase by one-half inch, how do the bottom width values change?
   
   \textbf{The bottom width decreases by 1 inch for every half inch that the side length increases.}

6. As the side length values increase by one inch, how do the bottom width values change?
   
   \textbf{The bottom width decreases by 2 inches for every 1 inch that the side length increases.}

7. Write an algebraic expression to represent how the change in side length affects the change in the bottom width.

   \( 8.5 - 2S \) where \( S \) represents the side length or

   \( W = 8.5 - 2S \) if students write an equation instead of an expression.

   \( W \) represents the bottom width.

When writing an algebraic expression or equation, continue to reinforce the need to include a written definition of the variables with units.
Constructing Customized Rain Gutters

8. For a side length of \(x\) inches, give a verbal description of the numerical process used to determine the bottom width measurement.

   *First double \(x\). Then, subtract 2\(x\) from 8.5 inches to get the bottom width.*

9. Let \(x\) express the side length. Write an algebraic equation for \(y\), the bottom width.

   \[ y = 8.5 - 2x \]
This problem is the students' formal introduction to functions. Functions will play a crucial role throughout the algebra curriculum. Students will be introduced to several families of functions and begin to understand that even though specific families of functions have certain properties, an underlying commonality exists among all functions. This will be made evident when quadratics are introduced in Unit 6.

It may take some time for students to understand and feel comfortable using function notation.

### Functions

In the problem on Constructing Customized Rain Gutters, the relationship between the side length and the bottom width can be described as a function.

A **function** describes a unique relationship between the input values (independent variables) and the output values (dependent variables), so that for every input value, there is exactly one output value. A special notation called **function notation** or **"f of x" notation** is used to write a functional relationship.

**Function notation** replaces the dependent variable with the expression \( f(x) \) and so we write

\[ f(x) = \text{an algebraic expression} \]

For example, if the algebraic equation was written as

\[ y = 2x + 1 \]

it would become

\[ f(x) = 2x + 1 \]

This expression is read "\( f \) of \( x \)" or "the value of the function \( f \) at \( x \)." It means that \( f \) is a function of the variable \( x \).

Note: The expression \( f(x) \) does not mean that \( f \) is multiplied by \( x \). The letter \( f \) as it is used in this notation is not a variable; it is merely a name for a function.

### Functions

1. Using the information in the Constructing Customized Rain Gutters problem, write the relationship between the side length and bottom width as a function. Use the letter \( f \) to represent the bottom-width function and the variable \( x \) to represent the side length.

\[ f(x) = 8.5 - 2x \]
2. \( y = f(x) = 8.5 - 2x \)

a. Find \( f(0.5) \).

The question being asked is “What is the bottom width when the side length is 0.5 inches?” In general terms, the question is “What is the output of the function when the input is 0.5?”

To find the answer, replace \( x \) in the function with the value 0.5, and perform the arithmetic:

\[
f(0.5) = 8.5 - 2(0.5) = 8.5 - 1 = 7.5
\]

Answer: The bottom width is 7.5 inches when the side length is 0.5 inches.

b. Find \( x \) when \( f(x) = 6.5 \).

The question being asked is “What is the side length when the bottom width is 6.5 inches?” In general terms, the question is “What is the input of the function when the output is 6.5?”

To find the answer, replace \( f(x) \) with 6.5, and solve the resulting equation for \( x \):

\[
6.5 = 8.5 - 2x
\]

\[
6.5 - 8.5 = -2x
\]

\[
-2 = -2x
\]

\[
\frac{-2}{-2} = x
\]

\[
x = 1
\]

Answer: The side length is 1 inch when the bottom width is 6.5 inches.
In problem 3b, if functional notation is hindering students’ ability to interpret the problem and to set it up, refer them to example 2b on the previous page.

Problems 3c and 3d continue to reinforce the distinction between the algebraic representation and the problem situation.

<table>
<thead>
<tr>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. ( f(x) = 8.5 - 2x )</td>
</tr>
<tr>
<td>In each of the following, state what question is being asked and find the numerical answer.</td>
</tr>
<tr>
<td>a. Find ( f(1.75) ).</td>
</tr>
<tr>
<td><strong>What is the bottom width, if the side length is 1.75 inches?</strong></td>
</tr>
<tr>
<td>( f(1.75) = 8.5 - 2 \times 1.75 )</td>
</tr>
<tr>
<td>( f(1.75) = 5 )</td>
</tr>
<tr>
<td><strong>If the side length is 1.75 inches, the bottom width is 5 inches.</strong></td>
</tr>
<tr>
<td>b. Find ( x ) when ( f(x) = 2.5 ).</td>
</tr>
<tr>
<td><strong>What is the side length, if the bottom width is 2.5 inches?</strong></td>
</tr>
<tr>
<td>( 2.5 = 8.5 - 2x )</td>
</tr>
<tr>
<td>(-6 = -2x )</td>
</tr>
<tr>
<td>( 3 = x )</td>
</tr>
<tr>
<td><strong>If the bottom width is 2.5 inches, the side length is 3 inches.</strong></td>
</tr>
<tr>
<td>c. Find ( f(5) ). Does the numerical value make sense for the Constructing Customized Rain Gutters scenario? Explain.</td>
</tr>
<tr>
<td><strong>If the side length is 5 inches, what is the bottom width?</strong></td>
</tr>
<tr>
<td>( f(5) = 8.5 - 2 \times 5 )</td>
</tr>
<tr>
<td>( f(5) = -1.5 )</td>
</tr>
<tr>
<td>This numerical answer does not make sense because the gutter cannot have a bottom width that is a negative number.</td>
</tr>
<tr>
<td>d. Find ( x ) when ( f(x) = 10 ). Does the numerical value make sense for the Constructing Customized Rain Gutters scenario? Explain.</td>
</tr>
<tr>
<td><strong>If the bottom width is 10 inches, find the side length.</strong></td>
</tr>
<tr>
<td>( 10 = 8.5 - 2x )</td>
</tr>
<tr>
<td>( 1.5 = -2x )</td>
</tr>
<tr>
<td>(-0.75 = x )</td>
</tr>
<tr>
<td><em>This numerical answer does not make sense because the side length cannot have a negative value, and because the bottom width has a maximum value of 8.5 inches, while this bottom width starts at 10 inches.</em></td>
</tr>
</tbody>
</table>
Questions 4 and 5 are intended to drive the definition of domain as the set of all possible input values.

### Functions

4. In order to form a gutter, the side length must be less than a certain number. What is that number and how is it related to the total width of available sheet metal, 8.5 inches? Write your answer in a complete sentence. (Fill in the table below, if you need help.)

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Bottom Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>inches</td>
</tr>
<tr>
<td>4.22</td>
<td>0.06</td>
</tr>
<tr>
<td>4.23</td>
<td>0.04</td>
</tr>
<tr>
<td>4.24</td>
<td>0.02</td>
</tr>
<tr>
<td>4.25</td>
<td>0</td>
</tr>
<tr>
<td>4.26</td>
<td>-0.02</td>
</tr>
<tr>
<td>4.27</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

The side length must be less than 4.25 inches, which is half of the total width available.

5. In order to form a gutter, the side length must be greater than a certain number. What is that number? Explain why this number makes sense. Write your answer in a complete sentence. (Fill in the table below, if you need help.)

The side length must be greater than 0, because you must have a side to make a gutter.

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Bottom Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>inches</td>
</tr>
<tr>
<td>0.4</td>
<td>7.7</td>
</tr>
<tr>
<td>0.3</td>
<td>7.9</td>
</tr>
<tr>
<td>0.2</td>
<td>8.1</td>
</tr>
<tr>
<td>0.1</td>
<td>8.3</td>
</tr>
<tr>
<td>0</td>
<td>8.5</td>
</tr>
<tr>
<td>-0.1</td>
<td>8.7</td>
</tr>
</tbody>
</table>
The inequality $2x < 8.5$ can be used to describe one restriction on side length. Solving this inequality yields one of the inequalities in question 8.

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functions</td>
</tr>
<tr>
<td>6. In terms of the construction process, explain why it is impossible to form a gutter with a side length of 5 inches.</td>
</tr>
</tbody>
</table>

* A side length of 5 inches would require a sheet of metal that is more than 10 inches in length. However, the length of the entire sheet of metal available to form the gutter in this problem is only 8.5 inches.*

Domain and range are now formally introduced.

<table>
<thead>
<tr>
<th>input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functions</td>
</tr>
<tr>
<td>The values that you have considered are called the domain. The <strong>domain</strong> of a function is the set of all possible input values.</td>
</tr>
</tbody>
</table>

The domain is often expressed using an *algebraic inequality*. You already are familiar with algebraic equations like $2x = 10$, where there is one value for $x$, 5, that makes the equation true. For an algebraic inequality, like $2x \leq 10$, the solution will be any value of $x$ that is less than or equal to 5, expressed as $x \leq 5$.

What does the inequality $x > 7$ mean? *The solution will be any value of $x$ that is greater than 7.*
Students are introduced to inequalities and how they are used in problem situations.

Be sure to clearly make the distinction between > and ≥, as well as between < and ≤.

If students have trouble with the algebraic inequalities, take some time to review numeric inequalities and inequality symbols.

### Functions

7. For each inequality, write a description of the numbers that satisfy the inequality.
   a. \( x > 5 \)
      
      *All numbers greater than 5, but not equal to 5 satisfy this inequality.*
   
   b. \( y \leq -1 \)
      
      *This inequality is satisfied by all numbers less than or equal to –1.*
   
   c. \( x \neq 7 \)
      
      *Any number not equal to 7 satisfies this inequality. This might include numbers greater than or less than 7.*
   
   d. \( A \geq -2 \)
      
      *Any number greater than or equal to –2 will satisfy this inequality.*
   
   e. \( 8 \leq G \)
      
      *Any number greater than or equal to 8 will satisfy this inequality.*

---

### Functions

8. Using the variable \( x \) to represent side-length values, write a pair of inequalities that expresses all possible side lengths that can be used to construct a gutter.

\[
X > 0 \text{ inches} \\
X < 4.25 \text{ inches}
\]
Questions 9 and 10 are intended to drive the definition of range as the set of all output values. Have students refer back to the tables, if needed.

### Functions

<table>
<thead>
<tr>
<th>process: evaluate</th>
</tr>
</thead>
</table>
| 9. In order for a gutter to be formed, the bottom width will be less than a certain number. What is that number?  
*The bottom width must be less than 8.5 inches.* |
| 10. In order for a gutter to be formed, the bottom width must be greater than a certain number. What is that number?  
*The bottom width must be greater than 0 inches.* |

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
</table>
| 11. In terms of the construction process, explain why is it impossible to form a gutter with a bottom width of 10 inches.  
*A bottom width of 10 inches requires a larger piece of metal than is available for these gutters. Only metal pieces of 8.5 inches are available.* |
Note that domain and range are only being presented in terms of the problem situation. Point out that from a purely algebraic view, all linear functions have the set of all real numbers as the domain and range.

### Functions

The values that you have considered belong to the range. The **range** of a function is the set of all possible output values.

In the problem about rain gutters, when you found

\[ x \text{ when } f(x) = 10 \]

you found the side length that corresponded to a bottom width of 10 inches. Since this side length was a negative number, a value of 10 inches was not a possible output value. A value of 10 inches was not in the range of the bottom-width function.

Like the domain, the range is often expressed using an algebraic inequality.

<table>
<thead>
<tr>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Using the variable ( y ) to represent bottom-width values, write a pair of inequalities that expresses all possible bottom-width values.</td>
</tr>
<tr>
<td>( y &gt; 0 )</td>
</tr>
<tr>
<td>( y &lt; 8.5 )</td>
</tr>
</tbody>
</table>
Students are introduced to solving inequalities. You may want to generate an example in which the students solve an equation and then change the situation such that the solution represents an upper or lower bound. Discuss what that might mean in terms of a solution to the problem, i.e., will there be one value or many possible values?

---

### Solving and Graphing Inequalities

Solving a linear inequality is similar to solving a linear equation. You want to find the value or values that satisfy the statement. In the case of a linear inequality, a range of values will work, while for a linear equation, there is one value of y for a particular value of x.

**Example 1:**

a. Solve the equation \( x - 9 = y \) given the value of \( y = 5 \) and graph the solution.

\[
egin{align*}
  x - 9 &= y \\
  x - 9 &= 5 \\
  x - 9 + 9 &= 5 + 9 \\
  x &= 14
\end{align*}
\]

So the point (14, 5) is on the line \( x - 9 = y \) and is a solution to the equation.

b. Substitute the value for \( x \) from part a above into the original equation to verify that the value is a correct solution.

\[
(14) - 9 = 5
\]

\( 5 = 5 \)

**Example 2:**

a. Solve the inequality \( x + 9 < y \) given the value of \( y = 15 \).

\[
egin{align*}
  x + 9 &< 15 \\
  x + 9 - 9 &< 15 - 9 \\
  x &< 6
\end{align*}
\]

To graph the solution since we are working in one dimension, we use a number line.

b. Substitute any value for \( x \) that satisfies the final inequality in part a above into the original inequality to verify that the value is one of the correct solutions.

There are many correct answers. For example, 0 satisfies the inequality \( x < 6 \), since \( 0 < 6 \).

Substitute 0 for \( x \) in the original inequality:

\[
(0) + 9 < 15
\]

\( 9 < 15 \)

The inequality \( 9 < 15 \) is true, so 0 is one of the correct solutions.

The solution \( x < 6 \) means any value less than, not including 6.
Solving and Graphing Inequalities

Study the following examples of inequalities graphed on a number line:

Example 3:
\[ x > 2 \]

The graph of \( x > 2 \) includes all values greater than and not including 2.

Example 4:
\[ x \leq 2 \]

The graph of \( x \leq 2 \) includes all values less than or equal to 2.

Here are three other examples of inequalities graphed on a number line. Examine them and describe, in words, what is being represented.

Example 5:
\[ x < -6 \text{ or } x > 3 \]

The graph of \( x < -6 \) or \( x > 3 \) includes all values less than \(-6\) and greater than \(3\) and no values between \(-3\) and \(3\).

Example 6:
\[ x \leq -1 \text{ or } x > 3 \]

The graph of \( x \leq -1 \) or \( x > 3 \) includes all values less than or equal to \(-1\) and greater than \(3\) and no values between \(-.99...\) and \(3\).

Example 7:
\[ x \geq -1 \text{ and } x \leq 3 \]

The graph of \( x \geq -1 \) and \( x \leq 3 \) includes all values between \(-1\) and \(3\).
Solving and Graphing Inequalities

Graph each inequality on a number line, as in the examples on the previous page.

1. \( x \geq 3 \)

2. \( x < -4 \)

3. \( x \leq 0 \) or \( x > 5 \)

4. \( x > -5 \) and \( x < 1 \)

5. \( x < -4 \) and \( x > 2 \)

There are no points on the graph, because no numbers satisfy the inequality.
You may want to review the answers as a class. Some students may not realize why their results do not make sense in question 8.

<table>
<thead>
<tr>
<th>Solving and Graphing Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Show your work in each part of this problem.</td>
</tr>
<tr>
<td>a. Solve (-7x = 16).</td>
</tr>
<tr>
<td>(x = \frac{-16}{7})</td>
</tr>
<tr>
<td>b. Substitute the value for (x) from part a above into the original equation to verify that the value is a correct solution.</td>
</tr>
<tr>
<td>(-7 \left(\frac{-16}{7}\right) = 16)</td>
</tr>
<tr>
<td>7. Show your work in each part of this problem.</td>
</tr>
<tr>
<td>a. Solve (3x \leq 8).</td>
</tr>
<tr>
<td>(x \leq \frac{8}{3}) after dividing both sides of the inequality by 3.</td>
</tr>
<tr>
<td>(3(2) \leq 8) 6 (\leq 8) Two is a solution.</td>
</tr>
<tr>
<td>b. Substitute any value for (x) that satisfies the final inequality you wrote in part a above into the original inequality to verify that the value is one of the correct solutions.</td>
</tr>
<tr>
<td>c. Graph the solution.</td>
</tr>
<tr>
<td>8. Show your work in each part of this problem.</td>
</tr>
<tr>
<td>a. Solve (-7x &gt; 3).</td>
</tr>
<tr>
<td>(x &lt; -\frac{3}{7}) is the solution, but it is expected that students will mistakenly write (x &gt; -\frac{3}{7}) when dividing both sides of the inequality by (-7).</td>
</tr>
<tr>
<td>b. Substitute the value for (x) that satisfies the final inequality you wrote in part a above into the original inequality to verify that the value is one of the correct solutions.</td>
</tr>
<tr>
<td>If students make the typical error cited above, they may chose 0 as a number greater than (-\frac{3}{7}). When tested in the original inequality, they get 0 (\geq 3), which is not a true statement.</td>
</tr>
<tr>
<td>c. Were you surprised by your result in part b? Consider, for example, the fact that 6 is greater than 3. If you change the signs of both numbers, is –6 greater than –3?</td>
</tr>
<tr>
<td>If students make the error cited above, they may be surprised in b. -6 is less than -3.</td>
</tr>
<tr>
<td>d. If the signs on both sides of an inequality change, what other change needs to occur based on your answer to c?</td>
</tr>
<tr>
<td>The direction of the inequality will also have to change.</td>
</tr>
<tr>
<td>e. Graph the solution.</td>
</tr>
</tbody>
</table>
Changing the signs of both sides of an equation keeps it in balance. However, changing the signs on both sides of an inequality – by multiplying or dividing by a negative number – reverses the direction of their relationship. For that reason, we have to change the direction of the inequality symbol.

Encourage students to substitute several values in the original inequality to verify their solution, as demonstrated on the previous page.

9. Solve the following inequalities using algebraic techniques and graph the solution. Show your work.

   a. \( \frac{x}{-9} \geq 5 \)

   After multiplying both sides by \(-9\), \( x \leq -45 \)

   b. \( 5x > -12 \)

   After dividing both sides by \(5\), \( x > -\frac{12}{5} \) or \(-2.4\).

   c. \( 5x + 3 \leq 23 \)

   \( 5x \leq 20 \)

   \( x \leq 4 \)
### Solving and Graphing Inequalities

<table>
<thead>
<tr>
<th>analysis</th>
<th>10. How is the solution for a linear equation different from the solution for a linear inequality?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The solution for a linear equation is a single point, where the solution for a linear inequality is an infinite number of points.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>process: evaluate</th>
<th>11. Given the inequality $y &lt; 2x$, determine if the following ordered pairs are solutions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (3, 1)</td>
<td>$1 &lt; 2(3)$ is true so it is a solution.</td>
</tr>
<tr>
<td>b. (2, 5)</td>
<td>$5 &lt; 2(3)$ is true so it is not a solution.</td>
</tr>
<tr>
<td>c. (-3, 3)</td>
<td>$3 &lt; 2(-3)$ is not true so it is not a solution.</td>
</tr>
<tr>
<td>d. (0, 0)</td>
<td>$0 &lt; 2(0)$ is not true since it is equal and therefore is not a solution.</td>
</tr>
<tr>
<td>e. (3, -3)</td>
<td>$-3 &lt; 2(3)$ is true so it is a solution.</td>
</tr>
</tbody>
</table>

Describe, in writing, how you determined if an ordered pair was a solution or not?

**Sample response:**

I determined it was a solution by substituting in the point values and seeing if the inequality was true.
Solving and Graphing Inequalities

Determining if a point is a solution to an inequality can be done algebraically by testing if the point satisfies the inequality or “works” in the inequality. However, if we want to find all of the points that are solutions to a given inequality, (called the solution set), this “test each point” method becomes very inefficient and cumbersome. A more efficient way to find the solution set for an inequality is to do so graphically.

To determine the solution graphically:

• Graph the inequality as if it were an equation.
• Find any ordered pair that is not on the line.
• Substitute the ordered pair into the inequality.

If the ordered pair satisfies the inequality, then it is in the solution set. The solution set will be all points above or below the line if it is a strict inequality < or >, and the set of all points above or below and including the line if the inequality is of the form ≤ or ≥.

12. If the ordered pair satisfies the inequality, then is it a member of the solution set?

Yes, if the ordered pair satisfies the inequality it is a member of the solution set.

a. Based on this, make a conjecture about the whole solution set for the inequality.

Every point in a solution set satisfies an inequality.
### Solving and Graphing Inequalities

13. Try this graphical procedure to find the solution set for the inequality \( y > x + 4 \).
   
   a. Graph \( y = x + 4 \)

   ![Graph of \( y = x + 4 \)]

   b. Pick a point not on the line and test whether this point satisfies the inequality.

   **Sample response:**
   
   \((0,0)\) \(0 > 0 + 4\)
   
   \(0 > 4\) is not true

   c. Is the point you picked a part of the solution set? If so, use your reasoning to pick another point you think will also be included. If not, use your reasoning to pick a different point that should be in the solution set. Test whether this point satisfies the inequality.

   **Sample response:**
   
   \((5,0)\) \(5 > 0 + 4\)
   
   \(5 > 4\) is true

   d. Hypothesize about a third point that should be included in the set of points that satisfy \( y > x + 4 \).

   **Sample response:**
   
   \((0,-5)\) \(0 > -5 + 4\)
   
   \(0 > -1\) is true

   e. Based on your responses to b – d, shade the portion of the coordinate plane that shows all the points that “work” in the inequality \( y > x + 4 \).

   ![Shaded portion of the coordinate plane]

   f. Should the line you graphed in part a, \( y = x + 4 \), be included in the solution set for \( y > x + 4 \)? Explain why or why not.

   **No, because the points on the line don’t satisfy the inequality.**

To indicate that a line is included in the solution set, leave it solid. To indicate that a line is not included, make it dotted.
Solving and Graphing Inequalities

14. $y > 3x$

15. $y < x + 5$

16. $y > x - 4$

17. $y > -2x + 5$
### Solving and Graphing Inequalities

<table>
<thead>
<tr>
<th></th>
<th>18. $y &lt; 6$</th>
<th>19. $y &gt; -2x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="18.png" alt="Graph" /></td>
<td><img src="19.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>20. $y &lt; 3x - 2$</th>
<th>21. $y &lt; 4x - 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="20.png" alt="Graph" /></td>
<td><img src="21.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
In questions 22 and 23 you may want to let students know that they will have to change the size of the intervals.

### Solving and Graphing Inequalities

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>22. ( y &lt; -8x - 48 )</td>
<td>23. ( y &gt; 6x - 20 )</td>
</tr>
</tbody>
</table>

![Graph 22](image)

![Graph 23](image)

24. \( y < -6 \)

![Graph 24](image)
The four concluding problems in this unit are designed to solidify the concepts students have been developing.

### Widgets

<table>
<thead>
<tr>
<th>Widgets</th>
<th>Widgets cost $4 each with a shipping charge of $9 per order.</th>
</tr>
</thead>
</table>

1. Write a complete sentence describing how you can find the total cost for an order of:

   a. 12 widgets  I would multiply 12 by 4 and add 9.

   b. 257 widgets  I would multiply 257 by 4 and add 9.

   c. any number of widgets  I would multiply the number of widgets by 4 and add 9.

2. Write an algebraic equation for the total cost of an order of widgets.

   \[ y = 4x + 9 \]

   or

   \[ w(x) = 4x + 9 \]
3. Find the total cost of:
   a. 12 widgets  The cost is $57.
   b. 257 widgets  The cost is $1037.

4. Determine how many widgets you can order for:
   a. $89  Twenty widgets can be ordered.
   b. $2069  An order for 515 widgets can be placed.
   c. $7789  An order for 1945 widgets can be made.

by setting up and solving an equation. Describe, in writing, how you solve the equation. Write your answer to each question in a complete sentence.

   The equation is \( y = 4x + 9 \).
   Substitute the actual cost for \( y \). Subtract the initial cost of $9 from both sides of the equation and divide both sides by 4.

5. Using the information you found in the problems above, complete the following data table.

<table>
<thead>
<tr>
<th>Number of Widgets</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Widgets</td>
<td>$</td>
</tr>
<tr>
<td>( x )</td>
<td>4( x + 9 )</td>
</tr>
<tr>
<td>12</td>
<td>57</td>
</tr>
<tr>
<td>257</td>
<td>1037</td>
</tr>
<tr>
<td>20</td>
<td>89</td>
</tr>
<tr>
<td>515</td>
<td>2069</td>
</tr>
<tr>
<td>1945</td>
<td>7789</td>
</tr>
<tr>
<td>2000</td>
<td>8009</td>
</tr>
</tbody>
</table>
There is no upper bound specified for the number of widgets ordered. However, students may introduce a maximum allowed order. If they do so, this will also affect their range.

6. Create a graph using the data from the table.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>x: Widgets</td>
<td>0</td>
<td>2000</td>
<td>100</td>
</tr>
<tr>
<td>y: Total Cost</td>
<td>0</td>
<td>8000</td>
<td>400</td>
</tr>
</tbody>
</table>
7. What would be an appropriate set of input values (domain) and output values (range). Express your answers using inequalities based on the problem situation.

\[ x > 0 \]

\[ y > 9 \]

8. If your company will spend between $100 and $200 on widgets, write inequalities to express the number of widgets you can order. Graph the solution on a number line.

\[ 22 < x < 48 \]

or

\[ 23 \leq x \leq 47 \]
Dumbbells are sold by the pound. Dumbbells Unlimited sells dumbbells for $0.50 a pound, plus $18 per order for shipping and handling.

1. Complete the data table.

<table>
<thead>
<tr>
<th>Dumbbells</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Expression</td>
</tr>
<tr>
<td>Pounds</td>
<td>$</td>
</tr>
<tr>
<td>$x$</td>
<td>$0.5x + 18$</td>
</tr>
<tr>
<td>120</td>
<td>78</td>
</tr>
<tr>
<td>300</td>
<td>168</td>
</tr>
<tr>
<td>142</td>
<td>89</td>
</tr>
<tr>
<td>377</td>
<td>206.50</td>
</tr>
<tr>
<td>897</td>
<td>466.50</td>
</tr>
</tbody>
</table>
Dumbbells

2. Summarize, in writing, how you
   a. determined the cost given the number of pounds of dumbbells.

   The number of pounds was multiplied by 0.5. Then the $18 cost for
   shipping was added.

   b. determined the number of pounds, given the cost.

   18 was subtracted from the cost, and then this value was divided by 0.5.

3. Generate a graph using the data from the table.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>x: \text{Dumbbells}</td>
<td>0</td>
<td>1000</td>
<td>50</td>
</tr>
<tr>
<td>y: \text{Cost}</td>
<td>0</td>
<td>500</td>
<td>25</td>
</tr>
</tbody>
</table>

![Graph of Dumbbells](image)
Students were introduced to the substitution method to verify a solution on page 3-38. They can use this method in question 4. In addition, students can substitute just the x or y value and attempt to verify that the result, when solving for the other variable, is the same as the given value.

Although student responses will vary, they should all conclude that the answer to problems 4a and 4b is "no."

4. Using the graph, determine if the following number of pounds of dumbbells and cost fit with this model.

   a. 220 pounds at a cost of $121.00

      It is close, but the cost is a little too low.

   b. 780 pounds at a cost of $390

      It is close, but the cost is a little too low.

5. What would be an appropriate set of input values (domain) and output values (range) based on the problem situation? Express your answers using inequalities. Graph the solution set for the domain and range on number lines.

   \[
   \begin{align*}
   \text{Domain} & \quad x > 0 \\
   \text{Range} & \quad y > 18
   \end{align*}
   \]

6. If you know you will spend under $500 on dumbbells, write inequalities to express the number of pounds you can order. Graph the solution set on a number line.

   \[
   \begin{align*}
   \text{Domain} & \quad x > 0 \text{ and } x < 964
   \end{align*}
   \]
<table>
<thead>
<tr>
<th>extension</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dumbbells</strong></td>
</tr>
<tr>
<td>7. Refer back to question 4. If you want to determine if the data points fit your algebraic model, what methods, other than looking at the graph, could you try? What do you think might be the most reliable method?</td>
</tr>
<tr>
<td>It is possible to use your equation to solve for one of the givens.</td>
</tr>
<tr>
<td>The equation is more reliable than a hand-drawn graph since you often have to estimate the point values.</td>
</tr>
</tbody>
</table>
In the activity "Moving a Sand Pile," students were asked how to generate the algebraic equation and table when provided with the graph. Since students do not usually begin with the graphical representation in the text or software, refer back to that activity as a guide for solving this problem.

This line represents the relationship between the number of minutes a company used a Cray super-computer and the usage cost. The usage cost included an Internet access fee of $47.13.

1. What can you say about the relationship between the time and the cost?

   *As the time increases, the cost also increases.*
Students may see the result in question 2 as being too high. It is expensive to access a super-computer.

### Computer Time

2. If the initial cost is $47.13 (Internet access fee) and the cost for two minutes is $110.03, find the cost per minute. Check a few other data points to verify the cost per minute.

\[
110.03 - 47.13 = 62.90; \quad \frac{62.9}{2} = 31.45
\]

It costs $31.45 per minute.

### Computer Time

3. Write the algebraic equation that uses the cost of Internet access, the number of minutes, and cost per minute to calculate the total cost.

\[
y = 31.45x + 47.13
\]
Questions 4 and 5 provide four rows in the table. Students may either use question 2 for the fifth row, or choose their own value to include.

There are no specific restrictions on the time in minutes. However, students may place restrictions if they interpret the equation to represent a function for the cost during a given month, for example.

Students might not use ≤ in question 7. Remind them that $160 or less implies that $160 is included.

4. Generate a table including the cost of:
   a. 11 minutes
   b. 32.17 minutes

5. Add to your table the number of minutes of computer time you can get for:
   a. $424.53
   b. $1902.68

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>minutes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>31.45x + 47.13</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>393.08</td>
</tr>
<tr>
<td>32.17</td>
<td></td>
<td>1058.88</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>424.53</td>
</tr>
<tr>
<td>59</td>
<td></td>
<td>1902.68</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>1305.13</td>
</tr>
</tbody>
</table>

6. What would be an appropriate set of input values (domain) and output values (range) for this problem. Express your answer using inequalities. Graph the solution set for the domain and range on separate number lines.

7. If the bill for computer usage has been $160 or less every month, use inequalities to express the number of minutes used (x should represent the time used). Graph the solution set on a number line.
In this activity, students begin with the tabular representation. They derive the algebraic and graphical representations.

### Dumpster

You have a job working for a hauling company. The company rents dumpsters. It delivers the dumpster to the site and hauls it away for a cost of $150. There is also a daily charge for the rental of the dumpster.

Your job is to answer the phone and provide information to potential customers. The company has given you a chart to keep near the phone. The chart shows the cost per week (7 days), without the delivery/hauling fee included.

<table>
<thead>
<tr>
<th>Length of Rental</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weeks</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>192.50</td>
</tr>
<tr>
<td>2</td>
<td>385.00</td>
</tr>
<tr>
<td>3</td>
<td>577.50</td>
</tr>
<tr>
<td>4</td>
<td>770.00</td>
</tr>
<tr>
<td>5</td>
<td>962.50</td>
</tr>
</tbody>
</table>

Many potential customers want to know the daily cost, so the chart is not very helpful.

1. What additional piece of information do you need to make this chart?

   It is necessary to know the cost per day to rent the dumpster.

2. How can you determine it?

   Divide the weekly charge by 7.

3. Generate the table for a rental period of 1 to 10 days. Include the cost of delivery and hauling of the dumpster.

<table>
<thead>
<tr>
<th>Length of Rental</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>177.50</td>
</tr>
<tr>
<td>2</td>
<td>205.00</td>
</tr>
<tr>
<td>3</td>
<td>232.50</td>
</tr>
<tr>
<td>4</td>
<td>260.00</td>
</tr>
<tr>
<td>5</td>
<td>287.50</td>
</tr>
<tr>
<td>Days</td>
<td>$</td>
</tr>
<tr>
<td>6</td>
<td>315.00</td>
</tr>
<tr>
<td>7</td>
<td>342.50</td>
</tr>
<tr>
<td>8</td>
<td>370.00</td>
</tr>
<tr>
<td>9</td>
<td>397.50</td>
</tr>
<tr>
<td>10</td>
<td>425.00</td>
</tr>
</tbody>
</table>
Students will need to perform a quick estimate to ensure that they are setting their upper bound for the independent variable high enough to find the solution in question 6.

### Dumpster

4. Write the formula you used to generate the information.

\[ y = 27.50x + 150 \]

5. After you made the table, someone called and asked you for the length of time he could rent a dumpster if he only had a maximum of $680.00. You realize the table does not provide sufficient information so you decide to make a graph.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>x: Length of Rental</td>
<td>0</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>y: Cost</td>
<td>0</td>
<td>1000</td>
<td>50</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between rental length and total cost](graph.png)
6. Based on the graph, what is the approximate number of days that the dumpster can be rented for a cost of no more than $680.00?

You can rent the dumpster for about 19 days.

7. What would be an appropriate set of input values (domain) and output values (range) based on the problem situation? Express your answers using inequalities. Graph the solution set for the domain and range on separate number lines.

Domain \( x > 0 \)

Range \( y > 150 \)
Modeling Situations Using Multiple Representations
Unit 4: Linear Equations and Systems of Equations

Unit Objectives and Skills

In addition to all the objectives and skills developed in the prior units, at the completion of this unit, students will be able to graph linear functions in two variables, compute x and y intercepts and find an equation of the line by finding the slope and identifying the y-intercept. Students will be able to determine the domain and range of a function as defined by the graph.

Students will be able to investigate, describe and predict the effects of changes in m and b on the graph of \( y = mx + b \) through the introduction of vertical and horizontal transformations. Representing a horizontal shift will introduce students to the equations of the form \( y = m(x - x_1) + y \). Students will be able to apply the Distributive Law. Finally, students will understand the relationship between parallel lines and their slopes.

Unit Overview

Building on several previous units, this unit focuses its attention on defining and interpreting variables, slopes, and intercepts and making sense of solutions. The majority of the work in this unit has students finding points, algebraically, on the line, given an input or output and determining whether the function is increasing or decreasing.

In general, the students work on several problems that are not set in context in order to focus full attention on important underlying mathematical characteristics and properties. It is at this point that the curriculum moves the students toward more formalized algebraic representations. Strong emphasis is placed on tying algebraic and graphical representations using numerical data as a mechanism to record the change in y with respect to the change in x.

Students develop a deeper understanding of slope as a constant rate of change and are able to interpret the meaning of the rate of change in a given situation.

Studying systems of equations represents an opportunity for students to fully exhibit their understanding of linear functions. Studying the relationship between functions requires students to demonstrate an understanding of the skills relating to setting up and solving equations. Further, performing comparative analyses requires that students develop their analytical reasoning skills, as well as skills relating to justification and communication.

Students will also be able to compare and contrast the function to the left and right of the point of intersection. Moreover, they will also be able to
represent the restricted domain and range mathematically (for the problem situation and to the left and right of the point of intersection) and interpret the relationship between the functions with respect to a given problem situation.

To extend students’ understanding of the linear equations, they will also be introduced to the general form for the equation of a line. Introducing the general form for the equation will allow for a smooth passage to studying functions in which the variables co-vary with respect to a fixed outcome. It will extend students’ ability to set up and solve linear equations independent of the symbolic form.

Unit Activities

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

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<th>Printed Classroom Activities</th>
<th>Homework Assignments</th>
</tr>
</thead>
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<td>pg. 79</td>
</tr>
<tr>
<td>Selling Balloons</td>
<td>pg. 83</td>
</tr>
<tr>
<td>Equations, Rates and Intercepts (Parts 1 – 4)</td>
<td>pg. 89</td>
</tr>
<tr>
<td>Generating and Interpreting Linear Equations</td>
<td>pg. 95</td>
</tr>
<tr>
<td>Spending Money</td>
<td>pgs. 99, 103</td>
</tr>
<tr>
<td>Comp-U-Us</td>
<td>pg. 107</td>
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<td>Mowing Lawns</td>
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<tr>
<td>The Distributive Property</td>
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<tr>
<td>Solving Equations Using Known Formulas</td>
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<tr>
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<td>pg. 119</td>
</tr>
<tr>
<td>Making and Selling Shirts</td>
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</tr>
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<td>Solving Systems of Two Equations Algebraically</td>
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</tr>
<tr>
<td>Finding the Better Paying Job</td>
<td>pg. 127</td>
</tr>
<tr>
<td>World Oil: Supply and Demand</td>
<td>pg. 127</td>
</tr>
<tr>
<td>Picking the Best Option</td>
<td>pg. 127</td>
</tr>
</tbody>
</table>

Suggestions for Classroom Implementation

As students are working with more out-of-context problems than previously, helping to bridge that transition will be important. Decorating the Math Lab and Selling Balloons help their process by offering relatively simple situations that can be modeled by a linear function almost on inspection.
The work on more abstract aspects begins after the students set up the equation. At this point, they define and detail aspects of the equation, generate input and output values, solve equations, graph the equation, and study the graph in relation to the equation. It is important to note that in some cases the input values provided are negative, which are, of course, correct algebraically, but do not make sense with respect to the situation. In addition, students are asked to think about the validity of their algebraic solution with respect to the problem scenario, therefore extending the ties between contextual and abstract problem solving. Ask students to justify their results in terms of the algebraic equation relative to the problem situation.

After completing these two problems, students are ready to move on to Equations, Rates and Intercepts Parts 1-4. This set of activities is designed to provide opportunities for students to identify and compute intercepts in Part 1, as well as to be introduced more formally to the concept of slope. How to find the slope of a line and how to write the equation of a line in the slope-intercept form is introduced in Parts 2 and 3. Problems in this unit activity are not as well suited to groups of three or four as they are for individual or paired problem solving. You may want to have students work for approximately ten minutes, then stop and hold a general class discussion to allow students to summarize their findings. Continue in this way, perhaps extending the amount of time they work on the activity. If you opt to do the extension activity, have students work in small groups and then present.

Part 4 of Equations, Rates and Intercepts extends students’ understanding of linear functions by introducing transformations through activities in which they compare and contrast the parent function with functions with greater or lesser slope or a vertical shift. Also introduced in this set of activities is the absolute value function as a new type of linear function. In keeping with the unit, comparisons are made between \( y = x \) and \( y = |x| \) as well as through transformations of the absolute value function.

Generating and Interpreting Linear Equations provides a group of practice problems where students, through multiple representations, can focus on the underlying mathematical properties of linear functions: slope, intercepts, domain, and range.

Spending Money, Comp-U-US, and Mowing Lawns let students explore vertical and horizontal shifts and how those shifts are interpreted. Spending Money is in many ways a capstone activity, as it has students apply each of the concepts highlighted throughout the course to this point, but also sets the stage for Comp-U-US and Mowing Lawns where the Distributive Law is introduced in context. The extension activity in Spending Money, which has the students generating a solution and series of questions, is an activity that will generate unique presentations worthy of discussion. You may want to use this extension problem at the end of this activity as a formative assessment.
As indicated, *Comp-U-US* and *Mowing Lawns* provide motivation for a formal presentation of the Distributive Property and practice on executing the rule. The activities in the section entitled *The Distributive Property* focus on developing the concept through symbolic manipulation and extend it to geometrical applications through computing areas.

Capping the discussion on single linear equations is a brief section on using known formulas to solve problems. To date, students have been given novel situations and have chosen an appropriate linear model. Students need to see that there are large classes of problems that can be modeled by an already specified linear model. By introducing the formulas and solving, students are simultaneously introduced to literal equations.

The final activities in the unit allow for a transition from defining and solving a single linear function to working with a system of two linear functions. The first two scenarios; *Producing and Selling Pens* and *Making and Selling Shirts*, are sequenced so that the first examines two problem situations individually and then through a modeling activity combines the two situations for interpretation. *Making and Selling Shirts* looks at two aspects of a situation simultaneously. The latter example matches more directly to the approach modeled in the software lessons.

As finding and interpreting points of intersection are critical to working with systems of equations, you will want to focus some time on *Connecting Algebraic and Graphical Representations*, as well as *Solving Systems of Two Equations Algebraically*. The focus when solving systems is on applying the method of substitution with equations in the Slope-Intercept Form or General Form. When addressing the different forms for the equations, you should note a couple of key facts. First, equations in the slope-intercept form and general form appear more often because real world situations are more aptly modeled by equations that describe a constant rate of change and initial start value or those that involve fixed outputs with constant co-varying independent variables. The other key fact is that, since all these forms represent a line, they are all equivalent. The difference is in form only, and this difference is achieved through symbol manipulation (applying what students have learned about solving literal equations).

**Unit Assessments**

*Balloon Problem*
*Television Problem and Climber Problem*
*Widgets-R-U*
*Computer Rental Problem*
*Systems Part 1*
*Systems Part 2*
*Systems Part 3*
Unit 4: Linear Equations and Systems of Equations

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To give the math computer lab more of its own identity, your teacher decides to order posters of great mathematicians. The posters cost $2 each and there is an additional charge of $4 per order for shipping and handling.

1. Using the variables $x$ and $y$, find an algebraic equation that represents this situation.

   Algebraic Equation: \[ y = 2x + 4 \]

Define each variable in your equation by writing a short phrase that describes what it represents.

   $x$: number of posters ordered measured in posters

   $y$: total cost measured in dollars.
Decorating the Math Lab

2. Graph the equation. Label your axes and use an appropriate interval.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>x: Number of Posters</td>
<td>-5</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>y: Cost of Order</td>
<td>0</td>
<td>40</td>
<td>2</td>
</tr>
</tbody>
</table>

Throughout the remainder of the unit, blank spaces will not be provided for labeling the axes. At this point in the curriculum, students should be in the habit of labeling axes with appropriate units.
To introduce the concept of intercepts, students will be expected to determine the intercepts from the graphical representation. The process of determining the intercepts algebraically will be covered later in this unit.

<table>
<thead>
<tr>
<th><strong>Decorating the Math Lab</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Where does the graph intersect the y-axis?</td>
</tr>
<tr>
<td><em>The graph intersects the y-axis at (0,4).</em></td>
</tr>
<tr>
<td>4. Where does the graph intersect the x-axis?</td>
</tr>
<tr>
<td><em>The graph intersects the x-axis at (-2,0).</em></td>
</tr>
</tbody>
</table>

Interpretation of the intercepts should be made from the graph (as in questions 3 and 4) and in terms of the problem situation (as in questions 6 and 7). In this example, only the y-intercept has meaning in the context of the problem.

<table>
<thead>
<tr>
<th><strong>Decorating the Math Lab</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Use complete sentences to describe your graph.</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td><em>The graph goes up as it travels from left to right. It starts above 0 on the y-axis. The line is straight and steep.</em></td>
</tr>
<tr>
<td>6. How do you interpret the point where the graph intersects the y-axis?</td>
</tr>
<tr>
<td><em>The point where the graph intersects the y-axis is the total cost of the order, if we do not order any posters.</em></td>
</tr>
<tr>
<td>7. How do you interpret the point where the graph intersects the x-axis?</td>
</tr>
<tr>
<td><em>The point where the graph intersects the x-axis is the number of posters ordered, if the cost is 0. In this problem, we cannot order -2 posters, so no order at all has been placed.</em></td>
</tr>
</tbody>
</table>
Students should understand that there will be an algebraic solution to a given question even when the solution is not reasonable in the context of the problem situation.

### Decorating the Math Lab

**8.** What is the value of \( y \) if:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>-9</td>
<td>-14</td>
</tr>
<tr>
<td>-12</td>
<td>-20</td>
</tr>
</tbody>
</table>

- a. \( x \) is 7? \( y \) is 18
- b. \( x \) is -9? \( y \) is -14

**9.** What is the value of \( x \) if:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( x )</td>
</tr>
<tr>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>-20</td>
<td>-12</td>
</tr>
</tbody>
</table>

- a. \( y \) is 16? \( x \) is 6
- b. \( y \) is -20? \( x \) is -12

**10.** Use your equation for this situation. Let \( y = -23.125 \). Solve the equation to find \( x \).

\[
-23.125 = 2x + 4
\]

\[
-27.125 = 2x
\]

\[
x = -13.5625
\]

**11.** Find the value of \( y \) if \( x = -23,456 \).

\[
y = 2(-23,456) + 4
\]

\[
y = -46,912 + 4
\]

\[
y = -46,908
\]

### Decorating the Math Lab

**12.** Do all the values for \( x \) and \( y \) make sense in this problem situation? Why or why not?

*No, the values do not make sense since you cannot have a negative number of posters or a negative cost. It does not make sense for there to be a cost of $4 when no posters are ordered. The four dollars is a shipping cost and will not be charged if nothing is shipped.*

- a. Write an inequality to express which \( x \) values make sense.

\[
x > 0
\]

- b. Write an inequality to express which \( y \) values make sense.

\[
y > 4
\]
### Selling Balloons

Your local community group wants to raise money to fix one of the playgrounds in your area. Since balloons are popular with young children, your group decides to sell them to make money. You have bought a box of balloons for $10. You decide to sell the balloons for $1 each.

<table>
<thead>
<tr>
<th>Process: Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Using the variables x and y, find an algebraic equation that represents this situation.</td>
</tr>
<tr>
<td>Algebraic Equation: [ y = x - 10 ]</td>
</tr>
<tr>
<td>Define each variable in your equation by writing a short phrase that describes what it represents.</td>
</tr>
<tr>
<td>( x ): number of balloons sold measured in balloons</td>
</tr>
<tr>
<td>( y ): profit measured in dollars</td>
</tr>
</tbody>
</table>

### Selling Balloons

You may want to suggest that there is a limit to the number of balloons. For example, 235 may not be reasonable if the class has decided that the box of balloons contains only 200 balloons, but it may be reasonable if the class decides the box contains say 300 or 500 balloons.

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. What is the value of ( y ) if:</td>
</tr>
<tr>
<td>a. ( x ) is 7? ( y ) is -3</td>
</tr>
<tr>
<td>b. ( x ) is -9? ( y ) is -19</td>
</tr>
<tr>
<td>c. ( x ) is 235? ( y ) is 225</td>
</tr>
</tbody>
</table>
3. What is the value of \( x \) if:
   
a. \( y \) is 16? \( x \) is 26
   
b. \( y \) is -20? \( x \) is -10
   
c. \( y \) is -4.57? \( x \) is 5.43

This is the first graph in the text that necessarily contains negative \( y \) values. Students will need to choose where to position the axes on the grid. Students have been working primarily in the first quadrant so they may continue to position the \( y \)-axis along the left edge of the grid. However, the position of the \( x \)-axis must be above the bottom edge of the graph.

5. Graph the equation. Label your axes and use an appropriate interval.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ): Balloons Sold</td>
<td>-20</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>( y ): Profit</td>
<td>-20</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>
Both the x and y intercepts have a realistic meaning in this activity. This is especially true with respect to the y-intercept since the scenario deals with the familiar concept of money.

This scenario provides an opportunity to use negative numbers where the interpretation makes sense algebraically and in a real world context.

<table>
<thead>
<tr>
<th>Selling Balloons</th>
</tr>
</thead>
</table>
| 6. Where does the graph intersect the x-axis? What does this point represent in the problem?  
   The graph intersects the x-axis at (10,0). This represents the number of balloons sold when the profit is 0. |
| 7. Where does the graph intersect the y-axis? What does the point represent in the problem?  
   The graph intersects the y-axis at (0, -10). This point represents the fact that the balloon sellers start in debt $10, after spending $10 on the box of balloons and not selling any. |
| 8. Use complete sentences to describe your graph.  
   Sample Responses:  
   The graph increases from left to right, and it is a straight line. The graph starts below zero on the y-axis. |
| 9. Do all the values for x and y make sense in this problem scenario?  
   No, it does not make sense to sell a negative number of balloons.  
   a. Write an inequality to express which x values make sense in this situation.  
      \[ x > 0 \]  
   b. Write an inequality to express which y values make sense.  
      \[ y > -10 \] |
| 10. If you ignore the scenario, and think only of the algebraic function, what is the domain and range?  
   The domain and range include all real numbers. |
Nearly everything that the students have done up to this point, with the exception of equation solver units in the software curriculum, has been presented in the context of a problem scenario. The next portion of this unit will focus on the discussion of linear functions out of context.

**Equations, Rates, and Intercepts: Part 1**

\( y = 3x - 5 \) is a linear equation. When it is graphed, all the points that fit into the equation form a line.

1. For this equation, make a table of values and construct a graph. Use both positive and negative values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
</tbody>
</table>

2. What is the domain and range for the equation?

*The domain and range include all real numbers.*
In previous activities in this unit, students have determined the intercepts using a graphical approach. Questions 3 and 4 introduce an algebraic approach to determining the intercepts.

**Intercepts** are the points where a graph crosses the coordinate axes. The **y-intercept** is the point where the graph intersects the y-axis. It is represented by the ordered pair (0, y). The **x-intercept** is the point where the graph intersects the x-axis. It is represented by the ordered pair (x, 0).

### Equations, Rates and Intercepts: Part 1

<table>
<thead>
<tr>
<th>input</th>
</tr>
</thead>
</table>

3. The graph of \( y = 3x - 5 \) crosses the y-axis when \( x = 0 \) and \( y = \_\_\_\_-5\_\_\_. \)

The ordered pair \((0, -5)\) is the **y-intercept**.

4. The graph of \( y = 3x - 5 \) crosses the x-axis when \( y = 0 \) and \( x = \_\_\_\_\_\_\_\_\_. \)

The ordered pair \((\_\_\_\_\_\_\_\_\_, 0)\) is the **x-intercept**.
Students have previously interpreted the rate of change in the context of a problem situation. Question 5 generalizes this interpretation. Slope will be formally defined on page 4-20.

For questions 6 through 9, students should apply what they know about ratio and proportions to answer each question.

### Equations, Rates and Intercepts: Part 1

<table>
<thead>
<tr>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. In the graph of $y = 3x - 5$, whenever $x$ increases (goes up) by 1, $y$ increases (goes up) by 3. Whenever $x$ decreases (goes down) by 1, $y$ decreases (goes down) by 3.</td>
</tr>
<tr>
<td>6. Whenever $x$ increases by 10, $y$ increases by 30.</td>
</tr>
<tr>
<td>7. Whenever $x$ increases by 30, $y$ increases by 90.</td>
</tr>
<tr>
<td>8. Whenever $x$ decreases by 97, $y$ decreases by 291.</td>
</tr>
<tr>
<td>9. Whenever $x$ decreases by 5, $y$ decreases by 15.</td>
</tr>
</tbody>
</table>

### Equations, Rates and Intercepts: Part 1

<table>
<thead>
<tr>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. If you wanted to restrict the graph to the first quadrant, how would you define the domain and range?</td>
</tr>
<tr>
<td>$x &gt; \frac{5}{3}$</td>
</tr>
<tr>
<td>$y &gt; 0$</td>
</tr>
</tbody>
</table>
For question 2 note that the graph of function $y = 3x - 5$ is on page 4-10.

**Equations, Rates, and Intercepts: Part 2**

$y = f(x) = -5x + 9$ is also a linear equation. This equation is written using function notation.

1. Why do you think this is a linear equation or function?

   *All the points create a straight line. The change is constant.*

2. For this equation, make a table of values and construct a graph. Use both positive and negative values for $x$. (Use the same bounds and intervals on this graph as you did when graphing $y = 3x - 5$.)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>19</td>
</tr>
<tr>
<td>-1</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

Note that the axes are drawn on the grid to encourage the use of both positive and negative values for $x$ and $y$.

After question 3, present the fact that the domain and range for a linear function out of context consist of the set of all real numbers.

3. What is the domain and range for the equation?

   *The domain is the set of all real numbers.*

   *The range is the set of all real numbers.*
The slope of this function is negative, so as x increases, y will decrease. Students must indicate that the graph increases or decreases and designate the rate.

<table>
<thead>
<tr>
<th>Equations, Rates and Intercepts: Part 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Indicate the y-intercept (make sure to write it as an ordered pair) <strong>(0, 9)</strong></td>
</tr>
<tr>
<td>Indicate the x-intercept (make sure to write an ordered pair). <strong>(1.8, 0) or (9/5, 0)</strong></td>
</tr>
<tr>
<td>5. Describe, in writing, the meaning of the y-intercept.</td>
</tr>
<tr>
<td><em>The y-intercept is the point at which the graph crosses the y-axis.</em></td>
</tr>
<tr>
<td>What is the meaning of the x-intercept?</td>
</tr>
<tr>
<td><em>The x-intercept is the point at which the graph crosses the x-axis.</em></td>
</tr>
<tr>
<td>6. Whenever x increases (goes up) by 1, y goes down by <strong>5</strong>.</td>
</tr>
<tr>
<td>7. Whenever x increases by 10, y <strong>goes down by 50</strong>.</td>
</tr>
<tr>
<td>8. Whenever x increases by 30, y <strong>goes down by 150</strong>.</td>
</tr>
<tr>
<td>9. Whenever x decreases by 97, y <strong>goes up by 485</strong>.</td>
</tr>
<tr>
<td>10. Whenever x decreases by 5, y <strong>goes up by 25</strong>.</td>
</tr>
</tbody>
</table>
To further extend this activity, have the students create questions to accompany their stories. Refer to previous activities for appropriate questions. These may include:

- solving for independent or dependent variables
- graphing the function
- finding and interpreting the intercepts.

11. Write a story that fits each function:

a. \( y = f(x) = 8x + 50 \)

**Sample Response:**
*I have earned $50 working at the local grocery store. I earn $8 per hour working there.*

b. \( y \leq f(x) \leq 4x + 12 \)

**Sample Response:**
*I have $12 in the bank, and I receive $4 per hour babysitting. I want to save enough money to buy a new pair of pants without going into debt. If I work 20 hours, I want to find the maximum amount I can spend.*
Questions 13 through 15 are designed to show the significance of the constants in the slope-intercept form of a linear equation.

<table>
<thead>
<tr>
<th>Equations, Rates and Intercepts: Part 2</th>
</tr>
</thead>
</table>
| 12. What are the similarities between the graphs of \( y = 3x - 5 \) and \( f(x) = -5x + 9 \)?

*They are both linear and have \( y \) intercepts. They both have the set of all real numbers as their domain and range.*

What are the differences between the graphs of \( y = 3x - 5 \) and \( f(x) = -5x + 9 \)?

*One way the graphs differ is that \( y = 3x - 5 \) is increasing (left to right) and \( f(x) = -5x + 9 \) is decreasing. Another way they differ is where they cross the \( y \)-axis. The function \( y = 3x - 5 \) crosses below the origin, while \( f(x) = -5x + 9 \) crosses above the origin.*

13. Describe how \( y \) changes for each unit change in \( x \) in \( y = 3x - 5 \). (You may want to generate a table of values for \( x = 0 \) through \( x = 6 \) to help you answer this question).

*As \( x \) goes up by 1, \( y \) goes up by 3.*

14. Describe how \( y \) changes for each unit change in \( f(x) = -5x + 9 \).

*As \( x \) goes up by 1, \( y \) goes down by 5.*

15. What do you think determines the rate of incline or rate of decline of the lines?

*The number before \( x \) (coefficient of \( x \)) determines the rate of incline or decline of the line.*

16. Give some examples of natural or person-made objects that can be described as having steepness or a rate of incline/decline.

*Sample Responses:*

Roads, jumps, mountains, hills.

17. Do you think it would be more difficult to ski down a very steep hill or one that is more gradual? How about biking up a steep short hill or a long gradual hill? Explain your reasoning.

*It would be harder to ski down a steep hill since you will travel at a higher speed, therefore making it hard to control your movements. It is harder to bike up a steep hill because more effort is required in a short amount of time.*

18. Based upon your response to question 17, how do you think the rate of increase or decrease of a line is measured?

*The rate of increase or decrease of a line is measured by how large the number in front of \( x \) (coefficient of \( x \)) is.*
19. How do you think you could measure the rate of ascent (increase) of a staircase? What observations can you make about a flight of steps that will help you?

*The upward (vertical) distance traveled could be compared to the distance moved forward (horizontal distance). Also, each step could be measured.*

20. If the part of the step you place your foot on (the run) is wider than the height of the step (the rise), what can you say about the steepness of the staircase? Create a picture of a staircase that fits this description to help you answer the question.

*The staircase will not be as steep.*

21. If the part of the step you place your foot on (the run) is narrower than the height of the step (the rise), what can you say about the steepness of the staircase? Create a picture of a staircase that fits this description to help you answer the question.

*The staircase will be steeper.*

22. Have you ever observed a staircase in which the relationship between the width of the step and the height varied along the staircase? Draw a picture of a staircase that fits this description. Why do you think most staircases have a fixed relationship between the width of the step (the run) and the height of the step (the rise)?

*Sample Response:*
*In a pair of steps going from the driveway to our front door.*

*The regularity will make building the steps easier. Set width and height make it easier for people to walk up and down safely.*

23. Draw the steps that represent the horizontal change (the run) and the vertical change (the rise) in each of the following three linear graphs. Describe what the steps mean in the picture.

- *y = x normal*
- *y = \( \frac{x}{2} \) wider run*
- *y = 2x higher rise*

*Each step shows a unit change in x with respect to y.*
Slope will be formally defined on page 4-20. In preparation for this, have students compute the ratio of rise to run (change in y to unit change in x) for questions 24 through 27.

Questions 24 through 27 focus solely on generating the graph from information about where it crosses the axes and the change in x with respect to y. To help students focus on the slope, the graphs all pass through the origin. The first two problems involve positive slopes, while the next two have negative slopes.

**Equations, Rates and Intercepts: Part 2**

24. Use the given information to plot at least four points to generate the graph.
   a. The line intersects both the x and y axes at the origin.
   b. For every unit increase in x, y increases by 4 units.

   **Sample Graph:**

![Sample Graph 1](image)

25. Use the given information to plot at least four points to generate the graph.
   a. The line intersects both the x and y axes at the origin.
   b. For every 3 units that you move horizontally (along the x-axis), you move up 2 units vertically (along the y-axis).

   **Sample Graph:**

![Sample Graph 2](image)
26. Use the given information to plot at least four points to generate the graph.
   a. The line intersects both the x and y axes at the origin.
   b. For every unit increase in x, y decreases by 4 units.

   Sample Graph:

27. Use the given information to plot at least four points to generate the graph.
   a. The line intersects both the x and y axes at the origin.
   b. For every 3 units that you move horizontally (along the x-axis), you move down 2 units vertically (along the y-axis).

   Sample Graph:
For additional practice computing the slope, have students go back to previous graphs or tables and compute the slope. By computing the slope in a particular problem using several different points, you are able to reinforce that linear functions have a constant rate of change. This will be one of the major differences between linear functions and other functions discussed throughout the curriculum.

Equations, Rates and Intercepts: Part 2

The rate of incline or decline of the line is called the **slope**, which is the ratio of the change in y with respect to the change in x. We may represent this relationship as

\[
\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}
\]

If you look at the equations for each of the graphs in problem 23, the coefficients of x represent the slope.

In general, the equations you have written to model situations or to represent the graphs of lines in the problems you have done, have been of the form

\[y = mx\]

where m is the slope of the line, or in the form

\[y = mx + b\]

where m is the slope of the line and b represents the y-intercept.

Think back to some of the problems you modeled using an algebraic equation. For example, in the $8 an Hour Problem, you wrote something like

\[E = 8H\]

while in the U.S. Shirts and Hot Shirts problems, the equations were of the form

\[y = 8x + 15\]

and

\[y = 5.50x + 49.95\]
Equations, Rates and Intercepts: Part 2

28. Look at each of these equations. Use complete sentences to answer the following questions.

a. What does the number 8 represent in the $8 an Hour Problem?

   The number 8 represents the slope (the amount earned per hour).

b. What do the numbers 8 and 15 represent in the U.S. Shirts problem?

   The number 8 represents the slope (the cost per shirt) and 15 represents the y-intercept (the initial start-up fee).

c. What do the numbers 5.50 and 49.95 represent in the Hot Shirts problem?

   The number 5.50 represents the slope (the cost per shirt) and 49.95 represents the y-intercept (the start-up cost).

29. How do you interpret the meaning of the slope in the $8 an Hour Problem? How do you interpret the meaning of $49.95 in the Hot Shirts problem?

   In the first problem, it means that for each hour you work, your earnings will increase by $8.

   In the Hot Shirts problem, $49.95 represents the initial cost before any shirts are sold.

Equations, Rates and Intercepts: Part 2

30. Based on this information, write the equations that represent the graphs of the lines on pages 4-18 and 4-19.

   24 b. \( y = 4x \)

   25 b. \( y = \frac{2}{3} x \)

   26 d. \( y = -4x \)

   27 d. \( y = -\frac{2}{3} x \)
Questions 1 through 3 have the same slope but varying $y$-intercepts. Assign each question to 1/3 of the groups. After groups present results for questions 1 through 3, use question 4 to summarize results.

**Equations, Rates, and Intercepts: Part 3**

You have just taken a job in which you will earn $10 per hour. Determine your total earnings if you work 4 hours, 10 hours, 22 hours, 50 hours.

1. Model the situation numerically, graphically, and algebraically. Complete a table of values, draw the graph of the function, and provide the algebraic equation that represents the situation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>22</td>
<td>220</td>
</tr>
<tr>
<td>50</td>
<td>500</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

   $X$ is the time worked in hours, and $y$ is the money earned in dollars.

   Algebraic equation: $y = 10x$
Even though it is not explicitly requested, students should continue to label the graphs' axes and determine appropriate bounds and intervals.

In addition, when writing the algebraic equations, make sure to include a written definition of variables with units.

**Equations, Rates, and Intercepts: Part 3**

You have $20 in the bank. You want to save some more money, so you continue to work at your job that pays $10 per hour. Assuming you put all of your earnings in the bank, how much will you have if you work 4 hours, 10 hours, 22 hours, 50 hours?

2. Model the situation numerically, graphically, and algebraically. Complete a table of values, draw the graph of the function, and provide the algebraic equation that represents the situation.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>22</td>
<td>240</td>
</tr>
<tr>
<td>50</td>
<td>520</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

Algebraic equation: \[ y = 10x + 20 \]
You spent all of your savings and even had to go into debt by $30 to buy the newest CDs. You are still earning $10 per hour. Determine how much money you will have if you work 4 hours, 10 hours, 22 hours, 50 hours?

3. Model the situation numerically, graphically, and algebraically. Complete a table of values, draw the graph of the function, and provide the algebraic equation that represents the situation.

\[
\begin{array}{c|c|c}
\hline
x & y & \\
\hline
4 & 10 & \\
10 & 70 & \\
22 & 190 & \\
50 & 470 & \\
0 & -30 & \\
\hline
\end{array}
\]

Algebraic equation: \( y = 10x - 30 \)

4. What are the similarities and differences between the graphs in the last three situations? Consider the slope, intercepts, or anything else you think is important.

All have the same slope. Each has a different y-intercept. The first line crosses the y-axis at 0, the second crosses the y-axis above the origin, while the third graph crosses the y-axis below 0.
Note that no intervals or bounds are provided in question 1. For this problem, assume that the same interval and bounds are used on both axes in all three examples provided. Students should be aware that this is not the case in general.

### Equations, Rates and Intercepts: Part 4

1. Compare the following graphs and indicate what you believe are their similarities and differences?

   ![Graphs](image)

   - **a)**
   - **b)**
   - **c)**

   \[ y = x \]

   *All three graphs have the same slope. Graph (a) has a y-intercept of 0. Graph (b) has a positive y-intercept. Graph (c) has a negative y-intercept.*

2. If (a) is the graph of \( y = x \), what is a possible equation for (b)? For (c)?

   - **Graph (b):** \( y = x + 3 \)
   - **Graph (c):** \( y = x - 3 \)
Have students use an interval size of 1 for both the x and y-axes.

3. Place the x and y axes on the grid below. Draw the line $y = x$. Then graph both of the following equations.
   
a. $y = x + 5$
   
b. $y = x - 2$
Equations, Rates, and Intercepts: Part 4 serves to introduce and analyze the concept of transformation with respect to linear functions. The concept of transformation will be revisited when introducing quadratic functions. Transformation behavior is consistent across all families of functions.

In order to recognize and understand some transformations, it is helpful to have students generate and examine a numerical representation in addition to the symbolic and graphical representations.

Make sure students realize that \( b \) can represent a positive or a negative number.

### Equations, Rates, and Intercepts: Part 4

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
</table>
| 4. What can you say about these lines with respect to the line \( y = x \)?

*They have the same slope but different y-intercepts.*

The function \( y = x \) is defined as the **parent function** for all linear functions. The parent function represents a function type in its most basic form. Use this information as you think about the answers to the questions below.

5. In general, how would you describe the difference between the two functions \( y = x \) and \( y = mx \) if the slope is positive?

*The line \( y = x \) passes through the origin, bisecting the first quadrant (45° degree angle is formed). The line \( y = mx \) also passes through the origin; however it will lie between the line \( y = x \) and the y axis for \( m > 1 \) (the angle formed will be between 45° and 90°). When \( 0 < m < 1 \), the line will be between \( y = x \) and the x axis (the angle formed will be between 0° and 45°).*

6. In general, how would you describe the difference between the two functions \( y = x \) and \( y = x + b \) if the slope is positive?

*The difference between \( y = x \) and \( y = x + b \) is that \( y = x \) passes through the origin while \( y = x + b \) crosses the y-axis \( b \) units above the origin.*

7. In general, how would you describe the difference between the two functions \( y = x \) and \( y = mx + b \) where \( m \) is positive?

*The difference between \( y = x \) and \( y = mx + b \) is the combination of responses in questions 5 and 6, meaning the graph may have a slope other than 1 and a y-intercept other than 0.*
8. Compare the graphs below. Indicate the similarities and differences.

\[ y = x \quad y = -x \]

The line \( y = x \) increases from left to right. The line \( y = -x \) decreases from left to right, but it decreases at the same rate as \( y = x \) increases. Both of these lines intersect the origin, so they have \( y \)-intercepts of \((0,0)\).

9. Describe the relationship between the change in \( y \) and the unit change in \( x \) in each of the graphs.

In the graph of \( y = x \), for each positive/negative unit change in \( x \), there is a positive/negative unit change in \( y \). In the graph of \( y = -x \), for each positive unit change in \( x \), there is a negative unit change in \( y \). Likewise, for each negative unit change in \( x \), there is a positive unit change in \( y \).
Questions 11 through 13 begin to combine several transformations at once. It may be useful to include secondary equations to examine each transformation one at a time. For example, in question 13, examine the equations:

\[ y = x \]
\[ y = -x \]
\[ y = \frac{x}{2} \]
\[ y = -\frac{x}{2} + 7. \]

Either have students create the graphs of the functions in questions 11 through 13 on graph paper before answering the questions, or have students use the graphing calculator to generate the graphs and transfer them to graph paper before answering the questions.

10. Place the x and y axes in the grid below. Draw the line \( y = -x \).

11. Graph the equation \( y = -2x \). What can you say about the graph of this equation? How does it compare with the parent function \( y = x \)?

The slope of the line is negative and the line passes through (0, 0). This line has the same intercepts as \( y = -x \), but its slope is greater.

12. Graph the equation \( y = -\frac{x}{2} \). What can you say about the graph of this equation? How does it compare with the parent function \( y = x \)?

The slope of the line is negative and the line passes through (0, 0). This line has the same intercepts as \( y = -x \), but its slope is lesser.

13. Graph the equation \( y = -\frac{x}{2} + 7 \). What do you know about the graph of this equation? How does it compare with the graph of \( y = x \)?

The line passes through (0, 7) as its y-intercept, not (0,0). It still has a negative slope like the other graphs, and it has the same slope as \( y = -x/2 \).
During the time when your teacher taught this unit in class, there was a flu epidemic in your town. Many students were absent. Your teacher wants to help the students who were ill, so she teams with the media teacher to produce a two-minute video to summarize linear functions. You and your group members were not afflicted by the flu, so your teacher involves you in the project instead of giving a quiz. Based on what you know about linear functions, write the script for this video.

Answers will vary, but should include some of the following points:

- Linear functions are represented graphically by straight lines.
- Linear functions can be written in the form \( y = mx + b \).
- The \( y \)-intercept or initial value is represented by the point \((0, b)\).
- The slope of a line \((m)\) represents the change in \(y\) with respect to the unit change in \(x\).
- If a point lies on the line, then those coordinates represent a solution to the equation \( y = mx + b \).
- The domain and range of all linear functions is the set of all real numbers, unless restricted by a problem situation.
- Given the independent variable, you can evaluate the expression to find the dependent value.
- Given the equation of the line and given the \(y\) value, you can solve for \(x\).
- Any situation involving a constant rate of change can be modeled by a linear function.
- All linear functions are continuous.
Absolute value is introduced as an extension of linear functions. Treatment is similar in terms of the study of transformations and graphing and in terms of understanding dependency relationships and domain and range.

After students derive the definition of absolute value, you may want to ask them to think of a situation that may be modeled by absolute value.

14. Compare the graphs and describe the difference between the two functions. To support your description, generate a table of values from the coordinate points for the values of \( x \) between –10 and 10.

\[
\begin{array}{c|c|c|c|c}
\hline
\text{ } & \text{ } & y = x & y = |x| \\
\hline
-10 & -10 & -10 & 10 \\
-8 & -8 & -8 & 8 \\
0 & 0 & 0 & 0 \\
4 & 4 & 4 & 4 \\
8 & 8 & 8 & 8 \\
\hline
\end{array}
\]

The function \( y = x \) passes through the origin and is in the first and third quadrant. When \( x \) is positive, so is \( y \). When \( x \) is negative, so is \( y \). The absolute value graph emanates from the origin. It is in the 1st and 2nd quadrants. This means that when \( x \) is positive so is \( y \), but when \( x \) is negative, \( y \) is still positive.

15. Define the domain and range of both functions.

The domain and range for \( y = x \) is the set of reals. The domain of the absolute value is the set of reals, but the range is non-negative reals.

Equations, Rates, and Intercepts: Part 4

Formally, the graph of the new function is defined by

\[ y = |x| \]

where \( | \cdot | \) represents the absolute value of \( x \).
16. Based on your analysis above, how would you define the absolute value of a number.

   **Sample Response:**
   
   The absolute value of a number is either 0 or positive.

17. Does the following statement support your definition of absolute value? The absolute value represents the number of units a number is from the origin no matter what direction. Justify your answer.

   **Sample Response:**
   
   Yes, because the answer is always positive whether the number is greater than or less than 0 or to the right or the left of the origin.

18. Given the parent function for the absolute value function, compare each of the graphs below to the parent function and indicate how the graphs are similar to or different from it.

   a. \( y = |x| \) compared to \( y = |x + 2| \)

   ![Graph](image1)

   The graph is shifted two units left.

   b. \( y = |x| \) compared to \( y = 2|x| \)

   ![Graph](image2)

   The graph is narrower than the parent.

If needed, have students create a table of input and output values before comparing each of the transformed functions to the parent function.
Note that $|2x - 2|$ is the same as $|2(x-1)|$ so that this function has actually been sifted right and then dilated. Because the transformations of the function are grouped with the variable $x$ in the absolute value quantity, there is no movement vertically.

However, in the function $y = |2x| - 2$, the graph has been dilated and moved down two units. Because only the 2 which is being multiplied by $x$ is grouped with it, the second 2, which is subtracted from the absolute value quantity, does not affect the graph as a horizontal transformation. Because this second 2 is not grouped with the variable $x$, it affects the graph with a vertical transformation. Note that this function could be rewritten as $y+2 = |2x|$, whereas the function in d could not be rewritten as such.

c. $y = |x|$ compared to $y = \frac{|x|}{2}$

The graph is wider than the parent function.

d. $y = |x|$ compared to $y = |2x - 2| = |2(x-1)|$

The graph shifted one unit to the right and is narrower than the parent function.

e. $y = |x|$ compared to $y = |2x| - 2$

The graph is narrower and shifted down 2.
19. Graph the following pairs of absolute value functions on the same set of axes. Compare and contrast each pair of functions. If needed, generate a table of values for each.

a. \( y = -|x| \) and \( y = 2|x| \)

b. \( y = 3|x| \) and \( y = 2|x| \)

c. \( y = -3|x| \) and \( y = -3|x| \)

- The graphs are mirror images.
- These graphs are identical.
- These graphs are mirror images.
Equations, Rates, and Intercepts: Part 4

d. \[ y = |x| + 2 \quad \text{and} \quad y = |x + 2| \]

The first graph is shifted up two, but the second graph is shifted left by 2 units.

20. Sketch the following absolute value graph without creating a table of values. Use a graphing calculator to confirm that your sketch is correct.

a. \[ y = \frac{x}{2} + 2 \]

Compared to the parent function, this graph is vertically stretched by a factor of 1/2 (wider), and it is shifted to the left by 2.

b. \[ y = |x - 2| \]

Compared to the parent function, this graph is shifted by 1 unit to the left. (This is because -x-1 is the same as -(x+1), and because the negative sign in the absolute value quantity has no effect on the graph of the function.)
### Equations, Rates, and Intercepts: Part 4

#### process: practice

c. \( y = |2x + 2| \)

Compared to the parent function, this graph is shifted left by 1 unit, dilated by a factor of 2, and then reflected over the x-axis.

### Equations, Rates, and Intercepts: Part 4

#### process: summary

21. Describe the differences between \( y = x \) and \( y = |x| \) with respect to the domain and range. Why do these differences exist?

**Sample Response:**

*The domain for both \( y = x \) and \( y = |x| \) is the set of all real numbers, but the range for each function is different. The range for \( y = x \) is all reals, but for the absolute value function, the range is only the non-negative real numbers. This occurs because the absolute value function measures distance from the origin and the distance cannot be negative. Thus, no negative numbers are included as outputs of the function.*

22. Describe any similarities or differences between \( y = x \) and \( y = |x| \) with respect to the effect of changes in the equations on the graph.

**Sample Response:**

*Most of the changes in the equations seem to affect both parent functions similarly. For example, when a number is added or subtracted from \( x \) in the equation \( y = x \) and from \( |x| \) in the equation \( y = |x| \), the graph of the function moves up or down. Similarly, when a number is multiplied by \( x \) or \( |x| \) the “slope” of the graph changes.*

*However, when the negative sign is in the absolute value quantity or when a constant is added to \( x \) within the absolute value quantity, the changes differ from the changes seen with a line. The negative sign in the absolute value quantity does not make the graph reflect over the x-axis, while a negative sign outside the \( | | \) does. A constant added within the absolute value moves the graph horizontally, while a constant added outside the \( | | \) moves the graph vertically.*
Generating and Interpreting Linear Equations

In this next group of problems you will review key concepts about linear equations using numeric, graphic, and symbolic representations. You will also look at alternate ways to find the equation of a line.

1. Given \( y = 3x + 2 \), answer the following questions.

   a. Will the graph of this equation increase or decrease from left to right?

      \textbf{It will increase.}

   b. What are the coordinates of the intercepts? What do the intercepts represent?

      \begin{align*}
      \text{x-intercept} & = \left(-\frac{2}{3}, 0\right) \\
      \text{y-intercept} & = (0, 2)
      \end{align*}

      \textbf{The intercepts represent the points where the line crosses the axes.}

   c. Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

      \textbf{\( y \) increases by 3.}

   d. What is the slope of the line? What does it represent?

      \textbf{The slope is 3. The slope indicates that, as \( x \) increases by 1, \( y \) increases by 3.}

   e. Generate a table with three values for \( x \) and \( y \). Label each column of the table.

      \begin{tabular}{|c|c|}
      \hline
      \( x \) & \( y \) \\
      \hline
      0 & 2 \\
      1 & 5 \\
      -1 & -1 \\
      \hline
      \end{tabular}

   f. Sketch a graph of the equation.
Students may need to generate a table of values to answer question 2, 5, and 8.

### Generating and Interpreting Linear Equations

<table>
<thead>
<tr>
<th>Process: Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2. Use the graph to find:</strong></td>
</tr>
<tr>
<td>Slope: ( \frac{1}{1} )</td>
</tr>
<tr>
<td>( y )-intercept: ((0, 0))</td>
</tr>
<tr>
<td>Equation: ( y = x )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>3. Given the data table, determine the information requested below and sketch the graph of the equation.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Slope: ( 5 )</td>
</tr>
<tr>
<td>( y )-intercept: ((0, 0))</td>
</tr>
<tr>
<td>Equation: ( y = 5x )</td>
</tr>
</tbody>
</table>
4. Given \( y = f(x) = -2x + 3 \), answer the following questions.

a. Will the graph of this equation increase or decrease from left to right?

**It will decrease.**

b. What are the coordinates of the intercepts? What do the intercepts represent?

\[
x\text{-intercept } = \left( \frac{3}{2}, 0 \right) \quad y\text{-intercept } = (0, 3)
\]

*The intercepts are the points where the line crosses the axes.*

c. Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

**\( y \) decreases by 2.**

d. What is the slope of the line? What does it represent?

*The slope is -2. It indicates that, as \( x \) goes up by 1, \( y \) goes down by 2.*

e. Generate a table with three values for \( x \) and \( y \). Label each column of the table.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

f. Sketch a graph of the function.
5. Use the graph to find:
   - Slope: $\frac{1}{1}
   - y$-intercept: $(0, -3)
   - Function: $y = x - 3$

6. Given the data table, determine the information requested below and sketch the graph of the equation.

<table>
<thead>
<tr>
<th>Expression</th>
<th>2</th>
<th>550</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>1300</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>50</td>
</tr>
</tbody>
</table>

   - Slope: $\frac{250}{250} = 1$
   - $y$-intercept: $(0, 50)$
   - Function: $y = 250x + 50$
7. Given \( y = \left( -\frac{2}{3} \right) x + 7 \), answer the following questions.

   a. Will the graph of this equation increase or decrease from left to right?

   The graph will decrease.

   b. What are the coordinates of the intercepts? What do the intercepts represent?

   \[ \text{x-intercept} = (10.5, 0) \quad \text{y-intercept} = (0, 7) \]

   The intercepts are the points at which the line crosses the axes.

   c. Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

   \( y \) decreases by \( \frac{2}{3} \)

   d. What is the slope of the line? What does it represent?

   The slope is \( -\frac{2}{3} \). It indicates that, as \( x \) goes up by 3, \( y \) goes down by 2.

   e. Generate a table with three values for \( x \) and \( y \). Label each column of the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

   f. Sketch a graph of the equation.
8. Use the graph to find:

- Slope: \(-2\)
- y-intercept: \((0, -3)\)
- Function: \(y = -2x - 3\)

9. Given the data table, determine the information requested below and sketch the graph of the equation.

<table>
<thead>
<tr>
<th>x</th>
<th>20</th>
<th>-30</th>
<th>5</th>
<th>2x + 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>70</td>
<td>-30</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

- Slope: \(2\)
- y-intercept: \((0, 30)\)
- Equation: \(y = 2x + 30\)
It can be assumed that the spending pattern has been and will continue to be constant.

### Spending Money

Today you have $48 left from the money you got for your birthday. On average you have been spending $3 per day and you are not planning on changing this spending pattern.

1. Generate and graph an equation that represents your spending patterns. Define each variable and indicate the set of input (domain) and output (range) values.

   Equation: \( y = 48 - 3x \)
   
   Variables:
   - \( x \): time in days
   - \( y \): money remaining in dollars

   Domain: \( x \geq 0 \)
   
   Range: \( y \leq 48 \)
2. After representing the situation algebraically and graphically, indicate
   a. The slope and what it represents in general and specifically in this situation.
      The slope is -3. In this situation, it represents the rate of spending per day. In general the slope is the change in y for every unit change in x.
   
   b. The y-intercept and what it represents in general and specifically in this situation.
      The y-intercept is 48. It represents the amount of money you have remaining from your birthday at this moment in time. In general the y-intercept represents the point where the line crosses the y-axis.
   
   c. The x-intercept and what it represents in general and specifically in this situation.
      The x-intercept is 16. It represents the number of days until all of the remaining money is spent. In general, the x-intercept represents the value of the independent variable (x) when the dependent variable (y) is 0. It represents the point at which the line crosses the x-axis.

3. Explain why the graph of the equation increases (or decreases) in this problem.
   The graph of this equation decreases because the initial amount of money is decreasing from day to day as it is spent. The amount of money is decreasing at a constant rate of $3 per day.
Spending Money

4. How much money did you have when you started spending your birthday money 20 days ago?

   Twenty days ago, there was $108.

   \[ y = 48 - 3x = 48 - 3 \times (-20) = 108 \]

5. How much money will you have 6 days from now?

   Six days from now, there will be $30 left.

   \[ y = 48 - 3x = 48 - 3 \times 6 = 30 \]

6. If you spend at this rate for 20 more days, how much money will you have spent?

   In twenty more days, there will be $-12, meaning there will be a debt of $12.

   \[ y = 48 - 3x = 48 - 3 \times 20 = -12 \]

7. How long will it take to spend all your money, but not spend more than you have?

   It will take 16 days to spend the $48.

   \[ 0 = 48 - 3 \times 16 \]

Question 8 informally introduces the concept of a piecewise function. You may choose to explore this in more detail, including defining a piecewise function algebraically and generating the graph on a graphing calculator.

The graph for question 8 can vary. Student graphs should have a larger decreasing slope at first and a smaller decreasing slope after the horizontal break.

8. If after five more days you stop spending for five days and then reduce your spending to $1.50 per day, what will the graph of your spending patterns look like?

![Graph of Spending Money](image-url)
Another possible variation on question 9 would be to change the rate of spending and consider the effects on the graph, domain and range, etc.

Spending Money

9. How would the solution to this problem change if the original scenario were changed to, “Two days ago you had $48 left from the money you got for your birthday. On average you have been spending $3 per day and you are not planning on changing this spending pattern.” Make sure to consider the graph, domain and range, slope and intercepts in your response.

To represent two days ago requires shifting the graph to the left by two units. To represent the new equation for spending two days ago or when time = 0 and the amount of money is $48 the equation will be \( y = 48 - 3(x + 2) \). This is equivalent to \( y = 48 - 3x - 6 \) or \( y = 42 - 3x \) beginning today. This means the new y-intercept is at the point \((0, 42)\) with the same rate of spending or slope \(-3\). The new x-intercept is \((14, 0)\). The domain and range remain unchanged.

10. You are the teacher for the day. You have selected the following problem for your students to work on in class.

A scuba diver is 110 feet below sea level. She knows that to avoid suffering from the bends, she must come up at a rate of 7 feet per minute.

To prepare for the day’s activity you must come up with:

a. A set of questions that require finding several different locations of the scuba diver

Sample Responses:

- How deep will the diver be in 1 minute?
- How deep will the diver be in 5 minutes?
- How deep was the diver 3 minutes ago?
- When will the diver reach the surface of the water?
- When will the diver reach a depth of 50 feet below the surface?
- When was the diver at a depth of 150 feet below the surface?
Spending Money

b. A set of questions that will help you to determine the student’s understanding of the key mathematical concepts in the problem

**Sample Responses:**
- **At what rate is the diver ascending?**
- **What is the slope in this situation, what are its units, and what does it mean?**
- **What is the y-intercept and what does it tell you?**
- **What is the x-intercept and what does it mean in the problem?**

c. One question that requires some analysis

**Sample Response:**
- Why does this problem have a negative y-intercept, yet a positive slope?

d. A detailed solution of the problem

\[
x: \text{ the time in minutes} \\
y: \text{ the distance above the water in feet (negative numbers indicate that the diver is below the surface of the water)}
\]

The slope is 7 feet per minute.

The y-intercept is (0, -110) meaning the diver starts 110 feet below the surface of the water.

The x-intercept is (15.7, 0) meaning the diver reaches a height of 0 feet, which is the surface of the water, in 15.7 minutes.

The equation for the situation is \(y = 7x - 110\)
The upcoming activities, Comp-U-Us and Mowing Lawns, both involve equations that can be written in two different forms through the use of the distributive property. This property will be formally discussed beginning on page 4-56.

You and two of your friends have decided to start a new company, Comp-U-Us, to assemble and sell computers.

So far you have assembled 20 computers. Your friend has just taken an order from a large local company that has agreed to purchase a number of computers from your company. Your sales price is $1,800 per computer.

1. Define a variable for the additional number of computers you will be assembling (in addition to the 20 computers already assembled).

   \[ C \text{ will represent additional computers assembled.} \]

2. Use the variable to write an expression for the total number of computers that your company will have assembled.

   \[ \text{The total number of computers will be } C + 20. \]

3. Write two expressions for the total income your company will receive for all the computers that will be assembled.

   a. Multiply the money you’ll receive for each computer by the expression for the total number of assembled computers:

      \[ 1800 (C + 20) \]

   b. Add the expression for the money you’ll receive from selling the additional computers and the money you’ll receive from selling the 20 computers you’ve already assembled:

      \[ 1800 C + 1800 \cdot 20 \text{ or } 1800 C + 36000 \]
Students may choose to use either expression generated in question 3 to answer questions 4 through 7. Most students will choose the equation from part (b) since it is similar to other equations they have encountered. However, if students have completed Unit 7 of the software curriculum, which focuses on using the distributive property to solve equations, they may feel comfortable using the form in part (a).

4. How much will your company receive if you assemble and sell:
   a. 20 additional computers? **The company will get $72,000.**
   b. 50 additional computers? **The company will receive $126,000**
   c. 100 additional computers? **The company will get $216,000.**
   d. 200 additional computers? **The company will get $396,000.**

Write a sentence explaining how you solved the problems above.

**The number of additional computers was multiplied by $1800 and the result was added to $36,000.**

**OR**

**The additional computers were added to 20 and that number was multiplied by $1800.**

5. To have a total income of $576,000, how many additional computers must you assemble? Explain how you found this answer.

**There must be 300 additional computers assembled. First, the money already earned ($36,000) was subtracted from total income. Then, the result was divided by the amount received per computer ($1800).**

**OR**

**First, 576,000 was divided by the amount received per computer ($1800). Then the 20 computers already sold were subtracted from the total number of computers sold.**

6. Complete this table:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Computers to Be Assembled</th>
<th>Total Computers</th>
<th>Total Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>computers</td>
<td>computers</td>
<td>$</td>
</tr>
<tr>
<td>Expression</td>
<td>C</td>
<td>C + 20</td>
<td>1800(C + 20) or 1800C + 36,000</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>72,000</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>70</td>
<td>126,000</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>120</td>
<td>216,000</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>220</td>
<td>396,000</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>20</td>
<td>36,000</td>
<td></td>
</tr>
</tbody>
</table>
7. Construct a graph for this situation, using the computers to be assembled and total income.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>x: Additional computers</td>
<td>0</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>y: Income</td>
<td>0</td>
<td>400,000</td>
<td>20,000</td>
</tr>
</tbody>
</table>
### Comp-U-Us

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. Which of the two algebraic representations from question 3 did you use to complete questions 4-6? Why did you use that representation over the other choice?</td>
</tr>
</tbody>
</table>

**Sample Response:**

*I used the first representation because I thought adding the number of additional computers before I multiplied by the income per computer was an easier way to think about the problem.*
You and three of your friends have decided to start a lawn mowing business. To start your business, you purchase two lawn mowers, a string trimmer, a leaf blower, and various other tools, for a total of $650. You decide to split all the profit evenly among the four of you.

Mowing Lawns

1. Define a variable for the total income.
   
   Let I represent the total income.

2. Use your variable to write an expression for the total profit.
   
   Profit will be I – 650.

3. Write two expressions for the amount of profit each person will receive.
   a. Divide the expression for total profit by 4:
      
      \(\frac{(I - 650)}{4}\)

   b. Subtract one-fourth of the start-up cost from one-fourth of the expression for total income:
      
      \(\frac{I}{4} - \frac{650}{4}\) or \(\frac{I}{4} - 162.5\)
Mowing Lawns

4. How much profit will each of you receive if your total income is:
   a. $750? Each will receive $25.00 in profit.
   b. $1,000? Each person will get $87.50.
   c. $2,000? Each will get $337.50.
   d. $10,000? The profit for each will be $2337.50.

   In a sentence or two, describe how you found these answers.

   Sample response:
   To find these answers, subtract $650 from the total income to find the total profit. Then, divide the profit by four for the partners.

5. If each person’s profit is $400, how much was the total income? Explain how you found this result.

   Sample response:
   The total income was $2250. Get this result by multiplying each person’s profit by four to find the total profit. Then, add the $650 that was spent for supplies.

6. Complete this table:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Income</th>
<th>Total Profits</th>
<th>Each Partner’s Profit Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>dollars</td>
<td>dollars</td>
<td>dollars</td>
</tr>
</tbody>
</table>
| Expression | I | I - 650 | \((I-650)/4\) or 
|          |           |               | \((I/4) – 162.50\)           |
| 750    | 100         | 25            |                             |
| 1,000  | 350         | 87.5          |                             |
| 2,000  | 1350        | 337.5         |                             |
| 10,000 | 9350        | 2337.5        |                             |
| 2250   | 1600        | 400           |                             |
7. Construct a graph, using the total income and each partner’s profit share for this situation.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>x: Income</td>
<td>0</td>
<td>10,000</td>
<td>500</td>
</tr>
<tr>
<td>y: Each person’s profit</td>
<td>0</td>
<td>3000</td>
<td>150</td>
</tr>
</tbody>
</table>
**Mowing Lawns**

8. Cite at least two similarities between this Comp-U-U problem and the Mowing Lawns problem. Make sure to examine the algebraic expressions, the table of values, the graph, and the scenario itself.

   **Sample Response:**
   - Each scenario was modeled by a linear function of the form \( y = mx + b \) or \( y = m(x - x_1) + b \).
   - In each scenario an initial or start value was given and a positive rate representing a positive slope and increasing function.

9. Cite at least two differences between this Comp-U-U problem and the Mowing Lawns problem. Make sure to examine the algebraic expressions, the table of values, the graph, and the scenario itself.

   **Mowing Lawns had a negative y-intercept while Comp-U-U had a positive y-intercept.**
   
   **The slopes were both positive, but the slope in Mowing Lawns was a fraction while the slope in Comp-U-U was a large integer.**

10. Which of the two algebraic representations from question 3 did you use to complete questions 4-6? Why did you use that representation over the other choice?

    **Sample Response:**
    I used the second one. I divided the income by four first and then subtracted each person’s expense. I thought it was easier to do the division first.
Linear Equations and Systems of Equations

Solving equations using the distributive property is covered in Unit 9 of the software curriculum.

Students may have difficulty seeing the equivalence between the two expressions. Be sure students understand that both expressions are the same in question 1 before moving on to question 2.

The Distributive Property

Think about the situations presented in Comp-U-Us and Mowing Lawns. You used two different expressions to model each situation.

In Comp-U-Us, one expression you wrote was

1800(x+20).

The other expression you wrote was

1800x + 20(1800)

or

1800x + 36,000.

1. Are these expressions all equivalent? Do you get the same results from either algebraic expression? How can you determine if they are equivalent? Give an example that illustrates whether the two expressions are equivalent or not.

The expressions are equivalent, and the same results are obtained from either algebraic expression. If the expressions always give the same result, then they are equivalent.

If \( x = 1 \)

\[ 1800 \times (1 + 20) = 1800 \times 21 = 37,800 \text{ for the first expression.} \]

If \( x = 1 \)

\[ 1800 \times 1 + 36,000 = 37,800 \text{ for the second expression.} \]

2. Your example above should help to show that

\[ 1800(x + 20) = 1800x + 1800 \times 20 \]

The “law” or rule that says that these two expressions are equal is called the Distributive Property. Look at the left side of this equation. Explain what was done to arrive at the right side of the equation.

*The number outside the parentheses was multiplied by the \( x \) and by 20. Those two parts were added together.*
Students may not see the similarity between questions like number 5 and the other questions here. For questions 5, 11, and 12, students may only multiply the last term in the parentheses by the quantity outside the parentheses. Remind students that multiplication is commutative. For example, 5 times 3 is the same as 3 times 5, so \((x-10)\ 100 = 100 \ (x-10)\).

**The Distributive Property**

Use the Distributive Property to rewrite each expression as a sum or difference:

Example: \(5(x + 11) = 5x + 55\)

3. \(2(x + 3) = 2x + 6\)  
4. \(2(2x + 14) = 4x + 28\)

5. \((x - 10)\ \cdot\ 100 = 100x - 1000\)  
6. \(7(5x - 7) = 35x - 49\)

7. \(5(7 - 3x) = 35 - 15x\)  
8. \(3(2 - 24x) = 6 - 72x\)

9. \(x(x + 3) = x^2 + 3x\)  
10. \(3(5x - 11) = 15x - 33\)

11. \((30 - 20x)\ \cdot\ 5 = 150 - 100x\)  
12. \((12x - 23)\ \cdot\ 3 = 36x - 69\)

In the Mowing Lawns problem, one expression you used was \(x\ -\ 650\ \frac{4}{4}\). The other expression you used was \(\frac{x}{4} - 162.50\).

13. Do you get the same results with either of these algebraic expressions? Are they equivalent? How can you determine if the expressions are equivalent? Give an example that will illustrate whether the two expressions are the same or not.

*Either expression gives the same results, so the expressions are equivalent. If the results obtained are identical for every x, the expressions must be equivalent.*

For \(x = 700\), \(\frac{x - 650}{4} = \frac{50}{4} = 12.5\)

For \(x = 700\), \(\frac{x}{4} - 162.50 = 175 - 162.5 = 12.5\)
### The Distributive Property

Your example should help to show that

\[
\frac{x - 650}{4} = \frac{x}{4} - 162.50
\]

14. The left side is equal to the right side because of the Distributive Property. Look at the left side. Why is it equivalent to the right side? Explain how this can be the same property (the Distributive Property) as was used in the Comp-U-Us problem.

*The left side is equivalent to the right side because, if \( x \) is divided by 4 it becomes \( x/4 \), and if 650 is divided by 4 it becomes 162.5. These two expressions will give the same result no matter which value of \( x \) is used.*

*This is the same property as was used in Comp-U-Us because you can think of this as multiplying \((x-650)\) by one-fourth instead of dividing by 4.*

---

The simplification of fractions may need to be reviewed prior to completing questions 15 through 20.

### The Distributive Property

Use the Distributive Property to rewrite each expression as a sum or difference:

**Examples:**

\[
\frac{x + 15}{5} = \frac{x}{5} + 3
\]

\[
\frac{3x - 27}{3} = x - 9
\]

15. \[
\frac{30 - 20x}{5} = 6 - 4x
\]

16. \[
\frac{12x - 23}{3} = 4x - \left(\frac{23}{3}\right)
\]

17. \[
\frac{16 - 2x}{2} = 8 - x
\]
Do not be concerned about teaching the laws of powers at this point. Questions 19 and 20 will test the students’ intuitive sense about dealing with variables raised to a power, but it is not yet expected that students know how to formally deal with such algebraic expressions.

### The Distributive Property

<table>
<thead>
<tr>
<th>Process: Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. ( \frac{300x - 700}{100} = 3x - 7 )</td>
</tr>
<tr>
<td>19. ( \frac{x^2 + 5x}{x} = x + 5 )</td>
</tr>
<tr>
<td>20. ( \frac{12x^2 - 15x}{3x} = 4x - 5 )</td>
</tr>
</tbody>
</table>

### The Distributive Property

In the last few process windows, you practiced using the distributive property. An example of this property is:

\[ 2(x + 3) = 2x + 6 \]

21. If you are given an expression like \( 4x + 8 \), could you “undistribute” to write this expression as a product? If so, show this product below. If not, explain why not.

\[ 4x + 8 \text{ can be “undistributed” by writing } 4(x + 2) \]

In these problems, “undistributing” or using the distributive property in reverse is called factoring or factoring out a common term. To undistribute or factor out a common term, you find the greatest common factor. Write your expression as the product of the greatest common factor and what remains from each term. (A term is a variable, a constant, or a product of variables and constants.)
### The Distributive Property

#### Examples:
- $5x + 20$ can be written as $5(x + 4)$ because 5 is the greatest common factor between $5x$ and 20.
- $14x + 35 = 7(2x + 5)$ because 7 is the greatest common factor between $14x$ and 35.
- $x^2 + 4x$ is factored as $x(x + 4)$ because $x$ is the greatest common factor.

Notice that when you apply the distributive property you return to the original expression:
- $5(x - 4) = 5x - 20$
- $7(2x + 5) = 14x + 35$
- $x(x + 4) = x^2 + 4x$

#### The Distributive Property

**Factor these expressions, as was done in the examples.**

<table>
<thead>
<tr>
<th>Input</th>
<th>Process: Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>22. $2x + 6 = 2(x + 3)$</td>
<td>23. $-4x + 6 = -2(2x - 3) \text{ or } 2(-2x + 3)$</td>
</tr>
<tr>
<td>24. $7x - 21 = 7(x - 3)$</td>
<td>25. $4x + 28 = 4(x + 7)$</td>
</tr>
<tr>
<td>26. $100x - 1000 = 100(x - 10)$</td>
<td>27. $-5 + 15x = -5(1 - 3x) \text{ or } 5(-1 + 3x)$</td>
</tr>
<tr>
<td>28. $-14x + 21 = -7(2x - 3)$</td>
<td>29. $32 - 24x = 8(4 - 3x)$</td>
</tr>
<tr>
<td>30. $3x + 5 = \text{ cannot factor using whole numbers}$</td>
<td>31. $15 + 25x = 5(3 + 5x)$</td>
</tr>
<tr>
<td>32. $35x - 112 = 7(5x - 16)$</td>
<td>33. $30 - 20x = 10(3 - 2x)$</td>
</tr>
<tr>
<td>34. $x^2 + 3x = x(x + 3)$</td>
<td></td>
</tr>
</tbody>
</table>
You may decide to mention to students that constants are terms for which the variable portion can be thought of as raised to the zero power. Of course, this means all constants are like terms.

The Distributive Property

On the last few pages, all of the expressions you saw contained two terms. Algebraic expressions that contain two terms are **binomials**. Algebraic expressions that contain only one term are **monomials**. Many algebraic expressions contain more than two terms. These expressions are called **trinomials** if they contain three terms. **Polynomial** is the word used to describe any algebraic expression with one, two, three, or more terms. Some polynomials can be simplified by combining like terms, which are terms that have the same variable portion raised to the same power.

For example, the polynomial

\[3x^2 - 9 - x^2 + 2\]

can be simplified to the binomial

\[2x^2 - 7\]

by combining the like terms.

35. Can you combine any like terms to simplify

\[7x^2 + 4x^2 + 2x + 7\]

Why or why not?

*This expression has no like terms because none of the variable portions are raised to the same power. Therefore, the expression can not be simplified.*

You can also use the distributive property to simplify polynomials that contain multiple terms.

Examples:

- \(5(x - 3) + 2x\) simplifies to \(5x - 15 + 2x\) using the distributive property, then to \(7x - 15\) by combining like terms.
- \(3x - 5(2 - 3x)\) simplifies to \(3x - 10 + 15x\) using the distributive property, then to \(18x - 10\) by combining like terms.
- \(12 + 5(2x - 3)\) simplifies to \(12 + 10x - 15\) using the distributive property, then to \(-3 + 10x\) or \(10x - 3\) by combining like terms.
To facilitate classroom interaction on this problem set, assign problems so that each student/group does three problems. Then have students present their explanations of the solutions.

### The Distributive Property

Simplify these expressions, as was done in the examples.

36. \(3(5x - 4) - 2x = \) \(13x - 12\)

37. \(5x - 2(3x - 4) = \) \(-x + 8\)

38. \(2x + 3(7 + x) = \) \(5x + 21\)

39. \(11x - 5x + 12 - 3x = \) \(3x + 12\)

40. \(12 - 3x + 7 = \) \(19 - 3x\)

41. \(5(x - 4) + 3(2 - x) = \) \(2x - 14\)

42. \(7(2x - 4) + 11 = \) \(14x - 17\)

43. \(2(3x + 5) - 3(2x + 5) = \) \(-5\)

44. \(5(x - 5) + 3(x - 2 - 5) = \) \(-15x^2 + 11x - 25\)

45. \(6(x - 7) - 6(x - 7) = \) \(0\)

46. \(8(2x + 4) - 8(2x - 4) = \) \(64\)
This section serves a dual purpose. First, it helps students see that many known formula are linear models that apply to situations involving a definable set of variables, like distance, rate and time, or area, length, and width. Second, it introduces the manipulation of equations, writing one variable in terms of another. When students reach the section on solving systems of equations, having a feel for equations in more than one variable and how to transform these equations will be helpful.

Students have worked “backwards” problems involving numbers and have an understanding of independence and dependence. The goal here is for students to apply and generalize their understanding.

### Solving Equations Using Known Formulas

While we have been modeling numerous novel situations, there are classes of problems that can all be modeled by the same formula. Name at least three formulas that you are familiar with:

**Students are likely to rely on formulas they know such as:**

- \( A = lw \)
- \( A = \frac{1}{2} bh \)
- \( A = br^2 \)

The equations you have identified are called **literal equations** because they all contain multiple variables. Interpreting literal equations is a powerful way to understand how one variable depends on another. For example, look at the equation for finding distance.

\[ d = rt \]

Interpreting it shows that the distance \( (d) \) is dependent on the rate \( (r) \) at which you travel and the period of time \( (t) \) you travel.

1. Based on the example above, how would you interpret the equation for the area of a rectangle?

   **The area is dependent on the length and width of the rectangle.**

2. Suppose you want to write the width of a rectangle as a function of the area and the length. How would you go about doing this? Write the transformed equation and write the new relationship in words.

   **You would write the function for the area of a rectangle as** \( A = lw \).

   **You will want to write** \( w = \) so you “solve” the equation for \( w \) by dividing both sides by \( l \), so \( w = \frac{A}{l} \). The width of the rectangle would be equal to the area divided by the length.

3. Suppose you want to write temperature in Celsius as a function of the temperature in Fahrenheit \( (F = \frac{9}{5} C + 32) \). How would you go about doing this? Write the transformed equation and write the new relationship in words.

   **You would write the temperature in Fahrenheit as** \( F = \frac{9}{5} C + 32 \). You will want to write \( C = \) so you must solve for \( C \). To do so, subtract 32 from each side. Multiply both sides by \( 5 \) and then by \( \frac{1}{9} \). The new equation is \( C = \frac{5}{9} (F - 32) \). This equation says Centigrade is five ninths the temperature in Fahrenheit minus 32 degrees.
Solving Equations Using Known Formulas

4. The formula for the volume of a rectangular box is
   \[ V = l \cdot w \cdot h \]
   Where \( V \) = volume, \( l \) = length, \( w \) = width, and \( h \) = height.
   a. Write the formula in words.
   \textbf{The volume of a rectangular box is equal to the length of the box times the width of the box times the height of the box.}
   
   b. Solve the formula for \( h \).
   \[
   \begin{align*}
   V &= l \cdot w \cdot h \\
   \frac{V}{l \cdot w} &= h
   \end{align*}
   
   c. Write the equation for \( h \) in words.
   \textbf{To find the height of the rectangular box, divide the volume by the product of the length and width of the box.}

5. The circumference of a circle is defined as
   \[ C = \pi d \]
   Where \( C \) is the circumference, \( \pi \) a constant, and \( d \) is the diameter.
   a. Write the formula in words.
   \textbf{Circumference (the distance around the circle) is equal to the product of \( \pi \) and the diameter of the circle.}
   
   b. Solve for \( d \).
   \[
   \begin{align*}
   C &= \pi d \\
   d &= \frac{C}{\pi}
   \end{align*}
   
   c. Write the equation for \( d \) in words.
   \textbf{The diameter of the circle is equal to the circumference divided by the constant \( \pi \).}
### Solving Equations Using Known Formulas

6. \( I = prt \) represents the formula for computing the interest earned on an investment. \( P \) represents the principle, \( r \) is the interest rate, and \( t \) is the time the money is invested. (The interest percentage is written as a decimal.)

   a. Write the formula in words.
   
   The amount of interest you earn is equal to the principle (amount of money you start with) multiplied by the interest rate percentage (written as a decimal) multiplied by the amount of time that passes.

   b. Solve for \( r \).

\[
I = prt \\
\frac{I}{pt} = r
\]

   c. Write the equation for \( r \) in words.

   The rate (expressed as a decimal) is equal to the interest earned divided by the product of the principle and the time that passes.

7. The area of a triangle is defined by the measure of the base times the measure of height divided in half.

   a. Write the formula symbolically.

\[
A = \frac{b \cdot h}{2} \quad \text{or} \quad b \frac{h}{2}
\]

   b. Solve for \( b \).

\[
A = \frac{b \cdot h}{2} \\
2A = b \cdot h \\
\frac{2A}{h} = b
\]

   c. Write the equation for \( b \) in words.

   The base of a triangle is equal to two times the area divided by the height.
At this point students are familiar with linear equations. The problems in the systems portion of the unit introduce the concept of examining two linear equations but are also a mechanism for transferring understanding to novel situations.

### Producing and Selling Markers

Since you are interested in learning about business, you take a part-time internship at a company that produces color art markers. In particular, you will be studying production costs at this company. You are told that it costs $2 to manufacture each marker and, before any marker is produced, there is a $100 start-up cost for setting the color.

The first problem facilitates the process by comparing two scenarios, while the second provides one scenario from which two equations can be generated and compared.

<table>
<thead>
<tr>
<th>Process: Model</th>
<th>Producing and Selling Markers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write an algebraic equation that represents the cost to produce the markers. Indicate what each variable and constant represents in the equation.</td>
<td></td>
</tr>
<tr>
<td>$C = \text{cost to produce markers measured in dollars}, \ m = \text{markers produced}$</td>
<td></td>
</tr>
<tr>
<td>$$2 \text{ is the cost to make each marker}$</td>
<td></td>
</tr>
<tr>
<td>$$100 \text{ is the start-up cost}$</td>
<td></td>
</tr>
<tr>
<td>$C = 2m + 100$</td>
<td></td>
</tr>
</tbody>
</table>

2. How much would it cost to produce 50 markers?

\[
2 \cdot 50 + 100 = 200.00
\]

3. How many markers can be produced for $474.00?

\[
474 = 2m + 100
\]
\[
474 - 100 = 2m
\]
\[
\frac{374}{2} = \frac{2m}{2}
\]
\[
187 = m
\]

You can produce 187 markers for $474.00.
### Producing and Selling Markers

4. Graph the equation representing the cost of production. Indicate the domain and the range for the function.

![Graph of linear equation](image)

<table>
<thead>
<tr>
<th>markers produced</th>
<th>cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>102</td>
</tr>
<tr>
<td>2</td>
<td>104</td>
</tr>
<tr>
<td>3</td>
<td>106</td>
</tr>
<tr>
<td>4</td>
<td>108</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
</tr>
<tr>
<td>6</td>
<td>112</td>
</tr>
<tr>
<td>7</td>
<td>114</td>
</tr>
<tr>
<td>8</td>
<td>116</td>
</tr>
<tr>
<td>9</td>
<td>118</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>11</td>
<td>122</td>
</tr>
<tr>
<td>12</td>
<td>124</td>
</tr>
<tr>
<td>13</td>
<td>126</td>
</tr>
<tr>
<td>14</td>
<td>128</td>
</tr>
<tr>
<td>15</td>
<td>130</td>
</tr>
<tr>
<td>16</td>
<td>132</td>
</tr>
<tr>
<td>17</td>
<td>134</td>
</tr>
<tr>
<td>18</td>
<td>136</td>
</tr>
<tr>
<td>19</td>
<td>138</td>
</tr>
<tr>
<td>20</td>
<td>140</td>
</tr>
</tbody>
</table>

**Domain** $x > 0$  
**Range** $y > 100$

### Producing and Selling Markers

5. To analyze the model for production cost in mathematical terms:

a. Define what the slope means with respect to this problem situation.

*The slope ($2 per marker) is the cost of one marker or the change in cost for each additional marker.*

b. Define what the y-intercept means with respect to this problem situation.

*The y-intercept ($100) is the initial (start-up) cost before any markers are produced.*

c. Determine the x-intercept. Does the x-intercept have meaning with respect to this problem situation? If so, how do you interpret its meaning?

(-50, 0) is not meaningful, you cannot produce a negative amount of markers

d. Does the graph of the equation increase or decrease from left to right? What does that mean with respect to this situation?

*The graph of the equation increases. It means the more pens you produce, the more the cost increases.*
In the next phase of your internship, you learn about income from sales. You are told that the company sells its markers to distributors for $3 per maker.

6. Write an algebraic equation that represents the company’s income from the sale of its markers.

\[ I = \text{income from marker sales measured in dollars, } m = \text{markers sold} \]

\[ I = 3m \]

$3$ is the amount for which each marker sells.

7. Graph the equation representing the income from the sale of the markers. Indicate the domain and range.

Domain \[ x \geq 0 \]  
Range \[ y \geq 0 \]
### Producing and Selling Markers

8. To get a better idea of the company’s income based on its sales determine:

   a. the income if 50 markers are sold.

   \[ I = 3m \]
   \[ I = 3 \times 50 \]
   \[ I = 150 \] The income from selling 50 markers is $150.

   b. the number of markers sold when the income is $474.

   \[ I = 3m \]
   \[ 474 = 3m \]
   \[ \frac{474}{3} = m \]
   \[ 158 = m \] You will make $474 if you sell 158 markers.

9. To analyze the model of income in mathematical terms:

   a. Define what the slope means with respect to this problem situation.

   The slope ($3 per marker) is the income from one marker or the change in income for every additional marker sold.

   b. Define what the y-intercept means with respect to this problem situation.

   The y-intercept (0,0) is the income when 0 (zero) markers are sold.

   c. Determine the x-intercept. Does the x-intercept have meaning with respect to this problem situation? If so, how do you interpret its meaning?

   Yes, that would be the number of markers (0) that gives an income of 0.

   d. Does the graph of the equation increase or decrease from left to right? What does that mean with respect to this situation?

   The graph of the equation increases. Income increases as the number of markers sold increases.
10. Using the information you know about production costs and income, complete the following table.

<table>
<thead>
<tr>
<th>Number of Markers</th>
<th>Production Cost</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2m + 100$</td>
<td>$3m$</td>
<td></td>
</tr>
<tr>
<td>$2(0) + 100$</td>
<td>$3(0) = 0$</td>
<td></td>
</tr>
<tr>
<td>$20$</td>
<td>140</td>
<td>60</td>
</tr>
<tr>
<td>$30$</td>
<td>160</td>
<td>90</td>
</tr>
<tr>
<td>$35$</td>
<td>170</td>
<td>105</td>
</tr>
<tr>
<td>$55$</td>
<td>210</td>
<td>165</td>
</tr>
<tr>
<td>$125$</td>
<td>350</td>
<td>375</td>
</tr>
<tr>
<td>$200$</td>
<td>500</td>
<td>600</td>
</tr>
<tr>
<td>$400$</td>
<td>900</td>
<td>1200</td>
</tr>
<tr>
<td>$2500$</td>
<td>5100</td>
<td>7500</td>
</tr>
</tbody>
</table>
11. Examine the relationship between the production costs and the income by graphing both equations on the grid below. Make sure to label each axis and to label each line with its equation.

Producing and Selling Markers

analysis

Based on the graph, what can you say about the relationship between the production costs and the income from sales?

- Both graphs increase from left to right.
- The lines cross at the point (100, 300).
- The area to the left of the intersection costs are more than income.
- The area to the right of the intersection income is more than costs.
- The slope of the income line is greater than the cost line.
- The "break even point" is where the lines cross.
- Points to the left of the y-axis have no meaning to this problem.
Making and Selling Shirts

After working at U.S. Shirts and at the marker company, you decide to go into business for yourself. Your plan is to sell customized T-shirts. Since you know U.S. Shirts does a good job making the shirts, you will have them produce your shirts. U.S. Shirts will charge you $7.50 per shirt, plus a set-up fee of $22.50 for each new design. You will charge $8.25 for each shirt that you sell.

| Students should work toward a method of finding the point of intersection. At this point, students have not set the equations equal to each other to find the intersection. They only have found the point of intersection graphically. |

<table>
<thead>
<tr>
<th>1. Write an equation describing the cost of production. Indicate what the variables and constants mean in this equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ C = \text{cost of production measured in dollars} ]</td>
</tr>
<tr>
<td>[ S = \text{number of shirts} ]</td>
</tr>
<tr>
<td>$7.50 is the charge for each shirt produced by U.S. Shirts. $22.50 is the set-up fee charged by U.S. Shirts.</td>
</tr>
<tr>
<td>[ C = 7.50S + 22.50 ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Write an equation describing the income from sales. Indicate what the variables and constants mean in this equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ I = \text{income from sales, measured in dollars} ]</td>
</tr>
<tr>
<td>[ S = \text{number of shirts} ]</td>
</tr>
<tr>
<td>$8.25 is the income from each shirt sold.</td>
</tr>
<tr>
<td>[ I = 8.25S ]</td>
</tr>
</tbody>
</table>
Making and Selling Shirts

3. How much will it cost to produce 15 shirts?

\[ C = 7.50(15) + 22.50 = 112.50 + 22.50 = 135.00 \]

It will cost $135.00 to produce 15 shirts.

4. What will your income be if you sell all 15 shirts?

\[ I = 8.25(15) = 123.75 \]

I will earn $123.75 from selling 15 shirts.

5. If your production costs are $165 for one type of customized shirt, how many shirts did you have produced?

\[ C = 7.50S + 22.50 \]
\[ 165 = 7.50S + 22.50 \]
\[ 165 - 22.50 = 7.50S + 22.50 - 22.50 \]
\[ 142.50 = 7.50S \]
\[ S = \frac{142.50}{7.50} = 19 \]

I had 19 shirts produced for $165.00.

6. If your production costs are $247.50, how many shirts did you have produced?

\[ I = 8.25S \]
\[ 247.50 = 8.25S \]
\[ \frac{247.50}{8.25} = 30 = S \]

The cost for 30 shirts is $247.50.

a. If your income is $247.50, how many shirts did you sell?

\[ C = 30 \cdot 7.50 + 22.50 = 225 + 22.50 = 247.50 \]

The income for 30 shirts is also $247.50. (break-even point)

7. Graph the two equations. Label each axis and each line on the graph.

![Graph of linear equations](image)
Making and Selling Shirts

8. Based on the information you have about production costs, income from sales, and from the graph, complete the table below.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Number of Shirts</th>
<th>Production Costs</th>
<th>Income Based on Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>$S$</td>
<td>$7.50S + 22.50$</td>
<td>$8.25S$</td>
</tr>
<tr>
<td>Expression</td>
<td>0</td>
<td>22.50</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>165</td>
<td>156.75</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>210</td>
<td>206.50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>247.50</td>
<td>247.50</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>337.50</td>
<td>346.50</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>502.50</td>
<td>528</td>
</tr>
</tbody>
</table>

Answer each of the following questions below in complete sentences.

9. Do the two lines intersect? If so, at what point?

   Yes. The lines intersect at the point (30, 247.50).

10. In the context of this situation, how do you interpret the point of intersection?

    This is the point where the income earned is equal to the money spent (costs). This point is called the "break-even point."

11. In general, what does it mean to say that two lines intersect at a point?

    It means the $(x, y)$ values are the same for both lines.

12. In the context of this situation, what can you conclude about the portion of the graph to the left of the point of intersection?

    The points to the left indicate where the costs of production are greater than the income earned from sales.

   a. What can you conclude about the portion to the right of the intersection?

    The portion to the right indicates where the income from sales is greater than the cost of production (you make a profit).
Connecting Algebraic to Graphical Representations

In the previous two problems about producing and selling markers and shirts, you studied the relationship between two related situations. From a mathematical perspective, you have examined two linear equations in two unknowns or a system of two linear equations.

A system of equations is said to have a solution if the graphs of the two equations intersect at a point \((x, y)\). The value of the point of intersection, when substituted into each of the equations, represents a solution to each linear equation, so it is a solution to the system of equations.

Connecting Algebraic to Graphical Representations

Recall that in Making and Selling Shirts, the equation used to model your costs of $7.50 per shirt with a start-up fee of $22.50 was:  
\[ C = 7.5s + 22.5. \]

The equation used to model your income from selling shirts at $8.25 per shirt was:  
\[ I = 8.25s. \]

At this selling price, no one seems to be buying your shirts. Your friend suggests that you lower your selling price to $7.50 per shirt.

1. At the reduced selling price, what is the new equation that will model income?

\[ I = 7.5s \]

2. Create a graph showing both the equation that models your costs and the new equation that models your income.
Students may discuss the fact that money will always be lost under this plan. If all shirts are sold, only $22.50 will be lost, but if less are sold, then more money will be lost.

<table>
<thead>
<tr>
<th>Connecting Algebraic to Graphical Representations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. How do the graphs of the two equations relate to one another? Why do the graphs relate to one another in this manner?</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td><em>The two graphs do not intersect. They are parallel to one another.</em></td>
</tr>
<tr>
<td><em>They relate to one another this way because in the problem scenario the shirts are now being sold for the same amount as the price per shirt. However, there is a set-up fee within the cost model, so that equation has a graph that is 22.5 units higher than the income graph.</em></td>
</tr>
<tr>
<td>4. Is there a solution to this system of equations? If so, what is the solution? If not, explain why there is no solution?</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td><em>There is no solution to this system because the lines are parallel to one another; the model representing production cost will always be higher, since there is an initial cost regardless of whether or not a shirt is produced.</em></td>
</tr>
<tr>
<td>5. Would reducing your selling price per shirt to $7.50 be a wise business decision? Why or why not? Base your explanation on the graphs as well as on other factors.</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td><em>Reducing the selling price is not a wise business decision. Even though more shirts might get sold at this lower price, there will never be any profit, no matter how many shirts are sold. This is because the purchase price and the selling price are the same.</em></td>
</tr>
</tbody>
</table>
Since you had difficulty selling your shirts at $8.25, and reducing the price will cause you to lose money, you decide to seek help from your old friends at U.S. Shirts.

In order to help you, U.S. Shirts decides that you will not be charged a set-up fee any longer. However, they will not reduce their charge of $7.50 per shirt.

6. Based upon U.S. Shirts dropping the set-up charge, what is the new equation that will model your costs?

\[ C + 7.5s \]

7. Based upon a reduced selling cost of $7.50 per shirt, what is the equation to model your income?

\[ I = 7.5s \]

8. Create a graph showing both the new equation that models your costs and the new equation that models your income.
## Connecting Algebraic to Graphical Representations

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 9. | How do the graphs of the two equations relate to one another? Why do the graphs relate to one another in this manner?  
*Sample Response:*  
*The two graphs are the same. They are right on top of one another.*  
They relate to one another this way because in the problem scenario the shirts are now being sold for the same amount as the price per shirt and there is no longer a set-up fee, so the cost graph is no longer 22.5 units higher than the income graph. |
| 10. | Is there a solution to this system of equations? If so, what is the solution? If not, explain why there is no solution?  
*Sample Response:*  
*Every point that is on one of the two lines is on the other line as well, so there are an infinite number of solutions to this system of equations.* |
| 11. | If U.S. Shirts removes your set-up fee, is reducing your selling price per shirt to $7.50 a wise business decision? Why or why not? Base your explanation on the graphs as well as on other factors.  
*Sample Response:*  
*Reducing the selling price is still not a wise business decision. Even though more shirts might get sold at this lower price, there will never be any profit, no matter how many shirts are sold. This will be the case because the purchase price and the selling price are the same. As long as you manage to sell all of the shirts that you purchase, you will always break even, which is better than the last situation where you would always lose money.* |
| 12. | Based only upon the graph, how can you determine if there are zero solutions, one solution, or many solutions to a system of equations?  
*Sample Response:*  
*If you see parallel lines, then there are no solutions. If the lines intersect, then there is one solution. If the lines coincide, then there are many solutions – an infinite number.* |
| 13. | Based only upon the problem scenario and the resulting equations, how can you determine if there are zero solutions, one solution, or many solutions to a system of equations?  
*Sample Response:*  
*If both parts of the scenario indicate that the slope is the same and the y-intercept is the same, then there are many solutions. If just the slope is the same, but not the y-intercept, then there are no solutions. If neither the slope or y-intercept is the same, then there is one solution.* |
Connecting Algebraic to Graphical Representations

14. Graph the following systems of equations:
   
   \[ y = 2x + 3 \]
   \[ y = -x + 5 \]

   a. Based on the graph, describe the relationship between the two lines.
   
   **These graphs intersect at a point.**

   b. Is there a solution to the system? If so, what is the solution? Explain why it is a solution to the system.
   
   **There is one solution to this system. The solution is \((2/3, 13/3)\), but students might approximate something a bit different upon inspection.**

Before having students graph the systems in numbers 14 – 16, have them examine the equations to hypothesize the number of solutions to the system.

Students should verify the point of intersection by substituting into the original equations.

It will be difficult for students to find the correct solution by graphing. The problem is designed with this purpose, in order to help motivate the need for solving the system algebraically.
15. Graph the following system of equations:

\[ y = -x - 1 \]
\[ y = x - 3 \]

a. Based on the graph, describe the relationship between the two lines.

These lines intersect at a point and they appear to meet at a 90 degree angle. They are perpendicular. (Students may not notice that the lines are perpendicular, which is acceptable as long as they can say the lines intersect.)

b. Is there a solution to the system? If so, what is the solution? Explain why it is a solution to the system.

There is a solution to this system. The point of intersection of the lines is (1, -2)
16. Graph the following system of equations:

\[ 2y = 2x + 8 \]
\[ 3y = 3x + 6 \]

a. Based on the graph, describe the relationship between the two lines.

*These two lines are parallel to one another.*

b. Is there a solution to the system? If so, what is the solution? Explain why it is a solution to the system.

*There is no solution to this system because the lines will never intersect and are not the same line. They both have a slope of one, but they have different y-intercepts, so they will never intersect.*
Students should be able to begin using slope to determine if the system has:

1. a point of intersection
2. no point of intersection
3. many points of intersection.

To make sure that students understand why some systems have one solution, some have none, and others have infinite solutions, ask them to think of situations in which each of these conditions will occur.

<table>
<thead>
<tr>
<th>Connecting Algebraic to Graphical Representations</th>
</tr>
</thead>
</table>
| 17. What does it mean if a system has only one solution?  
   There is one point of intersection. |
| 18. What does it mean if the system has no solutions?  
   There is no point of intersection. |
| 19. What does it mean if every point is a solution for each equation?  
   The lines are on top of each other. |
| 20. What does it mean graphically if a point is not a solution to a system of equations?  
   It means that the point is not on both graphs or that the point does not make both equations true. |
| 21. What does it mean algebraically if a point is not a solution to a system of equations?  
   A point that is not a solution, when substituted into the equation, will make the equation false, so a point that is not a solution to a system will make at least one equation in the system false. |
| 22. Can a point value satisfy one equation in a system and not the other? Explain.  
   Yes, when a graph shows a point on one line that is not on another line. |
<table>
<thead>
<tr>
<th>Connecting Algebraic to Graphical Representations</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. How can you use the equation to determine if lines are parallel (no points in common), intersecting (one point in common), or coincident (infinite number of points in common).</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td>You can use the equation to tell how the lines relate to one another by examining the slope and y-intercepts of the equations when they are in the form ( y = mx + b ).</td>
</tr>
<tr>
<td>If the slope and y-intercept are the same in the ( y = mx + b ), then the lines are the same line and coincide. Therefore, they have an infinite number of points in common.</td>
</tr>
<tr>
<td>If only the slope is the same, then the lines rise or fall at the same rate, but intersect the y-axis at different points, so the lines are parallel. This means there is no solution to the system.</td>
</tr>
<tr>
<td>If neither the slope nor the y-intercept is the same in the equations in the system, then the lines will have to intersect. In fact, the lines will intersect as long as the slopes are different, even if the y-intercepts are the same. If the y-intercepts are the same and the slopes are different, the solution will be the y-intercept.</td>
</tr>
<tr>
<td>24. How can you use the problem scenario to determine if lines are parallel, intersecting or coincident?</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td>If the scenario indicates that there are two different rates of change, then the lines will always intersect. If the scenario indicates that there is only one rate of change, even though the scenario may have two different parts, then the lines will either be parallel or coincide. If the two parts of the scenario are exactly the same, then of course the lines will coincide.</td>
</tr>
</tbody>
</table>
1. Graph the following systems of two linear equations.
   \[ y = -4x + 11 \]
   \[ y = 3x - 7 \]

2. Determine the point of intersection.
   The point of intersection is \( \left( \frac{18}{7}, \frac{5}{7} \right) \).

3. Verify your solution algebraically by checking the point in the two equations.
   \[ y = 3 \left( \frac{18}{7} \right) - 7 \quad y = -4 \left( \frac{18}{7} \right) + 11 \]
   \[ y = \frac{5}{7} \quad y = \frac{5}{7} \]
Solving Systems of Two Equations Algebraically

4. Were you able to verify your solution algebraically? If not, please explain why.
   
   Yes, but it was difficult to find graphically.

You may want to extend this section to include elimination as a method.

Solving Systems of Two Equations Algebraically

As you saw in the problem on the previous page, it can be difficult to estimate the point of intersection for a system of equations.

Why was it more difficult to find the solution in the last problem than in some of the previous problems you have examined?

*It was difficult because the answer was not a whole number for the coordinates of x or y.*

Because estimating the point of intersection from the graph is not always very efficient, we have some other techniques we can use to solve systems and find the point of intersection. The first of these methods is called the substitution method. As you work in this section, try to decide why this method is the substitution method.

In the problem on Making and Selling Shirts, the equations

\[ y = 7.50x + 22.50 \]  
\[ y = 8.25x \]

for the price of shirts were used to represent the cost to produce the shirts and the income from selling the shirts. The form of the equation used here is familiar to you. It is called the *slope-intercept form* of a linear equation.

Why is this form called the slope-intercept form?

*This form is called slope-intercept because the line is defined in terms of its slope and the y-intercept.*
The concept that the expressions representing \( y \) are equal, since \( y \) equals itself, is often difficult for students to understand. Stress that we are trying to find a solution \((x, y)\) that makes each equation true. This means that the same input value in each equation will get the same output, so \( y \) will be equal to \( y \).

Solving systems by elimination has not yet been covered.

Students should be familiar with the Solver steps, so “work” steps are not shown here.

You should require checking for all solutions.

If students have trouble understanding why it is valid to write \( 2x + 1 = x + 3 \), write \( 2x + 1 = y \) and then \( 2x + 1 = y = x + 3 \).

### Solving Systems of Two Equations Algebraically

<table>
<thead>
<tr>
<th>input</th>
<th></th>
</tr>
</thead>
</table>
| Here is a system of equations: | \[
| \begin{align*} 
| y &= 2x + 1 \\
| y &= x + 3. 
| \end{align*}
| In what form are these equations written? |
| The equations are in slope-intercept form. |
| Notice that both equations use “\( y \) equals”. |
| Is it valid to then write: |
| \( 2x + 1 = x + 3 ? \) |
| Why or why not? |
| It is valid to write this because if each of these expressions is equal to \( y \), then they must be equal to one another. |
| Now, we have one equation instead of two and only one variable instead of two. |
| Solve this equation for \( x \). |
| \( x = 2 \) |
| Now that we know the value of \( x \), find \( y \). Show all your work. |
| \( y = 5 \) |
| Using the information you just obtained, what is the point of intersection for this system of equations? |
| The point of intersection is \((2, 5)\) |
| As a final check, use the second equation to confirm that your point is actually on that line as well. |
| \( 5 = 2 + 3 \) |
Another form that is commonly used to model a linear relationship is the **general form** for the equation of a straight line, which is

\[ Ax + By = C \]

In this form, \( C \) represents a constant value or fixed output, \( A \) and \( B \) are non-zero constants, and \( x \) and \( y \) are variables.

Can you think of an example of a situation that can be modeled algebraically by an equation in this form?

*The snack bar is selling hot dogs \((x)\) and chips \((y)\). Hot dogs are $1.25 \((A)\) and chips are $.50 \((B)\). They can find the earnings \((C)\) using the equation:*

\[ Ax + By = C \]

\[ 1.25x + .5y = C \]

---

**Solving Systems of Two Equations Algebraically**

5. Given the following situations, which linear model do you think is most appropriate in this case, slope-intercept form or general form? Justify your choice.

Your club has $550 to spend on tickets to a major sporting event. Since it is a youth organization, the tickets are discounted. The cost of an adult ticket is $4 and the price of a student ticket is $3. You want to find out how many of each type of ticket you can buy and not exceed your spending limit.

*The General form is most appropriate because there are two different amounts that will vary from the input – number of adult tickets and number of student tickets.*
### Solving Systems of Two Equations Algebraically

We can use the substitution method to solve systems of equations no matter if the equations are in slope-intercept form or general form.

Here is a new system of equations:

\[
\begin{align*}
2x + 3y &= 7 \\
y &= x - 1
\end{align*}
\]

Replace the \( y \) in the first equation with the expression \( x - 1 \).

\[
2x + 3(x - 1) = 7
\]

Why is it valid to replace \( y \) with \( x - 1 \)?

*It is valid because the second equation says that \( y \) is equal to the expression \( x - 1 \).*

Solve the resulting equation for \( x \).

\[
x = 2
\]

Use the \( x \) value you found in either of the original equations to find \( y \).

\[
y = 1
\]

What is the point of intersection for these two equations?

*The point of intersection is \((2,1)\)*
Here is a third system of two equations:

\[ 4x + 2y = 550 \]
\[ 3x + y = 300 \]

6. In which form are each of these two equations written?

Both of these equations are in general form.

7. Solve one of these two equations for \( y \), so that it is written in slope-intercept form. (Hint: First think about which of the two equations will be simpler to solve for \( y \)).

The first equation becomes \( y = -2x + 275 \) in slope-intercept form.

The second equation becomes \( y = -3x + 300 \), if students choose it.

8. Now, use the substitution method to find the coordinates of the point of intersection for these two lines.

\[ 3x + (-2x + 275) = 300 \]
\[ X + 275 = 300 \]
\[ X = 25 \]

\[ Y = 225, \text{ so the point of intersection is } (25, 225) \]
### Solving Systems of Two Equations Algebraically

9. Solve the following systems of linear equations. Verify any solutions graphically by using your graphing calculator.

   a. \( y = x - 2 \) and \( 3x + 2y = 1 \)

   \[
   \begin{align*}
   3x + 2(x - 2) &= 1 \\
   3x + 2x - 4 &= 1 \\
   5x &= 4 + 1
   \end{align*}
   \]

   \[
   \begin{align*}
   y &= 1 - 2 \\
   y &= -1 \\
   5x &= 5 \\
   x &= 1
   \end{align*}
   \]

   Check:

   \[
   3 \cdot 1 + 2 \cdot (-1) = 1 \\
   3 - 2 = 1
   \]

   The point of intersection is \((1, -1)\).

   b. \( y = -5x + 7 \) and \( y = x - 11 \)

   \[
   \begin{align*}
   -5x + 7 &= x - 11 \\
   -6x &= -18 \\
   x &= 3
   \end{align*}
   \]

   \[
   \begin{align*}
   y &= 3 - 11 \\
   y &= -8
   \end{align*}
   \]

   Check:

   \[
   -8 = -5 \cdot 3 + 7 \\
   -8 = -8 \text{ is true}
   \]

   The point of intersection is \((3, -8)\).

   c. \( y = 10(3x - 4) \) and \( y = 10(-2 + 3x) \)

   \[
   \begin{align*}
   10(3x - 4) &= 10(-2 + 3x) \\
   3x - 4 &= -2 + 3x \\
   3x - 3x &= 4 - 2
   \end{align*}
   \]

   \[
   \begin{align*}
   0 &= 2
   \end{align*}
   \]

   No solution. There is no point of intersection. The lines are parallel.
Solving Systems of Two Equations Algebraically

**d.** \( x - 3y = 8 \) and \( y - 4x = 21 \)

\[
\begin{align*}
y &= 4x + 21 \\
y - 4\left(\frac{71}{11}\right) &= 21 \\
x - 3(4x + 21) &= 8 \\
x - 12x - 63 &= 8 \\
-11x &= 63 + 8 \\
x &= -\frac{71}{11} & y &= \frac{53}{11} \\
\text{Check: } \frac{71}{11} - 3\left(-\frac{53}{11}\right) &= 8 \\
8 &= 8 \text{ is true}
\end{align*}
\]

The intersection is the point \( \left( -\frac{71}{11}, -\frac{53}{11} \right) \).

**e.** \( x + y = 8 \) and \( 2x + 2y = 16 \)

\[
\begin{align*}
y &= 8 - x \\
2x + 2(8 - x) &= 16 \\
2x + 16 - 2x &= 16 \\
0 &= 0 \quad \text{These two lines are identical.}
\end{align*}
\]

Solving Systems of Two Equations Algebraically

10. Why is this method of solving a system of equations called the substitution method?

*Sample Response:*

This method is called substitution because an expression in one variable can be substituted into an equation in order to simplify it. This substitution allows two equations in two unknowns to be written in terms of one unknown.

For example, given two equations, you can take the equation where \( y \) is in terms of \( x \) and substitute that value of \( y \) into the second equation so that you have two equations in one unknown.
### Solving Systems of Two Equations Algebraically

<table>
<thead>
<tr>
<th>process: summary</th>
</tr>
</thead>
</table>
| 11. Can a system of two linear equations always be solved by substitution? Explain.  
*Sample Response:*  
*A system of linear equations can always be solved using substitution, since you can always write one equation as y in terms of x and then substitute that value for y to have two equations in one unknown.*  

*Sample Response:*  
*A system can theoretically always be solved using a graph, but this method is not always practical. When the coordinates of the point of intersection are not whole numbers, then it is difficult to name the exact solution, even when one exists.*  

13. When and why might it be more useful to solve a system graphically rather than algebraically using substitution?  
*Sample Response:*  
*It may be more useful to solve a system graphically when you want to see what is happening between the two equations.*  

14. Which of these two methods of solving systems of linear equations do you prefer and why?  
*Student answers and justifications will vary for this question.*
In each of the following problems, make sure that students provide a full comparison of the functions from the points below the intersection point to the values above the point of intersection.

Each problem has an extension question that will determine students' ability to transfer their knowledge of systems. They can be used for group challenge questions and presentations to culminate the unit.

### Finding the Better Paying Job

A friend of yours interviewed for two different sales positions in competing companies. The Stellar company pays $500 per week plus 10% commission on the total dollars from sales per week. The Lunar company pays $200 per week but offers a 20% commission on the total dollars from sales per week. Sales at both companies are seasonal. Your friend wants some help in determining which job option is best.

#### process: model

1. Write an equation that describes the salary model for Stellar. Indicate what the variables and constants mean in the equation.

   \[ E = \text{earnings measured in dollars.} \]

   \[ t = \text{total sales measured in dollars.} \]

   \[ 10\% \ (.10) \text{ is the commission.} \]

   \[ $500 \text{ is the base pay per week.} \]

   \[ E = 0.10t + 500 \]

2. Write an equation that describes the salary model for Lunar. Indicate what the variables and constants mean in the equation.

   \[ E = \text{earnings measured in dollars.} \]

   \[ t = \text{total sales measured in dollars.} \]

   \[ 20\% \ (.20) \text{ is the commission.} \]

   \[ $200 \text{ is the base pay per week.} \]

   \[ E = 0.20t + 200 \]
Finding the Better Paying Job

To determine which option is best, gather some data by answering the following questions.

3. If the total sales is $1,200 for a particular week, what salary will your friend receive for that week from:
   a. Stellar?
      
      \[ E = 0.10(1200) + 500 \]
      
      \[ E = 620 \]
      
      \[ E = \text{He will earn $620 if he sells $1200.} \]
   b. Lunar?
      
      \[ E = 0.20(1200) + 200 \]
      
      \[ E = 440 \]
      
      \[ E = \text{He will earn $440 if he sells $1200.} \]

4. If the weekly salary from Lunar is $1200, what were the total sales for the week?

   \[ 1200 = 0.20t + 200 \]
   
   \[ 1200 - 200 = 0.20t \]
   
   \[ 1000 = 0.20t \]
   
   \[ 1000 \div 0.20 = 5000 = t \]

   The total earnings from sales is $5000.

5. If the weekly salary for Stellar is $540, what will the salary be from Lunar?

   Stellar | Lunar
   --- | ---
   \[ 540 = 0.10t + 500 \] | \[ E = 0.20(400) + 200 \]
   \[ 540 - 500 = 0.10t \] | \[ = 80 + 200 \]
   \[ 40 = 0.10t \] | \[ = 280 \]
   \[ 400 = t \]

   If you made $540 from Stellar, you would only earn $280 from Lunar.

6. Will the weekly salaries ever be equal? If so, under what conditions will it occur? What will be the amount of the salary?

   \[ 0.10t + 500 = 0.20t + 200 \]
   
   \[ 500 - 200 = 0.20t - 0.10t \]
   
   \[ 300 = 0.10t \]
   
   \[ 300 \div 0.10 = 3000 = t \]

   The salaries can be equal if \( t = 3000 \). This will occur where the lines intersect.

   \[ 0.10(3000) + 500 \]
   
   \[ 300 + 500 = 800 \]

   The amount of the salary will be $800.
Finding the Better Paying Job

7. Verify your answer to the previous question by graphing the two equations.

\[ y = 0.2x + 200 \]
\[ y = 0.1x + 500 \]

Finding the Better Paying Job

8. Generate a table of values using the data gathered. Include at least four more values, where two sales totals generate a low weekly salary and two sales totals generate a high weekly salary.

<table>
<thead>
<tr>
<th>Total Sales</th>
<th>Stellar Earnings</th>
<th>Lunar Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$0.10t + 500$</td>
<td>$0.20t + 200$</td>
</tr>
<tr>
<td>1200</td>
<td>620</td>
<td>440</td>
</tr>
<tr>
<td>2000</td>
<td>700</td>
<td>600</td>
</tr>
<tr>
<td>5000</td>
<td>1000</td>
<td>1200</td>
</tr>
<tr>
<td>10000</td>
<td>1500</td>
<td>2200</td>
</tr>
<tr>
<td>3000</td>
<td>800</td>
<td>800</td>
</tr>
</tbody>
</table>
### Finding the Better Paying Job

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Based on your findings, which company would you recommend to your friend and why?</td>
</tr>
<tr>
<td><em>Answers will vary.</em></td>
</tr>
<tr>
<td><em>Sample Response:</em></td>
</tr>
<tr>
<td><em>My friend is a good salesman, so he can sell a lot and Lunar would therefore be the best.</em></td>
</tr>
<tr>
<td><em>My friend is quiet and will not sell much. Stellar would be better because he will get at least $500 per week.</em></td>
</tr>
<tr>
<td>a. Are there any conditions under which you would change your recommendation? If so, what are they?</td>
</tr>
<tr>
<td><em>Answers will vary.</em></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. Your friend had a third interview for a position with a company, called Solar. Solar pays a straight salary of $750 per week. Create the equation to represent this situation and graph it on the same set of axes where the salaries for Stellar and Lunar are graphed.</td>
</tr>
<tr>
<td>11. Under what conditions is Solar the best option?</td>
</tr>
<tr>
<td><em>Solar is best at the beginning when sales are low. At $2500 sales, Stellar and Solar are the same. Stellar is better from $2500 to $3000 sales. When Stellar and Lunar are equal, then Lunar is better for all sales over $3000.</em></td>
</tr>
<tr>
<td>12. You chose to take a job with Stellar and you want to earn at least $975 a week. Represent the situation algebraically. What is the solution set for the number of dollars in sales that you must earn weekly?</td>
</tr>
<tr>
<td><em>975 ≤ .10t + 500</em></td>
</tr>
<tr>
<td><em>975 – 500 ≤ .10t</em></td>
</tr>
<tr>
<td><em>475 ≤ .10t</em></td>
</tr>
<tr>
<td><em>4750 ≤ t</em></td>
</tr>
<tr>
<td><em>When sales are at least $4750 per week, then earnings will be at least $975 per week.</em></td>
</tr>
</tbody>
</table>
Students may need to separate the consumption data and the production data before beginning, to avoid confusion. Do not consider oil reserves when creating the equation for consumption. The oil consumed may come from new production or from reserves.

This problem may provide an opportunity to discuss the validity of certain mathematical models. For example, you might ask for how long the assumptions made in the problem are valid (if they are at all valid given the current world conditions).

---

### World Oil: Supply and Demand

Data on the world’s crude oil reserves in 2001 showed that there were about 1 trillion (1,004.1 billion) barrels of oil world-wide. In 1970, there were 933 billion barrels in reserve. This represents an average rate of change of about 2.3 billion barrels annually. Assume that this rate of increase in reserves will continue in the future.

In 2001, the world-wide consumption of oil was 27.74 billion barrels per year. (Incidentally, this was down from 33.5 billion barrels per year in 1970.) Assume that the rate of annual oil consumption is fixed at 27.74 billion barrels per year.

Under these conditions, study the relationship between the supply of oil versus the demand for oil.

---

**process: model**

1. Write the equation that models the world-wide supply of oil. Indicate what each variable and constant represents in the equation.

   \[ x = \text{time since 2001 in years} \]
   \[ y = \text{current supply in billions of barrels} \]

   \[ 2.3 \text{ billion barrels is the rate at which reserves increase each year.} \]
   \[ 1004.1 \text{ billion barrels is the amount of reserve in 2001} \]

   \[ y = 2.3x + 1004.1 \]

2. Write the equation that models the world-wide consumption of oil. Indicate what each variable and constant represents in the equation.

   \[ x = \text{time since 2001 in years} \]
   \[ y = \text{oil consumed in billions of barrels} \]

   \[ 27.74 \text{ billion barrels is the current consumption rate in 2001} \]

   \[ y = 27.74x \]
World Oil: Supply and Demand

3. Based on the 2001 data, in how many years from 2001 will the amount of the oil in the reserves reach 1250 billion barrels? What year will that be?

\[
1250 = 2.3x + 1004.1
\]
\[
1250 - 1004.1 = 2.3x
\]
\[
245.9 = 2.3x
\]
\[
x = 106.91\text{ years}\]

The reserves would reach 1250 billion barrels in 106.91 or 107 years.

4. Use the 2001 data to determine what year it would be if there were 1866 billion barrels of oil in reserve.

\[
1866 = 2.3x + 1004.1
\]
\[
1866 - 1004.1 = 2.3x
\]
\[
374.74 = x
\]

The reserves would reach 1866 billion barrels in 375 years.

5. At the 2001 rate of consumption, how much oil will be consumed by 2011?

\[
y = 27.74\text{ billion barrels per year} \times 10 = 277.4\text{ billion barrels}
\]
\[
y = 277.4
\]

6. At the 2001 rate of consumption, how long will it take to use 200 billion barrels of oil?

\[
200 = 27.74x
\]
\[
x = 7.21\text{ years}
\]

It would take about 7 years to use 200 billion barrels of oil.

7. Based on the 2001 data, how much oil will the reserves contain in 25 years?

\[
y = 2.3x + 1004.1 = 2.3 \times 25 + 1004.1 = 1061.6\text{ billion barrels}
\]

In 2026 there will be 1.06 trillion barrels in reserves.

8. At the 2001 rate of consumption, when will there be 500 billion barrels in the reserves?

\[
500 = 27.74x
\]
\[
x = 18.02\text{ years}
\]

There will be 500 billion barrels in the reserves in 2019.

The answer to question 6 will change depending upon what year it is currently. For example, the year 2003 is 2 years after 2001, so 27 would be used for x in the equation instead of 25.
9. Complete the following table using the data you generated.

<table>
<thead>
<tr>
<th>Number of Years from 2001</th>
<th>Amount of Oil Consumed</th>
<th>Amount of Oil in Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years</td>
<td>Billion Barrels</td>
<td>Billion Barrels</td>
</tr>
<tr>
<td>x</td>
<td>27.74x</td>
<td>2.3x + 1004.1</td>
</tr>
<tr>
<td>10</td>
<td>277.4</td>
<td>1027.1</td>
</tr>
<tr>
<td>15</td>
<td>416.1</td>
<td>1038.6</td>
</tr>
<tr>
<td>20</td>
<td>554.8</td>
<td>1050.1</td>
</tr>
<tr>
<td>30</td>
<td>832.2</td>
<td>1073.1</td>
</tr>
<tr>
<td>40</td>
<td>1109.6</td>
<td>1096.1</td>
</tr>
</tbody>
</table>

10. Based on the data, do you think supply will ever equal demand? If so predict when.

*Yes, the supply is not increasing as fast as the demand. Supply will equal demand in about 40 years based on the table data above.*

11. Verify your result by graphing the two equations.
12. Verify the result algebraically.

\[ 2.3x + 1004.1 = 27.74x \]

\[ 1004.1 = 25.44x \]

\[ x = 39.47 \text{ years} \]

13. Is the comparison being made in this problem between supply and demand of oil a valid comparison to be making? Justify your response.

*Answers will vary.*
Picking the Best Option

The Bici Bicycle Company is planning to build a low price ultra-light model. The two proposals being made require some analysis. One proposal suggests building a bicycle that will cost the company $125,000 initially for the design and prototype in addition to a $225 cost for material and labor for each bicycle. The second proposal has a lower start-up cost of $100,000, but materials and labor for each bike will be $275.

The company wants to know which is more economical to produce if the number of bicycles they plan to build is between 0 and 5000. They also want to know if there is some point at which the plans are the same. To be sure your analysis is complete, use algebraic, graphical, and numerical representations.

The Bici Bicycle Company is planning to build a low price ultra-light model. The two proposals being made require some analysis. One proposal suggests building a bicycle that will cost the company $125,000 initially for the design and prototype in addition to a $225 cost for material and labor for each bicycle. The second proposal has a lower start-up cost of $100,000, but materials and labor for each bike will be $275.

The company wants to know which is more economical to produce if the number of bicycles they plan to build is between 0 and 5000. They also want to know if there is some point at which the plans are the same. To be sure your analysis is complete, use algebraic, graphical, and numerical representations.

\[ Y = 125,000 + 225x \] is the model for production cost for plan 1,
\[ Y = 100,000 + 275x \] is the model for production cost for plan 2,
where \( x \) is the number of bicycles and \( y \) is the cost in dollars.

The solution to the system is (500, 237,500).
When the number of bikes is 500, the two costs of production are the same under both proposals. If the number of bikes exceeds 500, then the first proposal is the most cost effective.

<table>
<thead>
<tr>
<th>Number of bikes</th>
<th>Cost for proposal 1</th>
<th>Cost for proposal 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>125,000</td>
<td>100,000</td>
</tr>
<tr>
<td>500</td>
<td>237,500</td>
<td>237,500</td>
</tr>
<tr>
<td>1000</td>
<td>350,000</td>
<td>375,000</td>
</tr>
<tr>
<td>1500</td>
<td>462,500</td>
<td>512,500</td>
</tr>
</tbody>
</table>

$100,000 + 275x = 125,000 + 225x$

$50x = 25,000$

$x = 500$ bikes  
$y = 125,000 + 225(500) = \$237,500$

1. If the company plans to charge $525 per bicycle, under which plan can it make the most profit (not income)? What is the break-even point under each of the proposals?

Profit for proposal 1:  
$\text{Profit for proposal 1: } P = 525x - (125,000 + 225x)$

Profit for proposal 2:  
$\text{Profit for proposal 2: } P = 525x - (100,000 + 275x)$

$P = 300x - 125,000$

$0 = 300x - 125,000$

$300x = 125,000$

$x = 416.67$ bikes

$P = 250x - 100,000$

$0 = 250x - 100,000$

$250x = 100000$

$x = 400$ bikes

(x is number of bikes and P is profit.)
2. Suppose the company wants you to examine what happens if it drives the cost of materials and labor down by $22.50, while also increasing the sale price to $450. Examine each proposal under these conditions. How many bicycles will have to be produced in order for a profit to be made?

<table>
<thead>
<tr>
<th>Proposal 1:</th>
<th>Proposal 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P = 450x - (125,000 + 202.5x) )</td>
<td>( P = 450x - (100,000 + 252.5x) )</td>
</tr>
<tr>
<td>( P = 247.5x - 125,000 )</td>
<td>( P = 197.5x - 100,000 )</td>
</tr>
<tr>
<td>( 247.5x - 125,000 = 0 )</td>
<td>( 0 = 197.5x - 100,000 )</td>
</tr>
<tr>
<td>( 247.5x = 125,000 )</td>
<td>( 197.5x = 100,000 )</td>
</tr>
<tr>
<td>( x = 505.05 \text{ bikes} )</td>
<td>( x = 506.33 \text{ bikes} )</td>
</tr>
</tbody>
</table>
Unit 5: Statistical Analysis

Unit Objectives and Skills

At the completion of this unit, students will be able to organize data using stem and leaf plots, histograms and box plots. They will also understand measures of central tendency and dispersion and how to calculate and apply descriptive and inferential statistics. They will know how to design, conduct and interpret the results of a simple study. In addition, students will know how to model data using scatterplots and regression equations to find the best fit. Students will understand that a model is a best approximation and may be used for the purpose of prediction and estimation.

Unit Overview

The unit begins with a focus on visual displays through stem and leaf plots and histograms. A view of the distribution of data moves the students to a discussion of single variable data through the use of statistics such as mean, median and mode, range, percentiles, quartiles, and IQR. Variance and standard deviation are also introduced along with how to interpret variation in data.

The discussion then moves toward experimental design and modeling data using regression techniques.

Unit Activities

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

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<th>Homework Assignments</th>
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<tr>
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<tr>
<td>Box and Whisker Plot – Airline Industry</td>
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</tr>
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</table>
Printed Classroom Activities (continued)  

<table>
<thead>
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<th>Printed Classroom Activities (continued)</th>
<th>Homework Assignments</th>
</tr>
</thead>
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<td>Candy Sale</td>
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</tr>
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<td>pg. 151</td>
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<tr>
<td>Jumping</td>
<td>-</td>
</tr>
<tr>
<td>Hanford Nuclear Power Plant</td>
<td>pg. 155</td>
</tr>
<tr>
<td>Human Chain: Wrist Experiment</td>
<td>pg. 157</td>
</tr>
<tr>
<td>Human Chain: Shoulder Experiment</td>
<td>pg. 159</td>
</tr>
</tbody>
</table>

Suggestions for Classroom Implementation

Organizing and Summarizing Data

The activities initially focus on organizing data. Before beginning a discussion of stem and leaf plots and histograms, you might ask students to indicate what they know about visual displays, the purpose of the displays, and the different types they know. Emphasize the value of using stem and leaf plot and histogram displays when the data sets are larger, and the value of each in terms of what you can determine about the data, e.g. distribution. Once the students have a feel for working with larger data sets, you might begin asking a series of questions about other kinds of information that might be useful to know, and from there move to summarizing data through the use of statistics.

Summarizing data becomes more formalized beginning with **Measures of Central Tendency** and continuing through **Candy Sales**, where the focus is on mean, median and mode, quartiles, percentiles and IQR, variance and standard deviation. Have students work in groups to discuss the examples presented in each section of the unit and work at least one of the tasks relating to the example. Groups should then present their results and discuss differences in solutions and interpretations.

Experimental Design and Modeling Data

To start this portion of the unit, students work on modeling and interpreting data using **Mia’s Growing Like a Weed**, **College Students**, and **Retail Sales**. Comparing their results especially with respect to finding the line of
best fit will provide a basis for discussion. If differences exist with lines of best fit, spend time working with students on how to come to agreement. The discussion can also motivate a fruitful discussion on interpretation.

Successful completion of these activities will have the students primed for the Stroop Test. Gathering the data for this experiment is fun, but can be a bit challenging. It provides students with “real time” research and data collection and helps them appreciate the concept of approximation and the value of modeling to predict outcomes. Jumping, like the Stroop Test, has the students gather and analyze data with a special concentration on sampling.

In the Hanford Nuclear Power Plant activity, students are given the opportunity to learn the formal calculation for the equation of the linear regression line and linear regression formulas for the constants. Whether you have the students do the Hanford Nuclear Power Plant activity will depend on their readiness at this time.

To close this unit, have students do the data collection activities with the Human Chain: Wrist Experiment and Human Chain: Shoulder Experiment. Require students to collect the data and do a scatterplot. Using the overhead projector, have them place a line of best fit using a strand of spaghetti and then check their fit against that of the graphing calculator. Have a final discussion that focuses on comparing the results of the experiments and why they are similar or different. Finally, compare the results graphically to look at the relationship of the two linear models, supporting the work students have done in the unit that included systems of equations.

Unit Assessments

Baseball
Data
Running the Mile
Unit 5: Statistical Analysis

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At this point in the course, you have worked with algebraic functions through using representations and have generated data tables from situations. From the data, you were able to determine the relationship between variables and to represent the relationship with an algebraic model. In this chapter, you will examine data sets and ways in which you can organize and summarize data. You also will study how to generate models from data using visual displays, statistics, and statistical analysis.

When given a data set, it is often helpful to use a visual display. You are probably familiar with line graphs, number and dot plots, as well as bar graphs and pie charts.

The stem and leaf plot is a relatively new technique for presenting information. The advantage of a stem and leaf plot is that it displays the data with a picture, but retains all the individual values. With a stem and leaf plot, it is easier to determine the relative location of any particular data point in which you might be interested. The following example uses test scores from a 10th grade math class to show how data is plotted with a stem and leaf plot:

```
Stem Portion | Leaf Portion
-------------|------------
3            | 7          
4            | 6 7        
5            | 5 5 8      
6            | 1 1 3 5 7 7 9 
7            | 1 1 2 3 3 3 4 5 
8            | 2 2 4 4 4 8 
9            | 1 3 5      
```

The scores for this class are as follows:

```
61  55  71  84  58  93  
82  91  47  88  84  65  
46  61  84  55  69  67  
73  63  37  67  72  75  
73  74  95  82  73  71  
```

First, rearrange the test scores from smallest to largest:

```
37  58  67  72  75  84  
46  61  67  73  82  88  
47  61  69  73  82  91  
55  63  71  73  84  93  
55  65  71  74  84  95  
```
Stem and Leaf Plots – Math Grades

In this example, the stem part of your plot will represent the tens place of the test scores. Since the scores start in the 30s and end in the 90s, your stems will be 3, 4, 5, 6, 7, 8, and 9. Arrange them vertically in increasing order. Draw a vertical line to separate the stems from the leaves.

3
4
5
6
7
8
9

Next, place the leaves of your stem and leaf plot so they extend from the stem. For example, 37 would be represented in the plot like this:

3 7
4
5
6
7
8
9

There are two numbers in the forties, 46 and 47. Show 46 first by placing the 6 next to the 4 in the stem. Show the 47 by simply placing the 7 along the same horizontal row.

3 7
4 6 7
5
6
7
8
9
To make sure that each number "takes up the same amount of space", have students create stem and leaf plots on graph paper.

Stem and Leaf Plots – Math Grades

<table>
<thead>
<tr>
<th>input</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 7</td>
</tr>
<tr>
<td>4 6 7</td>
</tr>
<tr>
<td>5 5 8</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>

There are two fifty-fives, so we must represent 55 twice before representing the other score in the 50s, 58.

Complete the remainder of the stem and leaf plot, making sure that you order the numbers from smallest to largest, and that each number in the ones place takes up the same amount of space. Show your finished plot.
The New York theater – Broadway – has, along with the London theater, been the heart of stage productions. To understand the activity on Broadway, let’s look at the number of new Broadway productions each year from 1959 – 2000 using a stem and leaf plot.

<table>
<thead>
<tr>
<th>Season</th>
<th># New Shows</th>
<th>Season</th>
<th># New Shows</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963-1964</td>
<td>63</td>
<td>1984-1985</td>
<td>33</td>
</tr>
<tr>
<td>1964-1965</td>
<td>67</td>
<td>1985-1986</td>
<td>34</td>
</tr>
<tr>
<td>1974-1975</td>
<td>54</td>
<td>1995-1996</td>
<td>38</td>
</tr>
<tr>
<td>1979-1980</td>
<td>61</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The World Almanac and Book of Facts 2001

Note: If there are single digits, then the number in the tens place is 0. If there are three digits, place the hundreds and tens value in the stem and the units value in the leaf.
Stem and Leaf Plots – Broadway Theater

Show your complete stem and leaf plot for the New York theater data below. As you proceed, think carefully about whether the stem and leaf plot shows the numerical order or the order in which the data appears.

```
2 | 8
3 | 0 3 3 3 4 4 6 7 7 8 9 9
4 | 0 1 2 3 8 8 9
5 | 0 0 3 4 4 5 5 5 8
6 | 0 1 2 3 7 7 8 9
7 | 4
```

**stem width = 10**

There may be some disagreement on question 3 depending on how each group interprets the meaning of belonging to the data set.

Question 4 informally introduces the concept of mean. This concept will be approached again in the section on histograms and formalized in this unit.

You may want to offer the definition of outlier at this time.

---

**1.** What is the highest number of new productions? Lowest number of new productions?

*The highest number of new productions was 74; the lowest was 28.*

**2.** Are there any clusters? If so, where does the clustering occur?

*There is a slight cluster around 33-34 and around 54-55.*

**3.** Are there any data points that do not seem to belong? If so, what are they?

*There are no points that do not seem to belong. The only number that seems like it may not belong is 74, but it may not be large enough to exclude.*

**4.** Based on the stem and leaf plot you created, what would you estimate is the average number of new productions per year?

*Answers will vary, but should be approximately 48.*
5. What other observations can you make about the data?

Answers will vary.

6. In your own words, explain to the person sitting next to you how to interpret the data in a stem and leaf plot. Summarize and record what you told them.

Answers will vary.

7. Rotate your paper 90° in a counterclockwise direction so that the leaves go up instead of to the right. Does this presentation help you see the distribution of the data better? Why? Can you describe the shape of the distribution?

Answers will vary. The distribution is multi-modal.
Using the various data sets in the previous examples, ask the students if they can detect the shape of the distribution.

Distributions

The shape of the distribution can reveal a lot of information. There are many different distributions; however, the most common are: symmetric, skewed to the right, and skewed to the left.

1. What do you think it means when the distribution is symmetric?

   *Answers will vary. An appropriate answer is that it is bell-shaped.*

2. What do you think it means when the distribution is skewed to the right?

   *Answers will vary. An appropriate answer is that it has at least one outlier to the right of most of the data*

3. What do you think it means when the distribution is skewed to the left?

   *Answers will vary. An appropriate answer is that it has at least one outlier to the left of most of the data.*
4. If you had to describe to someone how to draw a symmetric distribution, what would you say?

Sample Response: To draw a symmetric distribution means making a curve that is a mirror image of itself starting at zero, reaching a maximum, and descending downward toward zero.

5. If you had to describe to someone how to draw a distribution that is skewed to the right, what would you say?

Sample Response: To draw a distribution skewed to the right means drawing a curve that ascends quickly and has its maximum toward the left. It then descends as you move toward the right.

6. If you had to describe to someone how to draw a distribution that is skewed to the left, what would you say?

Sample Response: To draw a distribution skewed to the left means drawing a curve that ascends moving left to right, reaching its maximum toward the right, and then descending.
Discuss different ways to define the stem and the leaf. This example is different from the previous two in terms of defining the stem and leaf portions.

### Distributions – ACT

1. Generate a stem and leaf plot using the state-by-state composite ACT scores for the period 1998-1999 shown in the following table. For this plot, use the whole number part of the data point as your stem, and use the tenths place as your leaf.

<table>
<thead>
<tr>
<th>State</th>
<th>Score</th>
<th>State</th>
<th>Score</th>
<th>State</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL</td>
<td>20.2</td>
<td>LA</td>
<td>19.6</td>
<td>OH</td>
<td>21.4</td>
</tr>
<tr>
<td>AK</td>
<td>21.1</td>
<td>ME</td>
<td>22.1</td>
<td>OK</td>
<td>20.6</td>
</tr>
<tr>
<td>AZ</td>
<td>21.4</td>
<td>MD</td>
<td>20.9</td>
<td>OR</td>
<td>22.6</td>
</tr>
<tr>
<td>AR</td>
<td>20.3</td>
<td>MA</td>
<td>22.0</td>
<td>PA</td>
<td>21.4</td>
</tr>
<tr>
<td>CA</td>
<td>21.3</td>
<td>MI</td>
<td>21.3</td>
<td>RI</td>
<td>22.7</td>
</tr>
<tr>
<td>CO</td>
<td>21.5</td>
<td>MN</td>
<td>22.1</td>
<td>SC</td>
<td>19.1</td>
</tr>
<tr>
<td>CT</td>
<td>21.6</td>
<td>MS</td>
<td>18.7</td>
<td>SD</td>
<td>21.2</td>
</tr>
<tr>
<td>DE</td>
<td>20.5</td>
<td>MO</td>
<td>21.6</td>
<td>TN</td>
<td>19.9</td>
</tr>
<tr>
<td>FL</td>
<td>20.6</td>
<td>MT</td>
<td>21.8</td>
<td>TX</td>
<td>20.3</td>
</tr>
<tr>
<td>GA</td>
<td>20.0</td>
<td>NE</td>
<td>21.7</td>
<td>UT</td>
<td>21.4</td>
</tr>
<tr>
<td>HI</td>
<td>21.6</td>
<td>NV</td>
<td>21.5</td>
<td>VT</td>
<td>21.9</td>
</tr>
<tr>
<td>ID</td>
<td>21.4</td>
<td>NH</td>
<td>22.2</td>
<td>VA</td>
<td>20.6</td>
</tr>
<tr>
<td>IL</td>
<td>21.4</td>
<td>NJ</td>
<td>20.7</td>
<td>WA</td>
<td>22.6</td>
</tr>
<tr>
<td>IN</td>
<td>21.2</td>
<td>NM</td>
<td>20.1</td>
<td>WV</td>
<td>20.2</td>
</tr>
<tr>
<td>IA</td>
<td>22.0</td>
<td>NY</td>
<td>22.0</td>
<td>WI</td>
<td>22.3</td>
</tr>
<tr>
<td>KS</td>
<td>21.5</td>
<td>NC</td>
<td>19.4</td>
<td>WY</td>
<td>21.4</td>
</tr>
<tr>
<td>KY</td>
<td>20.1</td>
<td>ND</td>
<td>21.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

American College Testing 2000

```
18 | 7
19 | 1 4 6 9
20 | 0 1 1 2 2 3 3 5 6 6 6 7 9
21 | 1 2 2 3 3 4 4 4 4 4 4 4 4 5 5 5 6 6 6 6 7 8 9
22 | 0 0 0 1 1 2 3 6 6 7

stem width = 1
```
### Distributions – ACT

2. Describe the distribution of the data (using the new terms you have learned). What do you notice about the ACT scores from the stem and leaf plot that you may not have noticed when the data was displayed in the chart?

*The scores are skewed to the left.*
Hold a discussion about the similarities and differences between histograms and stem and leaf plots, in addition to the similarities and differences discussed in questions 1 and 2 below.

The following histograms display the ages of Oscar Winning Actors and Actresses from 1970 – 1999:

A histogram is a visual display that resembles a bar graph and is quite similar to a stem and leaf plot. Histograms are more easily used with very large data sets than stem and leaf plots. The reason is that in constructing histograms, you divide the range of data into intervals of equal length and then count the number, or frequency, of observations in each interval.

The bars’ or rectangles’ heights correspond to the counts in each interval. Histograms may also display the relative frequency, which shows the percent of values that fall into a particular category. Frequency histograms and relative frequency histograms both differ from bar graphs.

1. Name two ways in which bar graphs and histograms are similar.  
   
   **Bar graphs and histograms are similar because they both use bars of varying heights to show data. They also show the frequency.**

2. Name two ways in which bar graphs and histograms are different.  
   
   **Histograms are different than bar graphs because histograms use intervals while bar graphs use descriptive categories. Histograms can also use relative frequency while bar graphs do not.**
Question 5 continues to introduce the concept of mean. Students have not learned how to calculate the average formally and do not need to do so since an estimate is required. In particular, the calculation is complicated by the use of intervals. The description of the method used is more important than the actual value.

### Histograms – Oscar Winners

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Describe the distribution of the ages of the female Oscar winners.</td>
</tr>
<tr>
<td>The distribution of the ages of the female Oscar winners is skewed to the right. Most of the ages are less than 50 years old.</td>
</tr>
<tr>
<td>4. Describe the distribution of the ages of the male Oscar winners.</td>
</tr>
<tr>
<td>The distribution of the ages of the male Oscar winners is also skewed to the right. All of the ages are at least 30 years old.</td>
</tr>
<tr>
<td>5. Can you estimate the average age of female Oscar Winners? Male Oscar Winners?</td>
</tr>
<tr>
<td>Female Oscar winners were about 40 years old on average.</td>
</tr>
<tr>
<td>Male Oscar winners were about 46 years old on average.</td>
</tr>
<tr>
<td>6. Compare the distributions. Why do you think there are differences?</td>
</tr>
<tr>
<td>Typically, the females were younger than the males when they won an Oscar.</td>
</tr>
</tbody>
</table>
The table below shows the mean verbal SAT score by state in 1999.

<table>
<thead>
<tr>
<th>State</th>
<th>VSAT</th>
<th>State</th>
<th>VSAT</th>
<th>State</th>
<th>VSAT</th>
<th>State</th>
<th>VSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>479</td>
<td>MD</td>
<td>507</td>
<td>OH</td>
<td>527</td>
<td>OK</td>
<td>567</td>
</tr>
<tr>
<td>HI</td>
<td>482</td>
<td>ME</td>
<td>507</td>
<td>CO</td>
<td>536</td>
<td>NE</td>
<td>568</td>
</tr>
<tr>
<td>GA</td>
<td>487</td>
<td>VA</td>
<td>508</td>
<td>ID</td>
<td>542</td>
<td>IL</td>
<td>569</td>
</tr>
<tr>
<td>NC</td>
<td>493</td>
<td>CT</td>
<td>510</td>
<td>MT</td>
<td>545</td>
<td>UT</td>
<td>570</td>
</tr>
<tr>
<td>TX</td>
<td>494</td>
<td>MA</td>
<td>511</td>
<td>WY</td>
<td>546</td>
<td>MO</td>
<td>572</td>
</tr>
<tr>
<td>NY</td>
<td>495</td>
<td>NE</td>
<td>512</td>
<td>KY</td>
<td>547</td>
<td>KS</td>
<td>578</td>
</tr>
<tr>
<td>IN</td>
<td>496</td>
<td>VT</td>
<td>514</td>
<td>NM</td>
<td>549</td>
<td>WI</td>
<td>584</td>
</tr>
<tr>
<td>CA</td>
<td>497</td>
<td>AK</td>
<td>516</td>
<td>MI</td>
<td>557</td>
<td>SD</td>
<td>585</td>
</tr>
<tr>
<td>NJ</td>
<td>498</td>
<td>NH</td>
<td>520</td>
<td>TN</td>
<td>559</td>
<td>MN</td>
<td>586</td>
</tr>
<tr>
<td>PA</td>
<td>498</td>
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<td>524</td>
<td>AL</td>
<td>561</td>
<td>ND</td>
<td>594</td>
</tr>
<tr>
<td>FL</td>
<td>499</td>
<td>WA</td>
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<td>LA</td>
<td>561</td>
<td>IA</td>
<td>594</td>
</tr>
<tr>
<td>DE</td>
<td>503</td>
<td>OR</td>
<td>525</td>
<td>AR</td>
<td>563</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RI</td>
<td>504</td>
<td>WV</td>
<td>527</td>
<td>MS</td>
<td>563</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

www.collegeboard.org
You may choose to vary the interval size. For example, you may assign different groups to create histograms with interval sizes of 5, 10 and 20. In addition, you can have one group create the frequency histogram and one group create the relative frequency histogram. The resulting discussion could focus on similarities and differences between the various histograms.

Allow students to use graph paper to create the histograms.

### Histograms – SATs

1. Display the data using a histogram where you examine:
   a. frequency (how many times a given score or group of scores occurred)
   b. relative frequency (divide the frequency of a score by the size of the sample)

(The interval should be 10 points – e.g., your interval might be 475-484.) To assist you in counting values for the histogram, use the following table to gather information:

<table>
<thead>
<tr>
<th>Verbal SAT Score Interval</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>475-484</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>485-494</td>
<td>3</td>
<td>0.06</td>
</tr>
<tr>
<td>495-504</td>
<td>8</td>
<td>0.16</td>
</tr>
<tr>
<td>505-514</td>
<td>7</td>
<td>0.14</td>
</tr>
<tr>
<td>515-524</td>
<td>3</td>
<td>0.06</td>
</tr>
<tr>
<td>525-534</td>
<td>4</td>
<td>0.08</td>
</tr>
<tr>
<td>535-544</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>545-554</td>
<td>4</td>
<td>0.08</td>
</tr>
<tr>
<td>555-564</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>565-574</td>
<td>5</td>
<td>0.10</td>
</tr>
<tr>
<td>575-584</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>585-594</td>
<td>4</td>
<td>0.08</td>
</tr>
<tr>
<td>595-604</td>
<td>0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

![Frequency Histogram](image1)

![Relative Frequency Histogram](image2)
2. Can you describe the two different distributions? Is there any difference? Can you explain why they may or may not be similar?

*The distributions are U-shaped or multi-modal. The only difference between them is the vertical scale. The relative frequency histogram uses a vertical scale that is the same as the original divided by 50, since 50 is the number of values sampled.*

3. Look at your state’s mean verbal score. Where does it fit in the group? Is it part of a cluster? Is it an outlier?

*Answers will vary according to the state of residence of the student.*

4. If you look at the data, can you guess at which point half the states have higher scores and half the states have lower scores? Where does your state fit?

*Half of the states have scores that are lower than about 530 points and half of the states have scores that are higher than about 530 points.*

*Answers will vary according to the state of residence of the student.*

5. What score do you observe most frequently?

*Scores between 495 and 504 occur the most frequently. (7 different values occur twice in the original data set.)*
Review distributions, outliers, and clusters before moving on to measures of central tendency.

Have students generate a list of different types of data where a mean score would be the best single value to represent the performance of a group.

<table>
<thead>
<tr>
<th>Measures of Central Tendency</th>
</tr>
</thead>
</table>
| In the last examples, you determined the average age or score, whether there was a point where half the data was below and half was above, or if there was a most common value. When we talk about an average score, or more precisely a **mean score**, we are trying to determine a single value that best represents the performance of a group. This single value is called a **measure of central tendency**. Being able to describe the performance of a whole group using a single statistic is important when comparing the performance of two or more groups. The **arithmetic mean** or **mean** represents the sum of the total number of values divided by the number of values. We write the arithmetic mean as follows:  

$$ \bar{X} = \frac{\Sigma X}{N} $$

where $\Sigma X$ is the symbol for the sum of all the X values and N is the number of values. The mean, in many ways, may be considered a “balancing out” or “evening out” of the data. For example, suppose everyone is given three scoops of M & Ms. You want everyone to get about the same number, but it is hard to make that happen when scooping them out. To estimate the number of M & Ms received by each person in this situation, we can find the average or “balance out” the difference between the amounts in the three scoops. To estimate the amount received by each person, we would figure out the average or balance out the differences between the amounts received.
### Measures of Central Tendency

1. Using the mean verbal score by state from the table, find the mean of those values.

<table>
<thead>
<tr>
<th>State</th>
<th>VSAT</th>
<th>State</th>
<th>VSAT</th>
<th>State</th>
<th>VSAT</th>
<th>State</th>
<th>VSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>479</td>
<td>MD</td>
<td>507</td>
<td>OH</td>
<td>527</td>
<td>OK</td>
<td>567</td>
</tr>
<tr>
<td>HI</td>
<td>482</td>
<td>ME</td>
<td>507</td>
<td>CO</td>
<td>536</td>
<td>NE</td>
<td>568</td>
</tr>
<tr>
<td>GA</td>
<td>487</td>
<td>VA</td>
<td>508</td>
<td>ID</td>
<td>542</td>
<td>IL</td>
<td>569</td>
</tr>
<tr>
<td>NC</td>
<td>493</td>
<td>CT</td>
<td>510</td>
<td>MT</td>
<td>545</td>
<td>UT</td>
<td>570</td>
</tr>
<tr>
<td>TX</td>
<td>494</td>
<td>MA</td>
<td>511</td>
<td>WY</td>
<td>546</td>
<td>MO</td>
<td>572</td>
</tr>
<tr>
<td>NY</td>
<td>495</td>
<td>NE</td>
<td>512</td>
<td>KY</td>
<td>547</td>
<td>KS</td>
<td>578</td>
</tr>
<tr>
<td>IN</td>
<td>496</td>
<td>VT</td>
<td>514</td>
<td>NM</td>
<td>549</td>
<td>WI</td>
<td>584</td>
</tr>
<tr>
<td>CA</td>
<td>497</td>
<td>AK</td>
<td>516</td>
<td>MI</td>
<td>557</td>
<td>SD</td>
<td>585</td>
</tr>
<tr>
<td>NJ</td>
<td>498</td>
<td>NH</td>
<td>520</td>
<td>TN</td>
<td>559</td>
<td>MN</td>
<td>586</td>
</tr>
<tr>
<td>PA</td>
<td>499</td>
<td>AZ</td>
<td>524</td>
<td>AL</td>
<td>561</td>
<td>ND</td>
<td>594</td>
</tr>
<tr>
<td>FL</td>
<td>499</td>
<td>WA</td>
<td>525</td>
<td>LA</td>
<td>561</td>
<td>IA</td>
<td>594</td>
</tr>
<tr>
<td>DE</td>
<td>503</td>
<td>OR</td>
<td>525</td>
<td>AR</td>
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<td></td>
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</tr>
<tr>
<td>RI</td>
<td>504</td>
<td>WV</td>
<td>527</td>
<td>MS</td>
<td>563</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean: \( \underline{533} \)

Have students determine when a mean and median score might be the same.

Ask when a mean value would be more informative and when a median value might be more useful.

### Measures of Central Tendency

The **median**, or middle value, of a data set is the value for which half the values in the set are less, and half are more. The median is another way to represent a group of numbers by a single value, and is found by putting the values of a set of data in increasing order from left to right. If there are an odd number of scores, then the middle score is the median. For example, the following series of numbers represent test scores:

\[
35 \quad 55 \quad 72 \quad 85 \quad 90
\]

The middle (or median) score among the five values is 72. If there are an even number of scores, for example,

\[
35 \quad 55 \quad 64 \quad 72 \quad 85 \quad 90
\]

then the median value is halfway between the scores closest to the middle (64 and 72), since there is no middle score.

\[
\frac{64 + 72}{2} = \frac{136}{2} = 68
\]

Half the test scores are less than 68 and half are more than 68.
Ask students to think about instances when assigning random numbers would be useful. Students should begin thinking about experimental designs and random sampling, as this will appear in several examples later in this unit.

You can generate random numbers on any graphing calculator. If you are using a TI-83 graphing calculator use the Math menu and the PRB submenu. Choose randInt. Enter 0,100 as the range of random numbers.

<table>
<thead>
<tr>
<th>Random Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask the person sitting nearest to you (in any direction) to give you 15 numbers between 0 and 100.</td>
</tr>
<tr>
<td>1. List the numbers</td>
</tr>
<tr>
<td><em>Answers will vary.</em></td>
</tr>
<tr>
<td>2. Place the numbers in increasing order and determine the median value.</td>
</tr>
<tr>
<td><em>Answers will vary.</em></td>
</tr>
<tr>
<td>3. Were the numbers random numbers? Explain.</td>
</tr>
<tr>
<td><em>The numbers are not truly random, although they may seem to be random. For example, students may have favorite numbers they included.</em></td>
</tr>
<tr>
<td>Use your graphing calculator or a computer to generate 15 random numbers between 0 and 100.</td>
</tr>
<tr>
<td>4. List the numbers in increasing order and determine the median.</td>
</tr>
<tr>
<td><em>Answers will vary.</em></td>
</tr>
</tbody>
</table>
Have students examine the stem and leaf plot they completed in this unit to find the mode of the data on page 5-19.

**Baseball History**

The mode is the value that appears most frequently in a set of data. The mode is the highest point, or bump, when looking at the distribution curve, and can be determined by observation. While it is considered a measure of central tendency, it is often not the most accurate measure and therefore not used as often as the mean or median. Nonetheless, calculating the mode can be extremely useful in some cases. For example, a clothing or shoe store would be interested in knowing the most popular sizes to ensure a sufficient inventory.

<table>
<thead>
<tr>
<th>Baseball History</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only 13 players in baseball have led their league in home runs, runs batted in, and batting average in one season. Look at the stem and leaf plot of the number of home runs hit:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1 6 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2 4 8 9 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Baseball History

1. Find the:
   a. mean
   
   *The mean is 36.4 or \( \bar{x} = 36.4 \)*
   
   b. median
   
   *The median value is 39.*
   
   c. mode
   
   *The modal value is 49.*

2. What type of distribution is represented by the data? Try sketching the shape of the distribution. (Recall the shapes of distribution discussed in Unit 1: symmetric, skewed to the left, and skewed to the right.)

   *The data set is skewed to the left.*
Breakfast Cereals

Calculating measures of central tendency is important to understanding data, but can you think of any characteristics of data that might be missed if you only study this aspect? While measures of central tendency help us to see what is typical, other aspects of data are important to identify too, for example, how the data is spread apart or how consistent it is.

One way we can get a quick and general idea of the spread of the data is to calculate the range. The **range** is the difference between the lowest value and the largest value.

<table>
<thead>
<tr>
<th>Cereal</th>
<th>Grams of Sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cocoa Rounds</td>
<td>13</td>
</tr>
<tr>
<td>Flakes of Corn</td>
<td>2</td>
</tr>
<tr>
<td>Frosty Flakes</td>
<td>11</td>
</tr>
<tr>
<td>Grape Nuggets</td>
<td>7</td>
</tr>
<tr>
<td>Golden Nuggets</td>
<td>10</td>
</tr>
<tr>
<td>Honey Nut O’s</td>
<td>10</td>
</tr>
<tr>
<td>Raisin Branola</td>
<td>9</td>
</tr>
<tr>
<td>Healthy Living Flakes</td>
<td>9</td>
</tr>
<tr>
<td>Wheatleys</td>
<td>8</td>
</tr>
<tr>
<td>Healthy Living Crunch</td>
<td>6</td>
</tr>
<tr>
<td>Multi-Grain O’s</td>
<td>6</td>
</tr>
<tr>
<td>All Branola</td>
<td>5</td>
</tr>
<tr>
<td>Munch Crunch</td>
<td>12</td>
</tr>
<tr>
<td>Branola Flakes</td>
<td>5</td>
</tr>
<tr>
<td>Complete Flakes</td>
<td>5</td>
</tr>
<tr>
<td>Corn Chrisps</td>
<td>3</td>
</tr>
<tr>
<td>Rice Chrisps</td>
<td>3</td>
</tr>
<tr>
<td>O’s</td>
<td>1</td>
</tr>
<tr>
<td>Shredded Wheatleys</td>
<td>0</td>
</tr>
<tr>
<td>Fruit Circles</td>
<td>13</td>
</tr>
</tbody>
</table>

In the above example, the range would be 13, because the lowest value is 0 and the largest is 13.

Another helpful way to see how the data set is spread is to divide it into equal parts. Recall that one of the measures of central tendency, the median, divides the data set into two equal parts. Breaking the data into parts with approximately the same number of data points in each is very useful.
### Breakfast Cereals

To determine the quartiles of a data set, begin by ordering the values from lowest (the lower extreme or minimum) to highest (upper extreme or maximum) and finding the range.

In the above problem, for example, you are looking at the number of grams of sugar per ounce in cereals. Given the range, follow these steps to determine the quartiles for this data. The places in the data where the divisions between each of four equal parts are located are called the **quartiles**.

**Step 1.** Arrange the numbers from lowest to highest as follows

```
0  1  2  3  3  5  5  5  6  6  7  8  9  9  10  10  11  12  13  13
```

**Step 2.** Find the median value and draw a vertical line through it. The **middle** (or second) **quartile** is the median.

```
0  1  2  3  3  5  5  5  6  6  7  8  9  9  10  10  11  12  13  13
```

**Step 3.** Count the number of values to the left of this vertical line and find their median value. Draw a vertical line through that median value as before. The median of these values is the **lower** (or first) **quartile** ($Q_1$), which in this example is 4.

```
0  1  2  3  3  5  5  5  6  6  7  8  9  9  10  10  11  12  13  13
```

**Step 4.** Count the values to the right of the median and find their median value. Draw a vertical line through that median value as before. The median of these values is the **upper** (or third) **quartile** ($Q_3$), which in this example is 10.

```
0  1  2  3  3  5  5  5  6  6  7  8  9  9  10  10  11  12  13  13
```

Using quartiles divides the data set into four equal parts. Percentiles also divide the data set into parts. Based on the root of this word, into how many parts do you think percentiles divide the data set?

**Percentiles divide the data set into 100 equal parts.**

The **$n^{th}$ percentile** for a data set is the value for which $n$ percent of the numbers in the set are less than that value. For example, the median in a data set often represents the $50^{th}$ percentile because 50% of the data is less than the median. In this data set, that is the case because 50% of the data is less than 6.5. Note that 6.5 itself is not a data point.

The **interquartile range (IQR)** is the difference between the upper and lower quartiles ($Q_3 - Q_1$) and represents the range of approximately the middle 50% of the data.
Breakfast Cereals

1. What percentile ranking is the first quartile in this data set?
   
   The first quartile is the 25th percentile.

2. The third quartile represents what percentile in this data set?
   
   The third quartile represents the 70th percentile. Note that this is not the 75th percentile because only 14 of the 20 data points are less than the number 10.

3. What percentile represents the minimum value in this data set? Can there be a 0th percentile? Why or why not?
   
   The 0th percentile represents the minimum value (0) in this data set. There can be a 0th percentile because there are no data points less than the minimum.

4. What percentile represents the maximum value in a data set? Can there be a 100th percentile? Why or why not?
   
   The maximum value in this data set would be the 95th percentile. There cannot be a 100th percentile because the percentile is determined by the percent of numbers in the set that are less than the value. In this case, 19 of 20 are less than the maximum.

5. Based on the data, find the IQR for sugar content in the cereal.
   
   The IQR is 6. IQR = 10 – 4 = 6.

6. State the benefit of the IQR and quartiles.
   
   - The IQR indicates the spread between the lower and upper quartiles. If it is a small number, then the middle 50% of the data is consistent. If it is a large number, then the middle 50% of the data is spread apart.
   
   - The outliers do not affect the IQR.
   
   - The quartiles indicate how the data is broken into quarters.
   
   - Together, the IQR and the quartiles can provide a quick impression of the spread of the data.
The top airlines in the U.S. are determined by the number of passengers, revenue, passenger miles, and miles flown. The following table shows the top 25 airlines in 1999 based on the number of passengers served (in millions).

<table>
<thead>
<tr>
<th>Airline Carrier</th>
<th>Number of Passengers</th>
<th>Airline Carrier</th>
<th>Number of Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Tran</td>
<td>6,458</td>
<td>Hawaiian</td>
<td>5,410</td>
</tr>
<tr>
<td>Air Wisconsin</td>
<td>3,373</td>
<td>Horizon Air</td>
<td>4,984</td>
</tr>
<tr>
<td>Alaska</td>
<td>13,604</td>
<td>Mesaba</td>
<td>5,453</td>
</tr>
<tr>
<td>Aloha</td>
<td>5,077</td>
<td>Midway</td>
<td>2,003</td>
</tr>
<tr>
<td>America West</td>
<td>18,686</td>
<td>Midwest Express</td>
<td>2,192</td>
</tr>
<tr>
<td>American</td>
<td>84,637</td>
<td>Northwest</td>
<td>54,692</td>
</tr>
<tr>
<td>American Eagle</td>
<td>11,449</td>
<td>Southwest</td>
<td>65,288</td>
</tr>
<tr>
<td>American Trans Air</td>
<td>5,022</td>
<td>Spirit Air</td>
<td>2,417</td>
</tr>
<tr>
<td>Atlantic Southeast</td>
<td>4,584</td>
<td>Trans States</td>
<td>2,384</td>
</tr>
<tr>
<td>Continental</td>
<td>43,880</td>
<td>Trans World</td>
<td>25,790</td>
</tr>
<tr>
<td>Continental Express</td>
<td>6,664</td>
<td>United</td>
<td>86,472</td>
</tr>
<tr>
<td>Delta</td>
<td>105,434</td>
<td>US Airways</td>
<td>55,812</td>
</tr>
<tr>
<td>Frontier</td>
<td>2,176</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students can put the data into the graphing calculator and generate a sorted list to help determine values of the quartiles.

### Airline Industry

1. Determine the median number of passengers (in millions) among the top 25 airlines. Indicate which airline is at the median. How many airlines are below the median? How many airlines are above the median?
   
   *The median is 6,458 million passengers. There are 12 airlines below and 12 airlines above the median value.*

2. Determine the lower extreme (minimum). Indicate which airline is at the lower extreme.
   
   *The lower extreme (minimum) value is 2,003 million passengers for Midway Airlines.*

3. Determine the upper extreme (maximum). Indicate which airline is at the upper extreme.
   
   *The upper extreme (maximum) value is 105,434 million passengers for Delta Airlines.*

4. Determine the lower quartile (Q₁). Indicate which airline is at the lower quartile.
   
   *The lower quartile is 3,978.5 million passengers, which is the mean of the number of passengers for Air Wisconsin and Atlantic Southeast.*

5. Determine the upper quartile (Q₃). Indicate which airline is at the upper quartile.
   
   *The upper quartile is 49,286 million passengers, which is the mean of the number of passengers for Continental and Northwest.*

6. Which airlines are below the 25th percentile? Which are above the 75th percentile?

   Below the 25th percentile are Midway (2,003), Frontier (2,176), Midwest Express (2,192), Trans States (2,384), Spirit Air (2,417) and Air Wisconsin (3,373).

7. Calculate the IQR.

   \[ IQR = 49,286 - 3,978.5 = 45,307.5 \text{ million passengers}. \]

8. Are there any outliers? If so, which airline carrier has this distinction? Is it for the lowest or highest number of passengers carried?

   *There are not outliers for this data, although some students may say Delta.*

Note: Although you can determine an outlier based on observation, you can also use a systematic method. To find the outlier, you first find the IQR, then follow these steps:

- **Step 1.** Add 1.5 X IQR to the third quartile – any number larger than Q₃ + (1.5 X IQR) is an outlier.
- **Step 2.** Add 1.5 X IQR to the first quartile – any number smaller than Q₁ - (1.5 X IQR) is an outlier.

9. Based on this formula, verify if the outliers you chose by observation were correct. If there are other outliers, please indicate them.

   *The 1.5*IQR = 67,961.25 million passengers. Q₃ + 1.5*IQR = 117,247.25 and none were higher. Q₁ – 1.5*IQR = - 63,982.75 and none could have a negative number of passengers.*
Box and Whisker Plot – Airline Industry

Box plots are a fairly recent invention in statistics. They were first used in the 1970s.

This activity guides students through the construction of a box and whiskers plot. The plot provided here will help students understand the desired outcome.

Discuss why the box and whiskers plot displays data in a more summative way than other visual displays.

Box and Whisker plots (also known as box plots) are useful when you are working with a large data set (hundreds or thousands of entries). Box and whisker plots allow you to focus on the relative positions of different sets of data, which allows you to make comparisons more easily. An example of a box and whiskers plot that compares the ages of the Oscar winning actresses and actors is shown below.

From the box and whiskers plot you can easily see the relative positions of the data, along with clusters and outliers.

Generating a box and whisker plot requires finding and using several pieces of information. In order to draw the plot, you will need to know the extremes, the upper and lower quartiles, and the median (these data points are sometimes referred to as a five-number summary).
Another method for creating the whiskers is to make the length of each whisker less than or equal to $1.5 \times IQR$ by extending it to the smallest or largest data point within $1.5 \times IQR$. This method allows outliers to be spotted quickly because they will be data points below or above the whiskers.

To generate a box and whisker plot, use the information you collected in the last section about airline carriers to complete the following steps:

Step 1. Fill in the information below using the previous task’s answers.

- Lower Extreme: 2,003
- Upper Extreme: 105,434
- Lower Quartile: 3,978.5
- Upper Quartile: 49,286
- Median: 6,458

Step 2. Draw a number line representing the full range of values and then draw dots below the number line to indicate the median, quartiles and extremes.

Step 3. Draw a box between the two quartiles. Mark the median with a vertical line through the box. Draw two whiskers from the quartiles to the extremes.
Be sure students realize that each ¼ of the data will not necessarily be equal in width (the range within each quartile will differ), even though each quartile represents approximately the same number of data points. For example, if there are 40 data points, the points in the 1st quartile may be spread quite a bit, while the points in the 2nd quartile may be clustered closely together.

<table>
<thead>
<tr>
<th>Box and Whisker Plot – Airline Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> From the box and whisker plot above, why is one whisker longer than the other? What does this mean?</td>
</tr>
<tr>
<td>The right whisker is longer since the value for Delta is far to the right of all the other values.</td>
</tr>
<tr>
<td><strong>2.</strong> What is indicated when the median is far to the left of center in the box?</td>
</tr>
<tr>
<td>The median is far to the left within the box since the third quartile of data has more spread than the second quartile of data. The number of data points in the second quartile and in the third quartile is the same. The data in the third quartile simply has a greater range of values than the data in the second quartile.</td>
</tr>
<tr>
<td><strong>3.</strong> What does the box and whisker plot tell you about the relationship between the top 25% of the carriers?</td>
</tr>
<tr>
<td>The top 25% of the carriers have passenger totals that vary greatly from each other as compared to the lower 50% of the values. The top 25% of the values are quite spread apart. This makes the right whisker long.</td>
</tr>
<tr>
<td><strong>4.</strong> How do you think the relationship between the airline carriers affects competition?</td>
</tr>
<tr>
<td>Answers will vary, but some students might say that airlines trying to increase their number of passengers might lower their prices forcing others to do the same.</td>
</tr>
</tbody>
</table>
These activities introduce measures of spread. Continue to stress that students should look at many aspects of data: how to organize the data, how to display the data and how to calculate measures of central tendency.

<table>
<thead>
<tr>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>As we continue to study measures of central tendency and spread, a key observation, independent of the manner in which the data is displayed, is that it has variability. We study the pattern of that variability by looking at the distribution of the values.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Some possible questions to ask include: “What does it mean for something to vary?” “How do the terms variable and variance relate?”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>input</th>
</tr>
</thead>
<tbody>
<tr>
<td>As we continue to study measures of central tendency and spread, a key observation, independent of the manner in which the data is displayed, is that it has variability. We study the pattern of that variability by looking at the distribution of the values.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
</table>
| 1. The census, mandated by the U.S. Congress to be conducted every ten years, is designed to determine congressional districts and number of representatives. Think about the collection of census data. What might cause variability in the data?  

**Answers will vary, but could include the notion that populations in areas change because children are born, people die, others move in or out of an area, etc.**  

2. If you were studying the test results for students, who are all of equal ability and in the same grade level but in different classes, what could cause variability in the results?  

**Answers will vary, but could include the following ideas:**  

**Various teachers may present information differently, they may give different tests, they may administer tests at different times of the day when students are more or less able to concentrate, they may state the directions differently, etc.** |
Variance – Basketball

The varsity basketball team has its first game of the season against a team about which they know very little. To get a sense of who might have the advantage, you look at the height of the players.

<table>
<thead>
<tr>
<th>Home Team Heights</th>
<th>Visiting Team Heights</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>68</td>
</tr>
<tr>
<td>68</td>
<td>68</td>
</tr>
<tr>
<td>67</td>
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<td>70</td>
<td>72</td>
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<td>70</td>
<td>71</td>
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<tr>
<td>71</td>
<td>66</td>
</tr>
<tr>
<td>71</td>
<td>67</td>
</tr>
</tbody>
</table>
The discussion leading up to the formal definition of standard deviation should help students construct an understanding of how to find variance and standard deviation, and the purpose for finding them. An important goal is that students know what these concepts mean and how we use them to interpret data. It is not as important that students actually calculate the standard deviation.

### Variance – Basketball

1. Determine the mean and range for each group.

**Home:**  
\[
\bar{x} = 68.5 \text{ inches} \quad n = 10
\]

**Range** = 6 inches \((71 - 65)\)

**Visiting:**  
\[
\bar{x} = 68.5 \text{ inches} \quad n = 10
\]

**Range** = 6 inches \((72 - 66)\)

Up to this point you have observed differences within a data set, or variability, by looking at its distribution. Variability can be measured in several ways. For example, interquartile range can be used as a measure of spread.

There are other measures that will help us to understand the spread of a data set. These include mean absolute deviation, variance, and standard deviation. A deviation from the mean represents the difference of each data value from the mean. Deviation is represented symbolically as \((x - \bar{x})\) where \(\bar{x}\) is the mean value.
It is important that these deviations include direction, so that they sum to 0. This will motivate the discussion on page 5-35 on absolute deviation.

### Variance – Basketball

2. 
   a. Determine the deviation of each value from the mean.

<table>
<thead>
<tr>
<th>Home Team</th>
<th>Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heights</td>
<td></td>
</tr>
<tr>
<td>69</td>
<td>0.5</td>
</tr>
<tr>
<td>68</td>
<td>-0.5</td>
</tr>
<tr>
<td>67</td>
<td>-1.5</td>
</tr>
<tr>
<td>68</td>
<td>-0.5</td>
</tr>
<tr>
<td>66</td>
<td>-2.5</td>
</tr>
<tr>
<td>65</td>
<td>-3.5</td>
</tr>
<tr>
<td>70</td>
<td>1.5</td>
</tr>
<tr>
<td>70</td>
<td>1.5</td>
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<tr>
<td>71</td>
<td>2.5</td>
</tr>
<tr>
<td>71</td>
<td>2.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Visiting Team</th>
<th>Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heights</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>-0.5</td>
</tr>
<tr>
<td>68</td>
<td>-0.5</td>
</tr>
<tr>
<td>69</td>
<td>0.5</td>
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<tr>
<td>69</td>
<td>0.5</td>
</tr>
<tr>
<td>67</td>
<td>-1.5</td>
</tr>
<tr>
<td>72</td>
<td>3.5</td>
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<td>71</td>
<td>2.5</td>
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<tr>
<td>66</td>
<td>-2.5</td>
</tr>
<tr>
<td>67</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

b. Determine the mean of the deviations for each team.

- **Home mean deviation = 0** *(The sum of the deviations is 0, n = 10.)*
- **Visiting mean deviation = 0** *(The sum of the deviations is 0, n = 10.)*

### Variance – Basketball

3. Is the average of the deviations meaningful? Why or why not?

*The deviations are quite meaningful. They indicate how far from the mean each data point is, but the average of the deviations is not meaningful. The sum of the deviations will always be 0, so the average of the deviations is always 0.*

4. Were any of your deviations negative? Why? Since deviation is independent of direction, how can you change your values to reflect that?

*Some of the deviations were negative. In computing the deviation (defined as \((x - \bar{x})\), some data values \(x\) were less than or below the mean value of 68.5 \((\bar{x})\). Thus, when the deviations were calculated, some negative values resulted. In order to make the values reflect deviation that is independent of direction, the absolute value of the deviations should be found.*
The absolute deviation is the distance from the individual data points to the mean of the data set. Absolute deviations do not indicate whether an individual data point is above or below the mean. Thus, students may correctly perceive that the average of the absolute deviations can be used to measure how spread out a data set is.

### Variance – Basketball

If you have not already considered finding the absolute value of the deviations, you may want to do so now. This is called the **absolute deviation**.

<table>
<thead>
<tr>
<th>Home Team Height</th>
<th>Deviation</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>68</td>
<td>-0.5</td>
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<td>65</td>
<td>-3.5</td>
<td>3.5</td>
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<td>70</td>
<td>1.5</td>
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<td>2.5</td>
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<tr>
<td>71</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Visiting Team Height</th>
<th>Deviation</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td>-0.5</td>
<td>0.5</td>
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<td>68</td>
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<td>-2.5</td>
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<tr>
<td>67</td>
<td>-1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

5. Having found the absolute value, find the average of the absolute deviations. Does finding this value make sense? How so?

   **Home Team:** \[
   \frac{17}{10} = 1.7
   \]

   **Visiting Team:** \[
   \frac{14}{10} = 1.4
   \]

   Finding the average of the absolute deviations makes sense because it indicates the average deviation of data points from the mean in a data set, which provides information about the spread of the data. (However, this statistic is not commonly used.)
Variance – Basketball

The value that you have calculated is called the mean absolute deviation. It is one measure of spread for a data set. While finding the mean absolute deviation seems like a sensible approach for measuring the variability, this method is not used very often. A more common measure of variability is standard deviation.

The standard deviation is found by finding the variance and then taking the square root of that variance. Variance and standard deviation are both measures of spread.

To find the variance of a sample of data, you square each deviation and add the squared values. Divide the sum of the squared deviations by the total number of observations minus one \((n - 1)\). The formula for finding the variance is:

\[
\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}
\]

To complete the process of finding the standard deviation, you take the square root of the variance. The formula for finding the standard deviation is

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}
\]

7. Why do you think that the deviations were squared and then the square root was taken in the formula? What purpose does this serve?

This technique is another method of representing only positive values (the distance from the mean irrespective of the direction) while making sure the standard deviation and data have the same units.

Variance – Basketball

8. Find the standard deviation of the players on each team.

The standard deviation of the home team heights is 2.07 (using the sample standard deviation formula given above. The population standard deviation is 1.962.)

The standard deviation of the visiting team heights is 1.841 (using the sample standard deviation formula given above. The population standard deviation is 1.746.)
This activity should be treated as a culminating activity for the introduction of concepts relating to organizing and summarizing data.

Following this activity, we will apply these concepts in experimental design settings.

**Candy Sale**

The members of a local youth organization decided that over the course of a year they would have four fundraisers to help various charities. For each fundraiser, they plan to sell different types of candy bars at several key locations over a period of one month. After the first fundraiser, a tally of all the candy bars sold was made. The data was collected to help the organization better understand how much money it could make and if it should change its fundraising plans in any way. The number of candy bars sold by each member is listed below.

<p>| | | | | | | | | | |</p>
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<td>37</td>
<td>44</td>
<td>48</td>
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<td>43</td>
<td>28</td>
<td>42</td>
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</tbody>
</table>

1. If you were asked to display the data visually, what type of display would you use? Why? Which type of visual display would be the most informative? Which would be the least informative? Draw the visual display that you have determined is the most informative.

*Answers will vary, but appropriate answers include a stem and leaf plot or a histogram as helpful visual displays for this data. They provide the opportunity to visualize the distribution of the data. With these, it is possible to see that the data is skewed to the right. A dot or line plot, in contrast, would be less informative for this data set.*

```
process: model

2  4  7  8  9  9
3  3  5  7  7  8  8  8
4  0  1  2  3  4  8
5  0  3
6  0
```

*stem width = 10*
Because students have calculated measures of center frequently, you may permit them to use the graphing calculator in performing these calculations. Follow with questions to assure a correct interpretation of these measures of center and spread.

| Candy Sale |
| 2. Describe the distribution of the data. Be sure to note whether clusters or outliers are present. |

There is a cluster of data from about 35 to 44.

There are no outliers since the IQR is 12.5 from $Q_2 - Q_1 = 43.5 - 31$. 1.5 times the IQR is 18.75. If 18.75 is subtracted from $Q_1$, 12.25 is the result. Thus, any number less than 12.25 would be an outlier. If 18.75 is added to $Q_3$, 62.25 is the result. Thus, any number greater than 62.25 would be an outlier.

The data is skewed somewhat to the right, meaning there are more low numbers in the data set.

| Candy Sale |
| 3. Find the mean, median, and standard deviation of the sales data. |

The mean is approximately 38.76.

The median is 38.

The standard deviation is 9.07.

| Candy Sale |
| 4. How would you explain the variation in the data? |

Answers will vary. Appropriate answers should include ideas such as variation results from the facts that different people are selling the candy, they are selling different kinds of candy, and they are selling them on different days and in different locations.
Refer back to the beginning of Unit 2 and the Left-Handed Learners activity. In that activity, students were asked about the generalization of their class sample to certain populations.

### Collecting, Modeling and Interpreting Data

When comparing single variable data or studying the relationship between variables through statistics, we also must understand why differences may exist and why some differences are small or large.

Differences will exist naturally, but they may be minimized or maximized due to error. Error may be associated with sampling techniques used. For example, a sample is biased if it tends to over-represent certain segments of the population while under-representing other segments.

In general, to avoid bias in a sample, experimenters use random sampling. For example, in a human subjects study, a random sample would be one in which subjects are arbitrarily assigned to experimental and control groups.

Another factor that may have an impact on statistical outcomes is the design of the study. It is possible for it to be flawed. For example, when designing an experimental study, you want to be able to control as many variables as possible, (i.e., make conditions basically the same across groups). Ideally, the only element that should vary is the one you are testing. Even when ideal conditions, variability will still occur and it must be understood in order to fully and accurately interpret results.

Finally, error may occur because correct statistical procedures and processes might not have been adhered to in the data collection process, or measurements might be imprecise.

Provide at least one example of a case where errors due to design or measurement could lead to faulty findings.

*Answers will vary. An example solution is that, in the process of administering an oral survey, the individual reading the question may inadvertently change its phrasing. This modification may serve to bias the response of the person being asked.*

Many applications in the social sciences, the biological sciences, business, and economics use linear equations as data models. Many times data does not fall precisely along a straight line. In those cases, we must find the best possible line to fit the data.
In this data, unlike the number sequences in Unit 1, a pattern may be less evident. Ask students if they think there might be some other visual display that might help to represent the data before getting to the graph. After the graph is complete, compare the visual display(s) they chose with the graph to see which worked the best.

Mia's Growing Like a Weed

Your cousin Mia was a healthy, happy baby girl. As most new parents do, Mia’s mom and dad kept records of her growth. At each doctor’s visit, Mia’s height and weight were recorded. Mia’s mom took a closer look at all these numbers after Mia was a year and a half old. She was amazed at how quickly Mia had grown and she wondered how tall Mia would be when she reached adulthood. Mia’s mom knew you were studying data analysis in school, so she asked you to help her figure out how tall Mia might grow.

Examine Mia’s height and weight chart.¹

<table>
<thead>
<tr>
<th>Quantity Name Units</th>
<th>Mia's age</th>
<th>Mia's weight</th>
<th>Mia's height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>months</td>
<td>pounds</td>
<td>inches</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>6.125</td>
<td>17.875</td>
</tr>
<tr>
<td>1</td>
<td>8.125</td>
<td>20.5</td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>2.25</td>
<td>10.313</td>
<td>21.75</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13.688</td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>25.75</td>
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<tr>
<td>8</td>
<td>21</td>
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<td>15</td>
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<td>30.5</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>25.125</td>
<td>32.5</td>
<td></td>
</tr>
</tbody>
</table>

¹ Actual records of Mia’s height and weight used with permission.
This is the first time in this course that students will plot data points that are not in a straight line. These points form a scatter plot from which students can approximate the best fitting line – one that will pass through or is closest to the points in the plane. Since we want students to construct this concept, caution them not to connect the points. In question 3, the concept of scatter plot will be formalized. Remind students that heights and weights do not change at a constant rate.

This is an opportunity to use the graphing calculator in the classroom. Students can view the data on the calculator, making sure to use the appropriate window.

Directions for using the graphing calculator with this activity are provided on the K-12 Community website at www.carnegielearning.com.

### Mia’s Growing Like a Weed

1. Plot the points from the data table of Mia’s age versus her weight. Make sure to label your axes appropriately and use an appropriate scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>Weight</td>
<td>0</td>
<td>40</td>
<td>2</td>
</tr>
</tbody>
</table>

---

**Directions:**
- **Process:** Model
- **1.** Plot the points from the data table of Mia’s age versus her weight. Make sure to label your axes appropriately and use an appropriate scale for each axis.
### Mia's Growing Like a Weed

<table>
<thead>
<tr>
<th>process: evaluate</th>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. How much did Mia’s weight increase between months 4 and 6?</td>
<td>5. Can you connect all the points on the graph with one straight line? Use your responses in questions 2 – 4 to justify your answer.</td>
</tr>
<tr>
<td>\textit{It increased 3.332 pounds.} [ 17 - 13.668 = 3.332 ]</td>
<td>\textit{No, not all of the points line up. The change in Mia’s weight is not constant.}</td>
</tr>
<tr>
<td>3. How much did Mia’s weight increase between months 6 and 8?</td>
<td></td>
</tr>
<tr>
<td>\textit{It increased 4 pounds.} [ 21 - 17 = 4 ]</td>
<td></td>
</tr>
<tr>
<td>4. Did Mia’s weight increase at the same rate during the two-month interval in question 2 as in the two-month interval in question 3?</td>
<td></td>
</tr>
<tr>
<td>\textit{No, it increased more the second two-month interval.} [ \text{The change is not constant.} ]</td>
<td></td>
</tr>
</tbody>
</table>
6. Plot the points from the data table of Mia’s age versus her height.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>Height</td>
<td>0</td>
<td>40</td>
<td>2</td>
</tr>
</tbody>
</table>

Use the same age bounds and intervals on this graph as you did on page 5-41.
### Mia's Growing Like a Weed

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. How much did Mia’s height increase between months 6 and 8?</td>
</tr>
<tr>
<td>$27 - 25.75 = 1.25$ inches</td>
</tr>
<tr>
<td>It increased 1.25 inches.</td>
</tr>
<tr>
<td>8. How much did Mia’s height increase between months 8 and 10?</td>
</tr>
<tr>
<td>$27 - 27 = 0$ inches</td>
</tr>
<tr>
<td>Her height did not increase.</td>
</tr>
<tr>
<td>9. Did Mia’s height increase at the same rate during the two-month interval in part a as in the two month interval in part b?</td>
</tr>
<tr>
<td>No, it increased more in the first two-month interval.</td>
</tr>
</tbody>
</table>

### Mia's Growing Like a Weed

<table>
<thead>
<tr>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. Can you connect all the points on the graph with one straight line? Use your responses in questions 7 - 9 to justify your answer.</td>
</tr>
<tr>
<td>No, not all of the points line up. The change in Mia’s height is not constant.</td>
</tr>
</tbody>
</table>

You may want to have students work with a couple of data sets and a string or a strand of uncooked spaghetti to understand what it means to find a line of best fit. You can discuss what points may be outliers. This is also a good time to discuss what it means to model. Namely, a model is often a best approximation.

### Mia's Growing Like a Weed

<table>
<thead>
<tr>
<th>Process: Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>The two plots you have just created are scatter plots. For each of the scatter plots, create a line that best approximates the data by laying your pencil down so that it has about half of the data points below it and half of the data points above it on the graph. This line will be called your line of best fit, and you should use it to answer the following questions.</td>
</tr>
<tr>
<td>Note: The lines of best fit are included in the graph plots on the preceding pages.</td>
</tr>
</tbody>
</table>
Final answers to these questions will vary depending on how each student drew the line of best fit for each graph. The intent of these questions is to get students to use some actual data to think proportionally. Focus on the written explanation of how they obtained each solution rather than the specific numerical answer.

If students use the graph to find the average rate of growth, that is the most appropriate. However, it is still appropriate for students to use the first and last data points to find an "average" rate of growth. Additionally, do not be overly concerned that students use Mia’s birth measurements as starting points in the calculations. Focus on students using the average rates of growth appropriately.

You may want to ask students about the reasonableness of the predictions after each of questions 13, 14, 15, and 16.

<table>
<thead>
<tr>
<th>Student Name: Mia’s Growing Like a Weed</th>
</tr>
</thead>
<tbody>
<tr>
<td>11. Approximately how many pounds has Mia gained each month of her first 18 months? How did you use your line of best fit to answer this question?</td>
</tr>
<tr>
<td><strong>Sample response:</strong> Mia gained about 1 pound each month. I used the line to find out how much her weight changed every month.</td>
</tr>
<tr>
<td>12. Approximately how many inches has Mia grown each month of her first year and a half? How did you use your line of best fit to answer this question?</td>
</tr>
<tr>
<td><strong>Sample response:</strong> Mia grew a little less than 1 inch per month (about ( \frac{3}{4} ) in.). I used the line to find out how much her height changed every month.</td>
</tr>
<tr>
<td>13. If Mia continues to grow at this rate, what will her height and weight be on her second birthday? Use complete sentences to explain how you used your lines of best fit to answer this question.</td>
</tr>
<tr>
<td><strong>Sample Responses:</strong></td>
</tr>
<tr>
<td>Her height will be about 37 inches.</td>
</tr>
<tr>
<td>Her weight will be about 31 pounds.</td>
</tr>
<tr>
<td>To find my lines of best fit, I added 6 pounds to her weight at 18 months and 5.5 inches to her height.</td>
</tr>
<tr>
<td>14. If Mia continues to grow at this rate, what will her height and weight be on her fifth birthday? Could you use your graph to answer this? If so, how? If not, what strategy did you use to answer the question?</td>
</tr>
<tr>
<td>Her height would be about 66 inches and her weight would be about 67 pounds. You could use the graph if you extended the x-axis to 60 months. Then you could find the point on the line and see what her height and weight would be. Another strategy is to add about 32 pounds to her weight at 18 months (or 45 pounds on to her birth weight).</td>
</tr>
<tr>
<td>15. What will Mia’s height and weight be on her 18th birthday?</td>
</tr>
<tr>
<td><strong>Her weight would be 223 pounds.</strong></td>
</tr>
<tr>
<td><strong>Her height would be about 191 inches.</strong></td>
</tr>
<tr>
<td>16. What will Mia’s height be on her 30th birthday?</td>
</tr>
<tr>
<td><strong>Her height would be about 306 inches.</strong></td>
</tr>
</tbody>
</table>
Students should now make the connection between this situation and constant rate of change.

### Mia's Growing Like a Weed

17. Is your model (the line of best fit) for Mia’s height reasonable to use to predict her height at two years of age? Why or why not? Is the model for Mia’s weight reasonable to use to predict her weight at two years of age?

*The model for height would not be reasonable; 3 feet is very tall for a two-year-old.*

*The model for weight may be close; 31 pounds sounds reasonable for a two-year-old.*

18. Is it reasonable to use these models to predict Mia’s height and weight when she is five? How about when she is 18? Explain your answer.

*No, after a while your growth slows down along with your weight.*

19. Write a note to Mia’s mom, summarizing the results of your analysis of this data. Make sure to include information about Mia’s rate of growth and what you think will happen as Mia continues to grow.

*Sample Response:*

Dear Mia’s Mom,

I found that, as Mia grows, her height and weight will have to slow down. Using her rate of growth from the first 18 months, I determined that Mia’s height and weight at 5 years of age would be 66 inches and 66 pounds. By her eighteenth birthday, she would be 191 inches tall. This measurement is not reasonable and therefore shows that her growth will have to slow down.
Supply students with graph paper for question 20. Their original plots will most likely not allow them to record to 66 months of age.

Students may have difficulty finding the equation. You may want to discuss average rate of change in general and specifically in this problem. Distance = RT is another good example where average rate applies.

Mia's Growing Like a Weed

Here is the rest of the data on Mia’s growth from age two through five and one-half years.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Mia's age</th>
<th>Mia's weight</th>
<th>Mia's height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>years</td>
<td>pounds</td>
<td>inches</td>
</tr>
<tr>
<td>Units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>27.25</td>
<td>34.5</td>
<td></td>
</tr>
<tr>
<td>2 ½</td>
<td>30.0</td>
<td>35.75</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>32.0</td>
<td>36.625</td>
<td></td>
</tr>
<tr>
<td>3 ½</td>
<td>33.0</td>
<td>38.0</td>
<td></td>
</tr>
<tr>
<td>4 ½</td>
<td>39.0</td>
<td>42.0</td>
<td></td>
</tr>
<tr>
<td>5 ½</td>
<td>44.0</td>
<td>45.0</td>
<td></td>
</tr>
</tbody>
</table>

20. Modify your scatter plots and your lines of best fit to include the new data points.

21. Are your new models any more reasonable to use to predict Mia’s height and weight when she reaches age 18? Explain in complete sentences.

They would be somewhat more reasonable because 5½ years is much closer to 18 years than is 18 months, but the prediction for height is still too much.

22. What are the equations for your lines of best fit with all the data included? How did you find these equations based upon the graphs of the lines?

\[
H = 0.4A + 22.5
\]
\[
W = 0.5A + 12.5
\]

I found these equations by looking at where the lines started and finding how much they changed each month.
Students will use a larger set of data points to generate a line of best fit.

If you choose to use the graphing calculator to process the data in the next few activities, you should still have students plot the data points and attempt to visualize the line of best fit on their hand-drawn plot for at least one of the activities. This allows students to compare their hand-drawn lines of best fit to those of other students to see that these approximations are not all consistent. The line of best fit created by the graphing calculator can also be contrasted with the manually created line of best fit.

Here is a table of the number of hours 20 college students spent studying for tests during the term and the average scores they received.

<table>
<thead>
<tr>
<th>Study Hours</th>
<th>Exam Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>86</td>
</tr>
<tr>
<td>20</td>
<td>91</td>
</tr>
<tr>
<td>9</td>
<td>65</td>
</tr>
<tr>
<td>45</td>
<td>98</td>
</tr>
<tr>
<td>32</td>
<td>83</td>
</tr>
<tr>
<td>21</td>
<td>72</td>
</tr>
<tr>
<td>11</td>
<td>74</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>72</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
</tr>
<tr>
<td>14</td>
<td>68</td>
</tr>
<tr>
<td>22</td>
<td>87</td>
</tr>
<tr>
<td>17</td>
<td>81</td>
</tr>
<tr>
<td>9</td>
<td>65</td>
</tr>
<tr>
<td>31</td>
<td>93</td>
</tr>
<tr>
<td>24</td>
<td>85</td>
</tr>
<tr>
<td>20</td>
<td>85</td>
</tr>
<tr>
<td>7</td>
<td>51</td>
</tr>
<tr>
<td>18</td>
<td>73</td>
</tr>
<tr>
<td>10</td>
<td>62</td>
</tr>
</tbody>
</table>
Help students by telling them that this line should be the one that minimizes the sum of the distances between the data points and the line.

**College Students**

1. Graph the data points and draw the line you think best fits the points.

   ![Graph of College Students data](image)

   y = 1.1x + 55.3

2. Find the slope, the y-intercept, and the equation of your line.

   - **Slope:** 1.1
   - **y-intercept:** (0, 55.3)
   - **Equation:** $y = 1.1x + 55.3$
These solutions will vary based on the equations of the student-created lines of best fit.

<table>
<thead>
<tr>
<th>College Students</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.</strong> If a student studied for 25 hours, approximately what score would she expect to get on the exam?</td>
</tr>
<tr>
<td>The student should expect to earn an 83 or 84 on the exam.</td>
</tr>
<tr>
<td><strong>4.</strong> If a student studied for 15 hours, approximately what score would she expect to get on the exam?</td>
</tr>
<tr>
<td>The student would expect to get a 72 or 73 on the exam.</td>
</tr>
<tr>
<td><strong>5.</strong> How many hours should a student study to get an A (a score of 90 or better)?</td>
</tr>
<tr>
<td>To earn a 90 or better, a student should study more than 30 hours.</td>
</tr>
<tr>
<td><strong>6.</strong> How many hours should a student study to get a C (a score of 70 to 79)?</td>
</tr>
<tr>
<td>To earn a C, a student should study between 13 and 22 hours.</td>
</tr>
<tr>
<td>College Students</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>7. Check with the other members of your group. Do they have the same equation you have? Why or why not?</td>
</tr>
<tr>
<td><strong>Sample Response:</strong></td>
</tr>
<tr>
<td>The other members of my group have different equations than I do because their lines of best fit that were drawn by hand have slightly different slopes or y-intercepts. They simply looked at the data a little differently than I did.</td>
</tr>
<tr>
<td>8. What does the slope represent in this problem situation?</td>
</tr>
<tr>
<td>The slope represents the amount of increase in a student’s score for every additional hour studied.</td>
</tr>
<tr>
<td>9. What does the y-intercept represent in this problem situation?</td>
</tr>
<tr>
<td>The y-intercept represents the score a student would earn if he or she did not study at all.</td>
</tr>
<tr>
<td>10. Does your model (your equation) provide reasonable answers to these questions? Why or why not?</td>
</tr>
<tr>
<td>Based upon the data, the answers are reasonable. For example, a score of 55 seems reasonable if no studying was done.</td>
</tr>
<tr>
<td>However, students may say that the answers are not reasonable based upon their experiences. For example, students may say that having to study 30 hours to earn an A on an exam is too much time.</td>
</tr>
</tbody>
</table>
Note that the dollar amounts spent on ads are in millions, while dollar amounts for sales are in billions.

This table shows the total retail sales in billions of dollars based on the amount spent in advertising (in millions of dollars).

### Data for Retail Sales

<table>
<thead>
<tr>
<th>Amount spent in advertising</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>millions of dollars</td>
<td>billions of dollars</td>
</tr>
<tr>
<td>100</td>
<td>157.8</td>
</tr>
<tr>
<td>150</td>
<td>158.2</td>
</tr>
<tr>
<td>200</td>
<td>159</td>
</tr>
<tr>
<td>250</td>
<td>158.8</td>
</tr>
<tr>
<td>300</td>
<td>160.7</td>
</tr>
<tr>
<td>350</td>
<td>161</td>
</tr>
<tr>
<td>400</td>
<td>162.2</td>
</tr>
<tr>
<td>450</td>
<td>164.8</td>
</tr>
<tr>
<td>500</td>
<td>165.1</td>
</tr>
<tr>
<td>550</td>
<td>166.8</td>
</tr>
<tr>
<td>600</td>
<td>167.1</td>
</tr>
</tbody>
</table>
1. Graph the data points and draw the line you think best fits the points.

2. Indicate the slope, the y-intercept, and the equation of the line.
   - Slope: 0.02
   - y-intercept: (0, 154.8)
   - Equation: \( y = 0.02x + 154.8 \)

In comparing equations in question 3, if students used a calculator to find the line of best fit, this information should also be included in their discussion.

3. Check with the other members of your group. Do they have the same equation you have? Why or why not?

   **Sample Response:**
   
   The other group members have different equations than I do because their lines of best fit that were drawn by hand have slightly different slopes or y-intercepts. They simply looked at the data a little differently than I did.
Retail Sales

4. Use your model (equation) to predict total retail sales if:
   a. 650 million was spent in advertising. The model predicts $168.1 billion in sales.
   b. 700 million was spent in advertising. The model predicts $169.1 billion in sales.

5. Predict the amount spent on advertising if retail sales reach:
   a. $170 billion dollars? The amount spent on advertising is $742 million.
   b. $200 billion dollars? The amount spent on advertising is $2.2 billion.

6. Use your model to find total retail sales if 350 million was spent on advertising. Does your answer match the value from the table?
   The model predicts that total retail sales would be $169.95 billion. This amount is higher than the $161 billion listed in the table.
Students are asked to interpret the slope in the context of the problem. Remind students that slope is the change in y for every unit change in x.

## Retail Sales

<table>
<thead>
<tr>
<th>Analysis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7. What does the slope represent in this problem situation?</td>
<td></td>
</tr>
<tr>
<td><strong>The slope in this problem situation represents the increase in retail sales (in billions of dollars) for every increase of $1 million in advertising money.</strong></td>
<td></td>
</tr>
<tr>
<td>8. What does the y-intercept represent in this problem situation?</td>
<td></td>
</tr>
<tr>
<td><strong>The y-intercept in this problem situation represents the amount of sales (in billions of dollars) if no money is spent on advertising.</strong></td>
<td></td>
</tr>
<tr>
<td>9. Will this model accurately predict retail sales for any amount spent in advertising? Why or why not?</td>
<td></td>
</tr>
<tr>
<td><strong>This model will not always be accurate. There is probably a point at which the amount spent on advertising could continue to increase without an increase in the sales figures. The model also may not be accurate for amounts lower than 100 million spent on advertising.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Stroop Test

One of the main applications of data analysis is to make predictions about real-world situations, using data from experiments, surveys, or measurements. The data you collect in this exercise originates from an experiment in cognitive psychology—the branch of psychology that tries to understand and explain how the human brain works. This experiment is called the Stroop Test.

The experiment uses lists of color words: red, green, black, and blue. Each list has a different number of words, and each word is written in ink that is red, green, black, or blue. There are two different kinds of lists:

- In Matching Lists, the color of the ink matches the color word: red written in red ink, for example.
- In Non-Matching Lists, the color of the ink does not match the color word: red written in blue ink, for example.

For the experiment, each student will see a different list. When it’s your turn, go down your list and say the color of ink in which each word is written. For example, if red is written in blue ink, you say blue. Three students will independently measure your time and calculate your average time.

Before you start this experiment, take a few minutes to think about what will happen and answer the following questions.

1. What do you think you will find when you perform this experiment?

   When this experiment is performed, the class will find that the amount of time it takes to read a matching list is less than the amount of time it takes to read a non-matching list with the same number of words.

2. What questions should you be looking to answer?

   Sample Responses:

   How are the times different between the matching and non-matching data? What does the difference in times mean?

3. What are the quantities that vary?

   The variable quantities in the experiment are the number of words in the list and the amount of time it takes to read the list.

4. Which quantity depends on the other? Why?

   The amount of time it takes to read the list depends on the length of the list.
Stroop Test

5. Use Matching Lists to collect data for the first part of the experiment. Record your data in this table.

Sample data is provided below. Do not use this data for your class' analysis. Students should perform the experiment and generate their unique class data.

<table>
<thead>
<tr>
<th>Length of List</th>
<th>Time #1</th>
<th>Time #2</th>
<th>Time #3</th>
<th>Average Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7.20</td>
<td>7.22</td>
<td>7.13</td>
<td>7.18</td>
</tr>
<tr>
<td>15</td>
<td>9.59</td>
<td>9.22</td>
<td>9.43</td>
<td>9.41</td>
</tr>
<tr>
<td>6</td>
<td>3.35</td>
<td>3.22</td>
<td>3.43</td>
<td>3.33</td>
</tr>
<tr>
<td>12</td>
<td>5.74</td>
<td>5.77</td>
<td>5.79</td>
<td>5.77</td>
</tr>
<tr>
<td>20</td>
<td>14.34</td>
<td>13.92</td>
<td>13.56</td>
<td>13.93</td>
</tr>
<tr>
<td>26</td>
<td>12.12</td>
<td>12.20</td>
<td>12.01</td>
<td>12.11</td>
</tr>
<tr>
<td>13</td>
<td>5.63</td>
<td>5.75</td>
<td>5.60</td>
<td>5.66</td>
</tr>
<tr>
<td>8</td>
<td>3.37</td>
<td>3.06</td>
<td>3.33</td>
<td>3.25</td>
</tr>
<tr>
<td>11</td>
<td>4.00</td>
<td>4.40</td>
<td>4.20</td>
<td>4.20</td>
</tr>
<tr>
<td>5</td>
<td>3.20</td>
<td>3.28</td>
<td>3.36</td>
<td>3.28</td>
</tr>
</tbody>
</table>

6. Use Non-Matching Lists to collect data for the next part of the experiment. Record your data in this table.

Sample data only.

<table>
<thead>
<tr>
<th>Length of List</th>
<th>Time #1</th>
<th>Time #2</th>
<th>Time #3</th>
<th>Average Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>6.85</td>
<td>6.78</td>
<td>6.49</td>
<td>6.71</td>
</tr>
<tr>
<td>13</td>
<td>9.76</td>
<td>9.87</td>
<td>11.09</td>
<td>10.24</td>
</tr>
<tr>
<td>12</td>
<td>9.91</td>
<td>9.41</td>
<td>9.20</td>
<td>9.51</td>
</tr>
<tr>
<td>6</td>
<td>6.70</td>
<td>6.68</td>
<td>6.53</td>
<td>6.64</td>
</tr>
<tr>
<td>10</td>
<td>8.12</td>
<td>8.50</td>
<td>8.27</td>
<td>8.3</td>
</tr>
<tr>
<td>26</td>
<td>21.31</td>
<td>21.85</td>
<td>21.90</td>
<td>21.69</td>
</tr>
<tr>
<td>20</td>
<td>16.72</td>
<td>16.70</td>
<td>14.60</td>
<td>16.01</td>
</tr>
<tr>
<td>7</td>
<td>8.49</td>
<td>7.63</td>
<td>7.12</td>
<td>7.75</td>
</tr>
<tr>
<td>15</td>
<td>10.01</td>
<td>9.99</td>
<td>10.03</td>
<td>10.01</td>
</tr>
<tr>
<td>11</td>
<td>4.9</td>
<td>4.94</td>
<td>4.98</td>
<td>4.94</td>
</tr>
</tbody>
</table>

7. Why do we take an “average time”? Which “average” is this (mean, median, or mode)?

The average used here is the mean. It is good experimental practice to run several trials under the same conditions. To save time, this practice is simulated with 3 timers. The average accounts for all three times.
Once students plot their data points and create the graph of the line of best fit, they should also create the equation of the line of best fit as part of question 8.

8. For the data on Matching Lists, plot your data points and draw a line of best fit on your graph. Use that line to make predictions.

One possible hand-drawn line of best fit using this data is $y = 0.5x + 0$. 
The meaning of the y-intercept may be elusive to students. They may say it should always be 0 because it takes no time at all to read a list of 0 words. If this is the case, question students about what is happening in the experiment before any names of color words are said aloud. Ask what they think happens once the projector is turned on and a student name is called?

Students may misinterpret the slope units as words per second. The independent or control variable in this situation is the length of the list, not time. Thus, the slope must be the change in the time per unit change in the number of words.

Responses here are based on the sample data. Your class answers may vary quite a bit.

### Stroop Test

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. What is the y-intercept of your line?</td>
<td>The y-intercept of the sample line is 0.</td>
</tr>
<tr>
<td>a. What are the units for the y-intercept?</td>
<td>The units on the y-intercept are seconds.</td>
</tr>
<tr>
<td>10. What does the y-intercept mean in this experiment?</td>
<td>It takes this amount of time for 0 words to be read. This is the average time it takes for the person’s mind to process the color of the first word before ever stating the color of the word.</td>
</tr>
<tr>
<td>11. What is the slope of your line?</td>
<td>The slope of the sample line is 0.5.</td>
</tr>
<tr>
<td>a. What is its unit of measure?</td>
<td>The units for slope are seconds per word.</td>
</tr>
<tr>
<td>12. Use your line to estimate how long it would take to read a Matching List that has:</td>
<td>Based on the sample line, it will take about 13 seconds to read 25 words.</td>
</tr>
<tr>
<td>a. 25 words</td>
<td>Based on the sample line, it will take about 5 seconds to read 10 words.</td>
</tr>
<tr>
<td>b. 10 words</td>
<td>Based on the sample line, in 120 seconds, 220 words will be read.</td>
</tr>
<tr>
<td>13. Use your line to estimate how many Matching words someone could read in:</td>
<td>Based on the sample line, in 300 seconds, 600 words will be read.</td>
</tr>
<tr>
<td>a. 2 minutes</td>
<td>Based on the sample line, in 120 seconds, 220 words will be read.</td>
</tr>
<tr>
<td>b. 5 minutes</td>
<td>Based on the sample line, in 300 seconds, 600 words will be read.</td>
</tr>
</tbody>
</table>
14. For the data on Non-Matching Lists, plot your data points, draw a line of best fit on your graph, and use that line to make predictions.

One possible hand-drawn line based on this data is \( y = 0.9x + 0.6 \).
### Stroop Test

15. What is the y-intercept of your line?

   *The y-intercept of the sample line is (0, 0.6).*
   
a. What are the units for the y-intercept?

   *The units for the y-intercept are seconds.*

16. What does the y-intercept mean in this experiment?

   *This y-intercept is the amount of time needed to read 0 words. It is the time needed to process information about the color of the first word being read.*

17. What is the slope of your line?

   *The slope of the sample line is 0.9.*
   
a. What is its unit of measure?

   *The units for slope are seconds per word.*

18. Use your line to estimate how long it would take to read a Non-Matching List that has:

   a. 25 words *Based on the sample line, it takes about 23 seconds for 25 words to be read.*

   b. 10 words *Based on the sample line, it takes about 9.6 seconds for 10 words to be read.*

19. Use your line to estimate how many Non-Matching words someone could read in:

   a. 2 minutes *Based on the sample line, someone could read 133 words in 120 seconds.*

   b. 5 minutes *Based on the sample line, someone could read 333 words in 300 seconds.*
Students will typically have less difficulty interpreting the slope than they have interpreting the y-intercept. The units of the slope make its meaning fairly straightforward.

As a possible extension, ask students what would happen if the matching and non-matching words were mixed in a list. Also, ask what would happen if all the words were written in a neutral color (e.g., black or gray).

In a large number of psychological experiments (over 700 as of 1991), the Stroop Test has been used. The cognitive scientists associated with Carnegie Learning® used the Stroop Test and found that:

1. Automaticity in reading is difficult to prevent.
2. Two processes taking place in the brain at once will interfere with one another.

### Stroop Test

20. What does the slope mean in each of these experiments? Explain.

The slope is the average amount of time required to say each new color of word. Once a color is said, the color of the next word on the list must be processed and stated aloud. The amount of time taken for each new word is the slope.

21. Compare the answers you found for each part of the experiment.

   a. Do they seem reasonable? Explain.

   *Sample Response: The answers seem reasonable because it should take slightly longer to say the color of a word that does not match the actual written word.*


   *Sample Response: This was not what I expected. I thought the differences in the times between matching and non-matching lists would be greater than they are.*

22. What conclusions might a cognitive psychologist draw from the results of this experiment? Be prepared to share your answer with the class.

   *Sample Responses:*
   - One conclusion might be that the color of the word and what the word reads are pieces of information that can conflict in a person’s mind.
   - If a person can say the colors in a non-matching list just as quickly as in a matching list, perhaps that person has a difficulty in reading (or can easily block certain information out of their mind).
   - If a person always makes errors on the non-matching list, perhaps he or she is color-blind.
As previously stated, directions for using the graphing calculator with the Stroop Test are posted on the K-12 Community website.

### Stroop Test

23. To validate your results, use a graphing calculator to generate the line of best fit for the Matching List and Non-Matching list data. Begin by:
   - Entering each data set
   - Generating a scatterplot for each data set
   - Performing a linear regression analysis for each data set.

24. Based on the output, answer the following questions:
   a. What is the linear regression equation for the matching data?
      \[ y = 0.54x + 0.06 \]
   b. What is the linear regression equation for the non-matching data?
      \[ y = 0.76x + 0.42 \]

Although student-created lines of best fit will most likely not match those generated by the graphing calculator, they should be fairly close. A significant difference in the line of best fit would occur if a student chose two data points that were not representative of the data.

25. How do these lines of best fit you found using the graphing calculator compare to the ones you found by hand? If there are significant differences, what do you think might have caused them? Should there be big differences? Explain.

   *Answers will vary.*

   **Sample Response:** Our equations were close to but not the same as the calculator-generated equations. The differences are probably due to human error. There should not be big differences between the hand-drawn lines of best fit and the calculator-generated lines because the class is viewing and using the same data as the calculator is.
Jumping

A controversy is going on in your class. Namely, some people think that the taller you are, the higher you can jump, while others think that the shorter you are, the higher you can jump.

To end this debate, your class decides to run an experiment to collect data. To run the experiment, you will have to figure out how to measure the height of each jump, and you will also need to measure the height of each student.

Data gathered here may lack an element of precision because it is difficult to accurately measure the jumping height. Two ways to attempt to measure jumping height are:

1. With feet flat on the floor and arms not bent and straight up, have students place a mark or piece of tape at the top of their extended fingertips. Have students make another mark at the top of their jump.

2. Have a second student sit on the floor to mark the place where the jumping student’s feet reach at the top of the jump.

You may choose to allow one to three trials per student, taking the highest jump or the average of their jumps.

1. Generate a table indicating name, gender, height in inches, and height jumped in inches for the students in your class.

2. Summarize your experimental design. How did you measure the heights the students jumped?

*Answers will vary, but sample data is presented below.*

<table>
<thead>
<tr>
<th>Name</th>
<th>Height (ins.)</th>
<th>Height Jumped (ins.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student A</td>
<td>67</td>
<td>13</td>
</tr>
<tr>
<td>Student B</td>
<td>65</td>
<td>9.5</td>
</tr>
<tr>
<td>Student C</td>
<td>71</td>
<td>7.5</td>
</tr>
<tr>
<td>Student D</td>
<td>67</td>
<td>10.5</td>
</tr>
<tr>
<td>Student E</td>
<td>62</td>
<td>12</td>
</tr>
<tr>
<td>Student F</td>
<td>68</td>
<td>9</td>
</tr>
</tbody>
</table>
Once students complete this activity, if time permits, you may want to introduce double histograms and split back-to-back stem and leaf plots, since they also work well to visually display the data comparing males and females.

3. What visual display would you choose in order to study the comparison between the heights jumped by males and females? Explain the reason for your choice. Generate the visual display.

   **Sample Response:**

   *I would choose a stem and leaf plot because then I can put the male jumps to the left and the female jumps to the right of a common stem. I can compare the two groups with the size of the leaves like the bars in a histogram, but I can still view the individual heights jumped.*

4. What statistics would you generate in order to compare the heights jumped by males and females? Which statistics provide the most information and why?

   **Sample Response:**

   *To compare the heights jumped, I would generate the mean and median as well as the standard deviation. These statistics will give me an idea of the average height of the jump for girls and boys as well as the spread of the data.*

5. Using your graphing calculator, make a scatterplot with the height jumped as a function of the height of the individual.

   **Students’ scatterplots will vary based on individual data collection.**

   **Sample Scatterplot:**

   ![Sample Scatterplot](image)

   \[ y = 0.8x - 42 \]

6. Model the data using an appropriate regression equation (line of best fit). What is that equation?

   **Answers will vary. See sample equation in above scatterplot.**
You may want to engage students in a conversation regarding cause and effect relationships to illustrate the difference from correlated data.

Correlation is a statistical technique that informs you about the relationship between two variables. If a relationship exists, then we say the variables are correlated. It is often important to note how strong the correlation is, which can be determined with the regression equation by examining the correlation coefficient “r”. When “r” is equal to 1.00, the correlation is a perfect positive correlation. A positive correlation means both variables increase together. If the correlation coefficient is equal to –1.00, then we call the relationship a perfect negative correlation. A negative correlation occurs when one variable increases as the other decreases.

Weaker correlation coefficients (say 0.6 or -0.4) mean that the relationship is not very strong. Therefore, it is more difficult to interpret the model, or less accurate to use it for prediction purposes.

It will be advantageous to use a graphing calculator to find the actual correlation coefficients for the data collected. However, if graphing technology is not available, students can still use their scatterplots to qualitatively evaluate whether the correlation is positive or negative, and strong or weak, based on the proximity of the points to the line.

Jumping

7. Is there a correlation between height and height jumped? If so, is it positive or negative? Is it strong or weak?

   **Sample Response:**
   
   There is a correlation between the height and height jumped. It is a positive correlation, but it is weak.

8. What does the correlation mean for your experiment? Do taller people jump higher, as some in your class suspected? Justify your response.

   **Sample Response:**
   
   This correlation means that, in general, taller people do seem to jump higher, but there are enough exceptions to make it impossible to make a generalization.

9. Indicate possible causes for error in this experiment.

   **Answers will vary but may include the following ideas:**
   
   - errors in the height measurements
   - errors in the jump height measurements
   - variation in shoe height
   - variation in each individual’s level of physical fitness
   - variation in the clothing worn by each individual

   These factors may cause errors because they affect the data numerically or because they simply render it misleading. For example, if a student were wearing sandals, he or she may not be able to jump as high as if the same student were wearing sneakers.
Although the data provided is relatively old, the issue addressed is still relevant and remains a concern of many people in today’s society.

Let’s investigate the health effects of radioactive wastes. The data in the table below is from an article in the Journal of Environmental Health from May-June 1965.

The author explains that the atomic energy plant in Hanford, Washington has produced plutonium since World War II. Some of the wastes have been stored in pits in the area. Radioactive waste has been seeping into the Columbia River since that time, and eight Oregon counties and the city of Portland have been exposed to radioactive contamination.

The table lists the number of cancer deaths per 100,000 residents for Portland and the eight counties. The exposure index measures the proximity of the residents to the contamination. This index is formulated on the assumption that exposure is based on residents’ distance from the plant and their frontage on the river.

### Data for Hanford Nuclear Power Plant

<table>
<thead>
<tr>
<th>County/City</th>
<th>Exposure Index</th>
<th>Rate of Cancer Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umatilla</td>
<td>2.5</td>
<td>147</td>
</tr>
<tr>
<td>Morrow</td>
<td>2.6</td>
<td>130</td>
</tr>
<tr>
<td>Gillian</td>
<td>3.4</td>
<td>130</td>
</tr>
<tr>
<td>Sherman</td>
<td>1.3</td>
<td>114</td>
</tr>
<tr>
<td>Wasco</td>
<td>1.6</td>
<td>138</td>
</tr>
<tr>
<td>Hood River</td>
<td>3.8</td>
<td>162</td>
</tr>
<tr>
<td>Portland</td>
<td>11.6</td>
<td>208</td>
</tr>
<tr>
<td>Columbia</td>
<td>6.4</td>
<td>178</td>
</tr>
<tr>
<td>Clatsop</td>
<td>8.3</td>
<td>210</td>
</tr>
</tbody>
</table>
1. Construct a scatter plot of the data in the table.

Next, use the following formulas to find the slope and y-intercept of the linear regression line for the data you just graphed.

**Equation of the Linear Regression Line**

\[ y = bx + a \]

**Linear Regression Formulas for a and b**

Slope: \[ b = \frac{\sum xy - \left(\frac{\sum x \sum y}{n}\right)}{\sum x^2 - \left(\frac{\sum x^2}{n}\right)} \]

y-intercept: \[ a = \bar{y} - b \bar{x} \]
Remind students of the meaning of $\sum$ and of the difference between $\sum x^2$ (the sum of the squares of each of the $x$ terms) and $(\sum x)^2$ (the square of the sum of all the $x$ terms).

### Hanford Nuclear Power Plant

2. To calculate $a$ and $b$ from the formulas, fill in this expanded table and the blanks in the equations below the table.

#### Data for Hanford Nuclear Power Plant

<table>
<thead>
<tr>
<th>County/City</th>
<th>Exposure Index</th>
<th>Rate of Cancer Deaths</th>
</tr>
</thead>
<tbody>
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<tr>
<td>Morrow</td>
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<td>130</td>
</tr>
<tr>
<td>Gillian</td>
<td>3.4</td>
<td>130</td>
</tr>
<tr>
<td>Sherman</td>
<td>1.3</td>
<td>114</td>
</tr>
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</tr>
<tr>
<td>Hood River</td>
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<td>162</td>
</tr>
<tr>
<td>Portland</td>
<td>11.6</td>
<td>208</td>
</tr>
<tr>
<td>Columbia</td>
<td>6.4</td>
<td>178</td>
</tr>
<tr>
<td>Clatsop</td>
<td>8.3</td>
<td>210</td>
</tr>
</tbody>
</table>

- $\Sigma x = \frac{\sum x}{n} = \frac{41.5}{9} = 4.61$
- $(\sum x)^2 = 1722.25$
- $\Sigma y = \frac{\sum y}{n} = \frac{1417}{9} = 157.44$

\[
b = \frac{\Sigma xy - (\Sigma x)(\Sigma y)}{\Sigma x^2 - (\Sigma x)^2}
\]

\[
a = \bar{y} - b \bar{x}
\]

**Calculations:**

\[
b = \frac{(7427.1) \cdot (41.5)(1417)}{(287.67) \cdot 96.25} = \frac{893.16}{96.25} = 9.28
\]

\[
a = (157.44 - b(4.61)) = 114.7
\]
Hanford Nuclear Power Plant

3. The equation of the regression line is $y = bx + a$.
   a. Fill in the values you calculated for $a$ and $b$.
      $$y = 9.28 x + 114.7$$
   b. Draw this line on your graph.
      *See the graph on page 68 for the line of best fit.*

4. What does the slope represent in this problem situation?
   *The slope represents the increase in the number of cancer deaths with every increase in one unit in the exposure index.*

5. What does the $y$-intercept represent in this problem situation?
   *The $y$-intercept in this situation represents the number of cancer deaths if the exposure index is 0.*

6. Use your model (equation) to predict the cancer rate per 100,000 residents of a community whose exposure index is:
   a. 7.2
      *The cancer death rate is 181 people per 100,000 if the exposure index is 7.2.*
   b. 6.5
      *The cancer death rate is 175 people per 100,000 if the exposure index is 6.5.*
   c. 9.7
      *If the exposure index is 9.7, then the cancer death rate is 205 people per 100,000.*
7. What does your model predict for the exposure index of a community that has a cancer rate of:
   a. 175 per 100,000 residents?
      *The model predicts an exposure index of about 6.5 if the cancer death rate is 175 per 100,000 people.*
   b. 150 per 100,000 residents?
      *The model predicts an exposure index of 3.8 for a cancer death rate of 150 per 100,000 residents.*

8. Use your model to find the predicted cancer rate for:
   a. Portland
      *Using the model, the predicted cancer rate for Portland, which has an exposure index of 11.6, is about 222 people who die from cancer per 100,000 residents.*
   b. Wasco
      *Using the model, Wasco, which has an exposure index of 1.6, has a cancer death rate of about 129 people per 100,000.*

9. How do your predicted rates for Portland and Wasco compare with the actual rates from the table?
   *The predicted rate for Portland was higher than the actual rate by 14 people per 100,000. The predicted rate for Wasco was lower than the actual rate by 9 people per 100,000.*
You may have performed the Human Chain experiments during your four days of training with Carnegie Learning. Recall that eyes must be closed so students do not see the squeeze coming and anticipate it before it actually reaches them. If you know of some students in your class who are self-conscious about touching others or being touched, have them serve as timers or recorders.

The Human Chain: Wrist Experiment is performed by making a human chain. Each student holds the wrist of the person to his or her right. Your teacher will pick a student to begin the chain and another to end it. The members of the chain must keep their eyes closed. When the teacher says go, the first student in the chain gently squeezes the wrist of the student to his or her right, who then squeezes the wrist of the next person, and so on through the chain. The last student should say stop when he or she feels the squeeze on his/her wrist.

As indicated you will pick someone to begin and end the chain. End points should be different. To signal the end tell the student who is the last person in the chain to let go of the wrist of the next person.

To get larger numbers, continue the squeeze around the chain more than once.

If there are individual times that seem too large or small, discuss what should be done with those times. If there are trials that seem to have average times that are too large, discuss what should be done with this data.

<table>
<thead>
<tr>
<th>Number of People</th>
<th>Time #1</th>
<th>Time #2</th>
<th>Time #3</th>
<th>Average Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7.22</td>
<td>7.50</td>
<td>7.85</td>
<td>7.52</td>
</tr>
<tr>
<td>10</td>
<td>2.96</td>
<td>3.63</td>
<td>3.60</td>
<td>3.40</td>
</tr>
<tr>
<td>15</td>
<td>4.50</td>
<td>4.29</td>
<td>4.44</td>
<td>4.41</td>
</tr>
<tr>
<td>17</td>
<td>5.56</td>
<td>5.17</td>
<td>4.19</td>
<td>4.97</td>
</tr>
<tr>
<td>5</td>
<td>1.53</td>
<td>1.64</td>
<td>1.48</td>
<td>1.55</td>
</tr>
<tr>
<td>40</td>
<td>10.62</td>
<td>10.69</td>
<td>11.00</td>
<td>10.77</td>
</tr>
<tr>
<td>30</td>
<td>8.50</td>
<td>8.39</td>
<td>9.68</td>
<td>8.86</td>
</tr>
<tr>
<td>60</td>
<td>16.12</td>
<td>16.43</td>
<td>15.60</td>
<td>16.05</td>
</tr>
<tr>
<td>22</td>
<td>7.69</td>
<td>(1.01)</td>
<td>7.74</td>
<td>7.72</td>
</tr>
<tr>
<td>44</td>
<td>(20.90)</td>
<td>12.84</td>
<td>12.20</td>
<td>12.53</td>
</tr>
</tbody>
</table>

(   ) indicates this time was thrown out.

This is sample data only! Run the experiment in your class to create your own class data. Do not use the data above.
2. Construct a graph of your data, using the number of people in the chain for \( x \) and the average time for \( y \). Sketch what you think is the line of best fit.

\[ y = 0.26x + 1.03 \]
Human Chain: Wrist Experiment

3. Use your graphing calculator to find the line of best fit.
   a. Write the equation for the line.
      This equation is based on the sample data.
      \[ y = 0.26x + 1.03 \]

   b. Draw this line on the graph.
      See the graph on page 61 for the line of best fit.

   c. Did your hand-drawn line match with the calculator-generated line? Explain any difference you see between the two lines.
      Answers will vary.
      Sample Response: The two lines are pretty close to matching. The difference was simply due to a lack of precision.

4. What is the value of the \( y \)-intercept on your graph?
   Based on the sample data, the \( y \)-intercept is 1.03.

   a. What is the unit of measure for the \( y \)-intercept?
      The units for the \( y \)-intercept are seconds.

5. What is the value of the slope of your line?
   Based upon the sample data, the slope is 0.26.

   a. What is the unit of measure for the slope?
      The units for the slope are seconds per person.
If the y-intercept is negative, ask students why this does not seem to make sense.

### Human Chain: Wrist Experiment

<table>
<thead>
<tr>
<th>Question</th>
<th>Analysis</th>
</tr>
</thead>
</table>
| 6. | What does the y-intercept mean in this experiment?  
   The y-intercept is the "start-up" time. It is the time from when the teacher says, "Go" to the time the first squeeze is felt. |
| 7. | What does the slope mean in this experiment?  
   The slope is the time it takes for the impulse to travel from one person’s wrist to the next person’s wrist. |
| 8. | Does this line provide a good fit to this data? Justify your answer, making sure to use the value of “r” (the correlation coefficient) as part of your reason.  
   Based upon the sample data, this is a good fit for the data. The value of r is 0.987, so about 98.7% of the data can be accounted for with this equation or model. |

Human Chain: Wrist Experiment

<table>
<thead>
<tr>
<th>Process: Evaluate</th>
<th>9. Use the linear regression equation to predict the time required to pass the squeeze through a chain of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Answers provided are based upon the linear regression equation found for the sample data. Your results will vary based upon your class data and corresponding regression equation.</td>
</tr>
</tbody>
</table>
|                  | a. 100 people  
   It will take about 27 seconds for the squeeze to pass through 100 people. |
|                  | b. 50 people  
   It will take about 14 seconds for the squeeze to travel through a chain of 50 people. |
|                  | c. 10,000 people  
   The squeeze takes about 2601 seconds or about 43 minutes to travel through a chain of 10,000 people. |
|                  | Note that answers for numbers 9 – 11 can vary depending upon where rounding is performed for the regression equation. These answers were calculated based upon the equation cited in question #3, which is rounded to the hundredths place. |
10. Use your linear regression equation to predict how long it would take to pass the squeeze through a human chain of the world’s entire population of 4 billion people.

*Based upon the sample data, this would take about 1,040,000,000 seconds, which is nearly 33 years.*

*(If the slope and y-intercept are used to 10 decimal places, the answer changes to nearly 42 years or 1,320,000,000 seconds.)*

11. How many people could the squeeze pass through in:

*Answers provided are based upon the linear regression equation found for the sample data. Your results will vary based upon your class data and corresponding regression equation.*

   a. one hour?

   *In 3600 seconds, the squeeze can pass through about 13,842 people.*

   b. one day?

   *In 86,400 seconds, the squeeze travels through about 332,300 people.*

   c. one year?

   *In 31,536,000 seconds, the squeeze can pass through about 121 million people.*

12. Could you actually perform the experiment in your classroom to check your predictions in problems 9 through 11? Why or why not?

*Answers will vary.*

*Sample Response: We could not check the problems in numbers 9 – 11 because it would take too long for most of those data points and because we could not fit that many people in our classroom.*

13. Why do you think it was a requirement to have three people timing the experiment?

*Sample Response: Three people timed the chain to protect against errors being made. Using the same timers over multiple trials would ensure consistency in the results.*
14. Why are there differences in the times recorded by each timer for an individual trial?

   *Answers will vary.*

   *Sample Response:* There are differences in the times because the three timers’ reaction times could have varied.

15. Why don’t the data points form a straight line?

   *Answers will vary.*

   *Sample Response:* These data points do not form a straight line because the speed of the squeeze was not the same in every trial and because not all of the same people were involved in every trial. Moreover, any person’s individual reaction time can vary from trial to trial.

16. Suppose you did the same experiment again. Would you get the same results? Would you get the same regression line? Why or why not?

   *Answers will vary.*

   *Sample Response:* The class would most likely not get the same results if this were done again, because the reaction times will not be identical to the last time the experiment was run.

17. Suppose you did the same experiment, but this time reversed the start and end person. Do you think you would get the same results? If so, justify. If not, tell how and why the results are different.

   *Answers will vary.*

   *Sample Response:* The results may be different because people would be using the opposite hand to do the squeezing. This may speed the process or slow the process.
18. Go back to the data table and add a column to record the median time. If you used the median value instead of the mean, how would your analysis change?

   *Answers will vary.*

   *Sample Response:* A few of the median values are lower than the mean values and a few are higher than the mean, so the overall analysis may not change much.

19. Assuming that the length of your arm span is equal to your height, what will the speed of the nerve impulse be as it travels through the chain? Indicate what assumptions you had to make in order to answer the question.

   *Answers will vary.*

   *Students can sum all the student heights in the class and find the average class height (or they can calculate the individual results based solely upon their own height). Divide the average height in feet (per person) by the slope in seconds per person to find the number of feet traveled in one second.*

   *The assumption that the nerve impulse speed is constant is made in this question.*

20. Using the speed you calculated above, how long will it take for the nerve impulse to travel around the world? (The approximate circumference of the earth is 25,000 miles.)

   *Answers will vary.*

   *Sample Response:* Using an average class height of 5' 6” and a slope of 0.26 seconds per person, the nerve impulse is traveling about 21.15 feet per second or 14.42 miles per hour. At this rate, it will take approximately 1733.7 hours or 72 days for the nerve impulse to travel around the world.
This experiment is almost identical to the last one, but the equation representing reaction time will have a different slope. The slope of this line will likely be between 0.08 and 0.15 seconds per person longer than in the Human Chain: Wrist Experiment. This difference can be attributed to the time it takes for a nerve impulse to travel up the average person's arm.

Human Chain: Shoulder Experiment

Here's an experiment that's similar to the Human Chain: Wrist Experiment. Again you will gather data, analyze the data, find the linear regression line, and use the regression line to make predictions. The Human Chain: Shoulder Experiment is performed by making a human chain. Each student holds the shoulder of the person to his or her right. Your teacher will pick a student to begin the chain and another to end it. The members of the chain must keep their eyes closed. When the teacher says go, the first student in the chain gently squeezes the shoulder of the student to his or her right, who then squeezes the shoulder of the next person, and so on through the chain. The last student should say stop when he/she feels the squeeze on his/her shoulder.

What do you think will be different about the results of this experiment? Explain why you think these differences will occur.

<table>
<thead>
<tr>
<th>Human Chain: Shoulder Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Three timers should record the elapsed time from the moment the teacher says go until the last student says stop. Calculate and record the average time of the three timers. Record the class data in the table.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of People</th>
<th>Time #1</th>
<th>Time #2</th>
<th>Time #3</th>
<th>Average Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>4.79</td>
<td>4.10</td>
<td>4.11</td>
<td>4.33</td>
</tr>
<tr>
<td>10</td>
<td>2.60</td>
<td>2.41</td>
<td>2.23</td>
<td>2.41</td>
</tr>
<tr>
<td>5</td>
<td>1.41</td>
<td>1.28</td>
<td>1.18</td>
<td>1.29</td>
</tr>
<tr>
<td>30</td>
<td>6.45</td>
<td>6.84</td>
<td>7.01</td>
<td>6.77</td>
</tr>
<tr>
<td>45</td>
<td>10.95</td>
<td>11.23</td>
<td>11.07</td>
<td>11.08</td>
</tr>
<tr>
<td>8</td>
<td>2.16</td>
<td>2.40</td>
<td>1.96</td>
<td>2.17</td>
</tr>
<tr>
<td>20</td>
<td>4.72</td>
<td>4.54</td>
<td>4.63</td>
<td>4.63</td>
</tr>
<tr>
<td>27</td>
<td>6.03</td>
<td>6.24</td>
<td>6.26</td>
<td>6.18</td>
</tr>
<tr>
<td>20</td>
<td>4.47</td>
<td>4.29</td>
<td>4.32</td>
<td>4.36</td>
</tr>
<tr>
<td>10</td>
<td>2.23</td>
<td>2.15</td>
<td>2.16</td>
<td>2.18</td>
</tr>
</tbody>
</table>
Human Chain: Shoulder Experiment

2. Construct a graph of your data, using the number of people in the chain for $x$ and the average time for $y$. Sketch what you think is the line of best fit.

*This is sample data only! Run the experiment in your class to create your own class data. Do not use the preceding data.*

\[ y = 0.24x + 0.06 \]
3. Use your graphing calculator to find the linear regression line.
   a. Write the equation for the linear regression line.
      This equation is based on the sample data.
      \[ y = 0.24x + 0.06 \]
   b. Draw this line on your graph.
      See the graph on page 68 for the line of best fit.
   c. Did your hand-drawn line match the calculator-generated line? Explain any differences you see between the two lines.
      Answers will vary.
      Sample Response: Originally, my hand-drawn line was incorrect because I plotted my data points using an incorrect scale. I fixed the points, and now my line looks closer to the linear regression line.

4. What is the value of the \( y \)-intercept on your graph?
   Based on the sample data, the value of the \( y \)-intercept is 0.06.
   a. What is the unit of measure for the \( y \)-intercept?
      The unit of measure for the \( y \)-intercept is seconds.

5. What is the value of the slope of your line?
   Based on the sample data, the value of the slope of the line is 0.24.
   a. What is the unit of measure for the slope?
      The unit of measure for the slope is seconds per person.
6. What does the y-intercept mean in this experiment?

   The y-intercept is the "start-up" time. It is the time from when the teacher says, "Go" to the time the first squeeze is felt.

7. What does the slope mean in this experiment?

   The slope is the time it takes for the impulse to travel from one person’s shoulder to the next person’s shoulder.

8. Does this line provide a good fit to this data? Justify your answer, making sure to use the value of "r" (the correlation coefficient) as part of your reason.

   Based on the sample data, this is a great fit. The r-value is over 0.99, so the data is almost perfectly correlated to the line.

9. Use the linear regression equation to predict how long it would take to pass the squeeze through a chain of:

   Answers provided are based upon the linear regression equation found for the sample data. Your results will vary based upon your class data and corresponding regression equation.

   a. 100 people

      It will take about 24.06 seconds for the shoulder squeeze to pass through 100 people.

   b. 50 people

      It will take about 12.06 seconds for the impulse to pass through a chain of 50 people.

   c. 10,000 people

      It will take about 2400 seconds for the squeeze to go through 10,000 people.

   Note that answers for numbers 9 – 11 can vary depending upon where rounding is performed for the regression equation.

10. Use your linear regression equation to predict how long it would take to pass the squeeze through a human chain of the world’s entire population of 4 billion people.

    Based upon the sample data, this would take about 960 million seconds, which is over 30 years.
11. How may people could the squeeze pass through in:
   a. one hour?
      *The squeeze can pass through about 15,000 people in an hour.*
   b. one day?
      *The squeeze can pass through about 360,000 people in a day.*
   c. one year?
      *The squeeze can pass through about 131,400,000 people in a year.*

To compare and contrast the two experiments, consider having students put the graph for the Wrist Experiment and the graph for the Shoulder Experiment on the same set of axes. Create an overhead transparency of each and place them on top of one another. Alternatively, you can have students use the calculator to do this.

12. How was this experiment different from the Human Chain: Wrist Experiment?
   *Answers will vary.*
   *Sample Response: This experiment was different from the wrist experiment because the nerve impulse did not have as far to travel between people.*

13. What did the slope in each experiment represent? What does the difference between these two slopes represent?
   *The slope represented the time between each successive squeeze in the chain.*
   *The difference between the two slopes is approximately the amount of time it takes for the nerve impulse to travel up a person’s arm from the wrist to the shoulder.*
In question 14, you can alternatively assume the average distance from wrist to shoulder in a teenager is 2 ft. Thus, you can simply subtract 2 ft. per student from the total distance traveled as it was calculated in the Wrist Experiment.

The speed of nerve impulses varies greatly from a low of 10 ft./sec. to a maximum of around 300 ft./sec.

### Human Chain: Shoulder Experiment

14. Assume the distance from your shoulder to the fingertip on your opposite hand is half your height. What was the speed of the nerve impulse as it traveled through the chain for this experiment?

   **Answers will vary.**

   **Sample Response (based on the sample data):** If the average class height is 5’ 6”, then the distance the nerve impulse travels per person is about 2.75 feet. With a slope of 0.236 seconds per person, this means the speed of the impulse is $\frac{2.75}{0.236} = 11.65$ feet per second.

15. Is the nerve impulse traveling faster, slower, or at the same rate as it was in the Human Chain: Wrist Experiment? What are some possible reasons for this?

   **Answers will vary.**

   **Sample Response:** This impulse is traveling a lot slower than the one in the Wrist Experiment. One possible reason may be that the nerve impulse starts traveling slowly and speeds up as it goes to the brain. Since this impulse is not traveling far to get to the brain, its average speed is slower.

16. How does your calculated speed of the nerve impulse compare to the actual speed of a nerve impulse (about 300 ft/sec)? What could account for any differences between the actual speed and the calculated speed?

   **Answers will vary.**

   **Sample Response:** The calculated impulse speeds are a lot slower than 300 feet per second. Perhaps the speed of 300 feet per second accounts for all the different kinds of nerve impulses, not just those from the sense of touch.
Unit 6: Quadratics

Unit Objectives and Skills

At the completion of this unit, students will be able to define and identify a quadratic equation, find the roots or x-intercepts by factoring or using the quadratic formula, and model and solve problems involving quadratic functions.

Unit Overview

The unit is introducing a new functional form and attempts to do so from a qualitative perspective by comparing and contrasting linear and quadratic functions. In particular, the unit presents the concept that rates of change are non-constant for quadratics versus constant for linear or, more specifically, that a quadratic function is not constantly increasing or decreasing.

The material focuses on developing students’ ability to understand the defining characteristics of a quadratic equation, i.e., rate of change, zeros vertex, and axis of symmetry.

Unit Activities

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

<table>
<thead>
<tr>
<th>Printed Classroom Activities</th>
<th>Homework Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animated gif Program</td>
<td>-</td>
</tr>
<tr>
<td>Setting Bounds</td>
<td>-</td>
</tr>
<tr>
<td>Exploring Quadratic Equations</td>
<td>pgs. 161, 163</td>
</tr>
<tr>
<td>Finding Zeros by Factoring</td>
<td>pgs. 165, 169</td>
</tr>
<tr>
<td>Finding Zeros Using the Quadratic Formula</td>
<td>pgs. 173, 175</td>
</tr>
<tr>
<td>Modeling: Vertical Motion</td>
<td>pg. 177</td>
</tr>
</tbody>
</table>

Suggestions for Classroom Implementation

The Animated gif Problem introduces quadratic functions in relation to linear functions. To represent the Animated gif Problem, students model linear and quadratic equations. Stress that, although introducing another type of function, we think about many of the same kinds of things; for example, the rate of change or how fast the function is growing.
Setting Bounds extends the introduction of quadratic functions by exploring a new situation in which rate of change, intercepts, vertex and axis of symmetry are defined. With key concepts and characteristics, students move on to Exploring Quadratic Equations, where they study a variety of quadratic equations in terms of transformations: dilation, vertical and horizontal shifts, and reflection. As an assigned activity, you can have the groups generate a transformed function and graph it. The groups then display their graphs to the class, with students having to determine the function. Have students articulate the differences between functions with one real root, two real roots and no real roots. Ask them to indicate what characteristics of the function make this occur.

The next problems, Finding Zeros by Factoring and Finding Zeros Using the Quadratic Formula, speak for themselves. Factoring is taught in this context, as students need to understand that it has a specific purpose and is not an arbitrary task. When students study rational expressions, factoring can be addressed again, and here too with a purpose in mind. While we want students to know why they use certain techniques and to make sense of the procedures, introduce the Quadratic Formula without deriving the formula by completing the square. At this time, it is acceptable to have students just apply the formula, as long as they understand that the zeros are solutions to the equation when y = 0 and that the zeros are represented on the graphs as x-intercepts.

The last section on Modeling: Vertical Motion presents a series of situations where certain quantities, like the initial height or initial velocity, are changed from one situation to another. The basic premise of the situations remains the same, so each problem can be assigned to a group. Presentations can focus on the differences in the models both symbolically and graphically. Students can then summarize the similarities and differences they see within the problem, explaining what parts of the equation account for these similarities and differences.

Unit Assessment

Squares: Part 1
Squares: Part 2
Quadratic Formula
Vertical Motion
Unit 6: Quadratics

Contents

- Animated gif Program.................................................................6-3
- Setting Bounds.................................................................................6-9
- Exploring Quadratic Equations......................................................6-17
- Finding Zeros by Factoring............................................................6-32
- Finding Zeros Using the Quadratic Formula.................................6-41
- Modeling: Vertical Motion.............................................................6-47
Students who have spent any time surfing the internet should be comfortable with the concept of a computerized image. If your classroom has a computer with internet access, display a gif image such as an online greeting card.

<table>
<thead>
<tr>
<th>Animated gif Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>As a web page designer, you get many clients who want to create a homepage that says something personal about themselves. A recent client, a dog lover, requested that her homepage open with a picture of her dog, but with the special condition that the image of the dog would grow over time.</td>
</tr>
<tr>
<td>You told your client that this could be accomplished easily through an animated gif. A gif is a computerized image. You explained that you would need a photograph of the dog to scan into the computer, and that you would need to generate a program to animate the image according to her specifications.</td>
</tr>
<tr>
<td>The design you presented to your client has the homepage opening with an image that begins to form into a square. The image of the dog, which will appear in the square box, is not visible when the page opens. For each second that passes, the side length of the box will increase by 1 inch, for a total of 5 seconds, during which the image of the dog will appear bigger in each box. (See scaled illustration below.)</td>
</tr>
</tbody>
</table>

![Scaled image drawings: 5 mm = 1 inch](image.png)
To develop the concept of the squaring function, the relationship between image length—a linear measure—and area—a squared measure—is examined first.

The questions on this page set the stage for a continuation of what began in Unit 3. Recall that when the concept of linear functions was introduced, the fact that functions have a certain set of common characteristics was stressed. By discussing growth patterns and rate of change, linear functions are contrasted with quadratic functions.

### Animated gif Program

When working on a web design project, you like to record all information relating to the initial plan, programming, and graphic displays. As part of that process, you track the data about the animation process.

<table>
<thead>
<tr>
<th>Time</th>
<th>Image Length</th>
<th>Image Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seconds</td>
<td>Inches</td>
<td>Square Inches</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>$x$</td>
<td>$x$</td>
<td>$x^2$</td>
</tr>
</tbody>
</table>

1. Describe the growth patterns in the image length and the image area.

   - The growth pattern of the image length shows that for each second, the length increases one inch, so the growth is steady and not too rapid.
   - The growth pattern of the image area shows that for each image length, the image area is the square of that number, so it grows rapidly.

2. Describe the rate of change in the growth patterns in the image length and the image area.

   - The rate of change in the growth pattern for the image length is constant. For each second that the time increases by one unit, the image length increases one unit.
   - The rate of change in the growth pattern for the image area is not constant. For each second that passes, the image area increases by the next odd number of units. For example, between seconds 1 and 2, the area increases by 3 units. Between seconds 2 and 3, the image area increases by 5. The next increase is by 7 and so on.
After students graph the equations, have them discuss the rate of change and compare this to their answers in question 2.

Ask students to describe each of the curves and try to draw out observations like "the curve has a low point." Ask students what the curves might look like if the value of $x$ was negative.

### Animated gif Program

3. To make sure the growth of the image is smooth and continuous, you want to graph the relationship between the time and the length of the sides of the box.

4. Similarly, you want to make sure that the change in the area of the image over time is smooth and continuous. Graph the relationship between time and area of the gif.
5. Make a conjecture about what type of function best models each of the curves pictured in the graphs.

*The function that best models the image length is a linear function or straight line. The function that best models the image area is a quadratic function. (Note that students may or may not know this terminology. At this point they may say parabola or "U-shaped" curve.)*

6. Write the function that describes the relationship between time and image length.

\[ y = x \]

*Image Length = Time*

7. Write the function that describes the relationship between time and image area. (If you have difficulty describing this relationship, think about how you determined the actual value of the area given a specific value for the time.)

\[ y = x^2 \]

*Area = Image Length • Image Length*

8. How does the curve describe the actual relationship between the linear side length of the image and the area of the image?

*The curve shows that the area grows at a more rapid and non-constant rate.*

If you have not already done so, complete the table by entering the function/expression.

The relationship you described in questions 7 and 8 is a **quadratic function**. The graph of a quadratic function is called a **parabola**.
In Unit 3, you discussed the solution to the problem in context versus a strictly algebraic solution. This problem provides the opportunity to discuss domain and range in context.

### Animated gif Program

<table>
<thead>
<tr>
<th>process: evaluate</th>
<th>animated gif program</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. You are experimenting with the image. You let it grow until it fills as much of the screen (15 in. x 13 in.) as possible. What is the maximum side length allowable? When will that length be achieved? What will the area be when the side is at its maximum length? Use complete sentences to answer these questions.</td>
<td></td>
</tr>
<tr>
<td><strong>Answers will vary.</strong></td>
<td></td>
</tr>
<tr>
<td><em>If we use the convention of length by width, then the maximum side length is 15. It will reach its maximum length in 15 seconds and the image area will be 195 square inches.</em></td>
<td></td>
</tr>
<tr>
<td><em>If students use 13 inches as the maximum side length, then the length of time required is 13 seconds and the image area is 169 square inches.</em></td>
<td></td>
</tr>
</tbody>
</table>

---

In question 11, students move from context specific to general understanding.

### Animated gif Program

<table>
<thead>
<tr>
<th>analysis</th>
<th>animated gif program</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. Given the maximum length, does the magnitude of the area surprise you? If so, explain why. If not, explain why not.</td>
<td></td>
</tr>
<tr>
<td><strong>Answers will vary.</strong></td>
<td></td>
</tr>
<tr>
<td><em>Sample Response:</em></td>
<td></td>
</tr>
<tr>
<td>Yes, the magnitude of the area surprises me. I did not expect it to get so big so quickly.</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td></td>
</tr>
<tr>
<td>No, the magnitude does not surprise me because I knew the squared numbers would get large pretty quickly.</td>
<td></td>
</tr>
</tbody>
</table>

11. How can you describe the relationship between this side length and the area? Do you think the relationship will hold true for any situation involving length and area? Be specific in your explanation.

**Sample Response:**

Area is dependent on side length. Find the area by multiplying the length by the width. The relationship we found will always be true for squares, since the area is always found by multiplying the length by the width.
You will want students to revisit this page as they work through this unit to compare their initial understanding and perceptions to any new understanding. Throughout Algebra I and Algebra II, new functions will be introduced in a similar manner. The pattern of introducing and formalizing functions through comparing characteristics is an important area of focus.

### Process: Summary

By definition, functions are special types of relationships, which means that specific characteristics define or describe them. Summarize your findings thus far regarding linear and quadratic functions.

1. Summarize what you know about linear functions.
   - Students may not use formal terminology.
   - Responses may include:
     - Linear functions have a constant rate of change.
     - Linear functions can be represented symbolically by $y = mx + b$.
     - Straight lines can represent linear functions.
     - Linear functions can have positive or negative slopes.
     - Linear functions are continuous.

2. Summarize what you know about quadratic functions.
   - Student may not use formal terminology.
   - Responses may include:
     - Quadratic functions are represented graphically by a curve (parabola).
     - Quadratic functions do not have a constant rate of change.
     - Quadratic functions grow rapidly.
     - Quadratic functions are continuous.

3. What are the similarities and differences between these two types of functions?
   - Both functions are smooth with no breaks in the curves.
   - Linear functions are represented by straight lines, while quadratics are represented by parabolas. Quadratic functions grow more rapidly than linear functions.
This activity will help students develop concepts of dependency, y-intercepts and zeros.

Setting Bounds

You and your client were talking after you showed her the web page you had designed. She was telling you about her dog, Fido. She said she wanted to build a dog run (fenced-in area) so the dog could stay outside during the day.

She said the idea of building the dog run came to her because she found 16 yards of fencing at her parents’ house that they said she could have. She was telling you about some of the problems she was facing in coming up with proper dimensions for the run, since she had to work around trees and bushes. She also indicated that she wanted to use the wall of the garage as one of the sides of the dog run.

If possible, have students create sketches for the different possible measurements of the sides on graph paper. This will help them to visualize the dependency relationship between the length, width, and area.

You do a rough sketch of her yard and begin drawing some configurations for the dog run. For simplicity, you consider only whole-yard measures. Record your measurements to keep track of the various possibilities.

1. One thing you must keep in mind is that the total amount of fencing is fixed at 16 yards, so the sum of the lengths of the three sides must equal 16 yards. **This table is a sample response. Students are likely to choose different values than seen here.**

<table>
<thead>
<tr>
<th>Units</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of Dog Run</td>
<td>yds</td>
</tr>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
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<tr>
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<td>14</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Length of Dog Run</th>
<th>yds</th>
</tr>
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<tbody>
<tr>
<td>8</td>
<td>7</td>
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<tr>
<td>7</td>
<td>6</td>
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<td>6</td>
<td>5</td>
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<td>2</td>
<td>1</td>
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<tr>
<td>1</td>
<td>0</td>
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<table>
<thead>
<tr>
<th>Area of Dog Run</th>
<th>sq. yds.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
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<tr>
<td>14</td>
<td>24</td>
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<td>24</td>
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<td>30</td>
<td>24</td>
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<tr>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
</tr>
</tbody>
</table>
### Setting Bounds

2. The size and shape of the dog run will have an impact on the dog's comfort, since in some cases trees and bushes will be enclosed and in others they will not. You want to clearly understand how changing the width affects the other measurements. What do you think happens to the length when you increase the width? [Hint: If your measurements for the width are not in order from smallest to largest, you may want to reorganize your data]. Describe why you think the change occurs as it does? Be specific.

   *Since there is a specified amount of fencing material (a fixed perimeter), if you change one measurement, you must change the other. As the width increases from 0 to 16, the length decreases from 8 to 0.*

---

3. What happens to the area when you increase the width? Describe why you think the change occurs as it does.

   *As the width increases, the area will increase until it reaches a maximum value and then it will decrease.*

   *As the run becomes more square, there is a greater area.*

---

4. What happens to the length and the area if you decrease the width?

   *As you decrease the width, the length will increase. As the width decreases, the area will increase until it reaches a maximum value and then it will decrease.*
<table>
<thead>
<tr>
<th>analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setting Bounds</td>
</tr>
</tbody>
</table>

5. What happens to the width and the area if you increase the length? What happens if you decrease the length?

*If you increase the length, you must decrease the width to keep the total amount of fencing equal to 16 yards. Area will increase until it reaches a maximum and then it will decrease.*

*If you decrease the length, the width will increase and the area will increase until it reaches its maximum. Then it will decrease.*

6. Are the changes to the area the same when you change the width as when you change the length? Explain your reasoning.

*Yes, the changes are basically the same, since the area will always increase until it reaches a maximum and then decrease or vice versa. However, an increase of one unit in width has the same effect on the area as a decrease by two units in length.*
7. You think you may want to do a more detailed set of plans and look at an additional set of measures. You figure it would be easier if you could model the data and then use the equation.

Using your graphing calculator to model the data, you want to construct a scatter plot for both the length as a function of width and the area as a function of width. Then draw the best-fitting curve and find the equation of that curve.

**Length vs. Width Relationship**

![Graph of Length vs. Width](image)

**Area vs. Width Relationship**

![Graph of Area vs. Width](image)
These activities will lead students beyond the parent function.

After students complete questions 8 through 11, return to page 6-8 and ask them how they might add to their summaries.

### Setting Bounds

<table>
<thead>
<tr>
<th><strong>analysis</strong></th>
<th><strong>Setting Bounds</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The graphs of the functions help to define the functions. Graphs can also provide specific details about functions. Consider the graphs of length vs. width and area vs. width and answer the following questions.</td>
</tr>
<tr>
<td>8.</td>
<td>Based on the graph of the curves, name the type of function associated with each.</td>
</tr>
<tr>
<td>9.</td>
<td>In general, for any value associated to the length, how many values may be assigned to the width?</td>
</tr>
<tr>
<td>10.</td>
<td>Using a graphing calculator, determine the x-intercepts (points where the curve crosses the x-axis). How do you interpret those values with respect to the situation?</td>
</tr>
<tr>
<td>11.</td>
<td>What is the y-intercept? How do you interpret this value with respect to the situation?</td>
</tr>
</tbody>
</table>

**Linear for length versus width.**

**Quadratic for area versus width.**

**For any value of length, only one value may be associated to the width.**

In looking at the linear function, the point where the graph crosses the x-axis occurs when the width is 16 and the length is 0. Therefore, no dog run can be formed.

For the quadratic equation, the graph will cross the x-axis when the width is 0 and when the width is 16. At both of these points, no dog run can be formed.

The y-intercept is the point where the graph crosses the y-axis. In a situation, it means the initial value. The y-intercept for the linear function is the point (0,8). This means the width is equal to zero and the length of one side is 8, so that the sum of the two lengths will equal a perimeter of 16 yds. However, without a width, no dog run can be formed.

For the quadratic function the y-intercept is the point (0,0). This means that the width is zero and so is the area. There is no space inside the fence.
12. Describe the rate of change for each of the curves in the problem.

*The rate of change for the linear function, as in every linear case, is constant, whereas for the quadratic function, the rate of change is not constant – the rate of change varies between any two points along the curve.*

13. What is the smallest side length possible? What is the largest side length possible? What is the smallest area possible? What is the largest area possible?

*The problem considers only whole-yard measurements, so the smallest side length possible is 1 yard, and the largest side length possible is 7 yards. The smallest possible area is 14 square yards, and the largest is 32 square yards.*

14. You have decided that with 16 yards of fencing, you will never be able to build and enclose the space you want for the dog run, so you buy an additional 14 yards. Given the additional fencing, what is the maximum possible area? Explain how you found your answer.

*The maximum area using only whole-number measurements for length and width is 112 square yards for a fence of length 16 yards and width 7 yards.*

15. As you were unrolling the original 16 yards of fencing, you discovered a segment of it was damaged, reducing the amount you have to 14 yards. Given the fact that you have 2 yards less fencing, what is the maximum possible area? Explain how you found your answer.

*The maximum area given 14 yards of fencing is 24 square yards.*
These concepts can still be treated informally. In the upcoming sections, the function will be represented formally and algebraic and graphing techniques will be introduced.

### Setting Bounds

Now that you have analyzed two situations (the gif program and the dog run), try to see what you understand about the mathematical concepts related to the situations and the linear and quadratic functions.

Based on each of the situations:

16. What are the two parent functions used in each of the last two problems (gif and dog run)? Recall that a parent function is the most basic form of the function.

   \[ y = x \]
   \[ y = x^2 \]

17. Describe, in detail, the rates of change for each of the parent functions.

   The rate of change for the linear function is constant whereas for the quadratic function, the rate of change varies along the curve.

18. Indicate what the x and y intercepts represent in each of the functions.

   The y-intercept represents the value of y when the value of x is zero.
   The x-intercept represents the value of x when y is zero and the image width when image area is zero.

19. Determine if a maximum or minimum value exists for each of the functions. The maximum or minimum value for a function is called the vertex.

   The linear functions do not have maximum or minimum values except within the context of the problem scenarios. The quadratic functions have either a maximum or a minimum value.

20. A function is symmetric if each half is a mirror image of itself. This means that for two different x values, the y value will be the same. The axis of symmetry is the line that acts like the mirror and divides the function into its two identical halves.

   Determine if either function is symmetric. Explain your answer.

   The quadratic function is symmetric. It has a vertex. In this problem, after the maximum area is reached, the function decreases and forms a mirror image with the increasing portion of the curve.
Once again, students can add to the list of common characteristics and differences between functions. Refer back to page 6-8 after completing questions 16 through 22.

### Setting Bounds

As you continue your study, you will be introduced to additional concepts that help to define both linear and quadratic functions, as well as several other functional types. At this point, see what your guess is on a few of these questions. As you continue the course, return to these questions to see if you still agree with your responses.

21. How many y-intercepts can a linear function have? How many y-intercepts can a quadratic function have?

   *A linear function can have one y-intercept. A quadratic function can have one y-intercept.*

22. How many x-intercepts can a linear function have? How many x-intercepts can a quadratic function have? Explain your reasoning.

   *A linear function can have one x-intercept, since wherever you draw the line, it can cross each of the axes only once. A quadratic function can have at most two x-intercepts. If you draw a parabola anywhere in the four quadrants it may not cross the x-axis, it may cross the x-axis once, but it will never cross the x-axis more than twice.*
Exploring Quadratic Equations

1. Examine each of the graphs below.

   \[ y = x^2 \quad \text{and} \quad y = 2x^2 \]

   a. What are the similarities and differences?

   The similarities between the curves are: they both open upward, both have their vertex at the origin and both are symmetric with respect to the y-axis. The difference is that \( y = 2x^2 \) is narrower.

   b. In the graph of \( y = 2x^2 \), identify:

      - y-intercept: (0,0)
      - x-intercept: (0,0)
      - vertex: (0,0)
      - axis of symmetry (the line that divides the parabola into mirror images of itself). The vertical line through the vertex is \( x = 0 \).

2. \[ y = x^2 \quad \text{and} \quad y = \frac{x^2}{2} \]

   a. What are the similarities and differences?

   The similarities between the curves are that they both open upward, both have their vertex at the origin and both are symmetric with respect to the y-axis. The difference is that \( y = x^2 \) is narrower.
As an activity, you may want to use the graphing calculator or overhead transparencies to show different equations. Ask the students to guess what the function might be. Toward the end of this activity, introduce one equation in the form \( y = ax^2 + bx + c \) and one quadratic equation that has no real roots.

### Exploring Quadratic Equations

<table>
<thead>
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<th>analysis</th>
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<tbody>
<tr>
<td>b. In the graph of ( y = \frac{x^2}{2} ), identify:</td>
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3. The graph of \( 2y = x^2 \):

|   | y-intercept \((0,0)\) |
|   | x-intercept \((0,0)\) |
|   | vertex \((0,0)\) |
|   | axis of symmetry The vertical line through the vertex is \( x = 0 \). |

3. The graph of \( y = 4x^2 \):

|   | y-intercept \((0,0)\) |
|   | x-intercept \((0,0)\) |
|   | vertex \((0,0)\) |
|   | axis of symmetry The vertical line through the vertex is \( x = 0 \). |

---

The similarities between the curves are:

* They both open upward, both have their vertex at the origin, and both are symmetric with respect to the y-axis.
* The difference is that \( y = 4x^2 \) is narrower.
a. What are the similarities and differences?

The similarities between the curves are: they both open upward, both have their vertex at the origin and both are symmetric with respect to the y-axis. The difference is that $y = x^2$ is narrower.

b. In the graph of $y = \frac{x^2}{4}$, identify:

- y-intercept (0,0)
- x-intercept (0,0)
- vertex (0,0)
- axis of symmetry The vertical line through the vertex is $x = 0$. 
Exploring Quadratic Equations

5. On the graph below, draw:
   a. \( y = 2x^2 \)
   b. \( y = \frac{x^2}{2} \)
   y = 4x^2
   y = \frac{x^2}{4}
   y = 6x^2
   y = \frac{x^2}{6}

After students graph these functions and answer question 6, ask, "How does the coefficient of \( x \) control the graph of a linear function?"

You may also ask students to graph \( y = \frac{3}{2}x^2 \), \( y = \frac{7}{3}x^2 \), or \( y = \frac{13}{4}x^2 \) and \( y = 0.05x^2 \), \( y = 0.72x^2 \), or \( y = 0.001x^2 \) to dispel the misconception that a "fractional" coefficient makes the parabola wider.

6. What can you say about the relationship between the two groups of functions you graphed? Summarize your findings about what the coefficient of \( x^2 \) controls in a quadratic function.

   The functions are similar to the parent function but will be narrower if the coefficient of \( x^2 \) is larger than one. The larger the coefficient the narrower the curve. If there is a coefficient of \( x^2 \) between 1 and 0, the curve is wider. The closer the coefficient is to 0, the wider the curve.
Exploring Quadratic Equations

7. Examine each of the graphs below.

\[ y = x^2 \quad \text{and} \quad y = x^2 + 2 \]

a. What are the similarities and differences?

The similarities between the curves are: they both open upward and both are symmetric with respect to the y-axis. One curve is not wider or narrower than the other.

The difference is that the curve \( y = x^2 + 2 \) does not have its vertex at the origin. It is shifted upward by 2 units, so its vertex is at (0,2).

b. In the graph of \( y = x^2 + 2 \), identify:

- y-intercept \((0,2)\)
- x-intercept \(\text{none}\)
- vertex \((0,2)\)
- axis of symmetry The vertical line through the vertex is \(x = 0\).
8. Examine each of the graphs below.

\[ y = x^2 \quad \text{and} \quad y = x^2 - 2 \]

a. What are the similarities and differences?

*The similarities between the curves are: they both open upward and both are symmetric with respect to the y-axis. One curve is not wider or narrower than the other.*

*The difference is that the curve \( y = x^2 - 2 \) does not have its vertex at the origin. It is shifted down by 2 units, so its vertex is at \((0, -2)\).*

b. In the graph of \( y = x^2 - 2 \), identify:

- y-intercept \((0, -2)\)
- x-intercept \((-1.414, 0)\) and \((1.414, 0)\)
- vertex \((0, -2)\)
- axis of symmetry The vertical line through the vertex is \(x = 0\).
### Exploring Quadratic Equations

9. Graph the equations. Indicate the y-intercept(s), x-intercept(s), vertex, and axis of symmetry.

   a. \( y = x^2 + 1 \)
      
      - **y-intercept:** \((0,1)\)
      
      - **x-intercept:** None
      
      - **vertex:** \((0,1)\)
      
      - **axis of symmetry:** \(y\)-axis; \(x = 0\)

   ![Graph of a quadratic equation with y-intercept at (0,1), x-intercept None, vertex at (0,1), and axis of symmetry at the y-axis.]

   b. \( y = x^2 - 3 \)
      
      - **y-intercept:** \((0,-3)\)
      
      - **x-intercept:** \((-1.732, 0);(1.732,0)\)
      
      - **vertex:** \((0,-3)\)
      
      - **axis of symmetry:** \(y\)-axis; \(x = 0\)

   ![Graph of a quadratic equation with y-intercept at (0,-3), x-intercepts at \((-1.732, 0)\) and \((1.732,0)\), vertex at (0,-3), and axis of symmetry at the y-axis.]

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Exploring Quadratic Equations

| process: practice | c.  $y = x^2 + 4$
|                  | y-intercept: **(0,4)**
|                  | x-intercept: **None**
|                  | vertex: **(0,4)**
|                  | axis of symmetry:
|                  | $y$-axis; $x = 0$
|                  | ![Graph of c. $y = x^2 + 4$](image)

| d.  $y = x^2 - 10$
| y-intercept: **(0,-10)**
| x-intercept: **(-3.16,0); (3.16,0)**
| vertex: **(0,-10)**
| axis of symmetry:
| $y$-axis; $x = 0$
| ![Graph of d. $y = x^2 - 10$](image)

10. Summarize your findings about what a number added to or subtracted from $x^2$ controls.

   **When a number is added to or subtracted from $x^2$, the curve is shifted up or down along the y-axis, changing the vertex from the origin.**

After students complete question 10, ask them how a number added or subtracted to $x$ controls the linear function. Is it the same effect, and if so, why? If it is not the same effect, why not?
11. Examine each of the graphs below.

\[ y = x^2 \quad \text{and} \quad y = (x - 2)^2 \]

**a.** What are the similarities and differences?

The similarities between the curves are: they both open upward and both are symmetric. One curve is not wider or narrower than the other.

The difference is that the curve \( y = (x - 2)^2 \) does not have its vertex at the origin. It is shifted horizontally to the right by 2 units, so its vertex is at \((2,0)\).

**b.** In the graph of \( y = (x - 2)^2 \), identify:

- y-intercept \((0,4)\)
- x-intercept \((2,0)\)
- vertex \((2,0)\)
- axis of symmetry \( \text{The vertical line through the vertex is } x = 2. \)
12. Examine each of the graphs below.

\[
y = x^2 \quad \quad y = (x + 1)^2
\]

a. What are the similarities and differences?

*The similarities between the curves are: they both open upward and both are symmetric. One curve is not wider or narrower than the other.*

*The difference is that the curve \( y = (x + 1)^2 \) does not have its vertex at the origin. It is shifted horizontally to the left by 1 unit, so its vertex is at \((-1,0)\).*

b. In the graph of \( y = (x + 1)^2 \), identify:

- y-intercept \((0,1)\)
- x-intercept \((-1,0)\)
- vertex \((-1,0)\)
- axis of symmetry *The vertical line through the vertex is \( x = -1 \).*
Exploring Quadratic Equations

13. Graph the equations. Indicate the y-intercept(s), x-intercept(s), vertex, and axis of symmetry.

- a. $y = (x - 3)^2$
  - y-intercept: $(0, 9)$
  - x-intercept: $(3, 0)$
  - vertex: $(3, 0)$
  - axis of symmetry: $x = 3$

- b. $y = (x + 2)^2$
  - y-intercept: $(0, 4)$
  - x-intercept: $(-2, 0)$
  - vertex: $(-2, 0)$
  - axis of symmetry: $x = -2$
Exploring Quadratic Equations

<table>
<thead>
<tr>
<th>process: practice</th>
</tr>
</thead>
</table>
| c. \( y = (x + 4)^2 \)  
  y-intercept: (0,16)  
x-intercept: (-4,0)  
  vertex: (-4,0)  
  axis of symmetry: \( X = -4 \)  

| d. \( y = (x - 2)^2 \)  
  y-intercept: (0,4)  
x-intercept: (2,0)  
  vertex: (2,0)  
  axis of symmetry: \( X = 2 \)  

14. Summarize your findings about what a number added to or subtracted from \( x \) before squaring controls.  

*The addition of a number to or subtraction from \( x \) before squaring shifts the curve to the left or right, respectively, along the horizontal axis.*

After question 14, see if students can discuss what happens to a linear function if you add or subtract a number from \( x \) when the slope is 1. (The result will be the same as a vertical shift. Students may not have realized this.)
**Exploring Quadratic Equations**

15. Graph $y = x^2 + 5x - 6$. How does this graph differ from the others you have studied?

*The curve is shifted down and to the left. Other curves have been shifted in one direction.*

16. Graph the following equations:

   a. $y = x^2 + 3x + 5$
   b. $y = x^2 + 2x + 1$
   c. $y = x^2 + 7x + 12$

What do these curves have in common? What is different about the graphs? *The curves all open up, and neither is wider nor narrower than the other. They are all symmetric. The curves differ because they have been moved horizontally or vertically by different amounts, so they have different axes of symmetry.*
17. What do you think the graph of 
\[ y = -x^2 \]
looks like? Describe it in complete sentences and verify if your description is accurate by graphing it.

*The curve is the same as \( y = x^2 \) except that it opens downward.*

18. What do you think the graph of 
\[ y = -x^2 - 5x - 6 \]
looks like? Describe it in complete sentences and verify if your description is accurate by graphing it. *It will open downward; it will be shifted and will not be stretched or shrunk.*
19. Graph \( y = (x - 2)^2 + 4 \) and \( y = x^2 - 4x + 8 \).

Describe the similarities and differences between these two equations and between their graphs.

The graphs are identical.

*If you expand the equation \( y = (x - 2)^2 + 4 \), you find it is \( y = x^2 - 4x + 8 \), so the curves must have the same intercepts, vertex points and lines of symmetry. (Students may not yet know how to expand \( y = (x - 2)^2 \). That skill will be developed in the next section.)*
Finding Zeros by Factoring

Notice in the last problem, the graphs were the same, which means that the equations were the same, just written in a different form. Factoring or writing a polynomial as a product can be useful for finding the x-intercepts or zeros. In this case, we are factoring a quadratic (or second-degree) polynomial. Solving a quadratic equation means finding the zeros or x-intercepts.

We’ll now look at finding zeros by factoring. First let’s review how to multiply polynomials using the Distributive Law.

Given an equation in the form:

\[ y = (x + 4)(x - 1) \]

to multiply two linear factors, you apply the Distributive Law twice. So, \( y = (x + 4)(x - 1) \) becomes:

\[
\begin{align*}
  y &= x(x - 1) + 4(x - 1) \\
  &= x^2 - x + 4x - 4 \\
  &= x^2 + 3x - 4.
\end{align*}
\]

The equation is now written in standard form. To see this in another way, let’s look at \( y = (x + 4)(x + 1) \) using the multiplication table below.

1. In the left-hand column, write the terms of the first factor, \( x \) and \( 4 \).

   \[
   \begin{array}{ccc}
   \cdot & x & -1 \\
   \hline
   x & x^2 & -x \\
   4 & -4 & \end{array}
   \]

2. In the first row, write the terms of the second factor, \( x \) and \(-1\).

   Note: It does not matter which factor you put in the row or the column.

3. Multiply the first term in the first column by the first term in the row.

4. Multiply the second term in the first column by the first term in the row.

5. Multiply the first term in the first column by the second term in the row.
6. Multiply the second term in the first column by the second term in the row to complete your table.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x^2</td>
<td>-x</td>
</tr>
<tr>
<td>4</td>
<td>4x</td>
<td>-4</td>
</tr>
</tbody>
</table>

We want to add the terms from the table:

\[ x^2 + 4x - x - 4 \]

And simplify by combining like terms:

\[ x^2 + 3x - 4 \]

Using this information, look at the various methods for factoring second-degree polynomials.

- **Common Factoring**
  
  Given a polynomial, finding a common factor means finding the largest value that will divide each and every term evenly (greatest common factor).
  
  Examples:
  
  a. \[ 6x - 6 = 6(x - 1) \]
  
  Since 6 is the largest value that is common to each and every term, it is the greatest common factor.
  
  b. \[ 4x^3 + 8x^2 + 16x = 4x(x^2 + 2x + 4) \]
  
  Since 4x is the largest value that is common to each and every term, it is the greatest common factor.

  You can think of factoring as “undistributing” (as you saw in unit 4).
If you think this more formal approach will be too difficult for your students, tie factoring to the multiplication tables that follow on the next page. This technique is introduced to help students "visualize" which factors may work and why.

### Finding Zeros by Factoring

**Trinomial Factoring**

A quadratic equation

\[ ax^2 + bx + c \]

may be factored as the product of two linear factors of the form

\[(kx + m)(lx + n)\]

where \(k, l, m\) and \(n\) are all integers or fractions. To factor,

1. List all the factors of \(a\) so that \(a = kl\)
2. List all the factors of \(c\) so that \(c = mn\)
3. Test the pairs of factors of \(a\) and \(c\) to determine if the sum of their products is equal to \(b = kn + lm\).

If the last condition can be met, then the polynomial is factorable. Not every trinomial is factorable.

Examples:

a. Factor: \(x^2 + 7x + 10\)

List all the factors of \(a\). In this example \(a = 1\) and the factors of 1 are

- 1 and 1
- -1 and -1

List all factors of \(c\). In this example \(c = 10\) and the factors of 10 are

- 10 and 1
- -10 and -1
- 5 and 2
- -5 and -2

Since \(a, b,\) and \(c\) are all positive, you will need to consider only the pairs of positive factors. Why do you think this is true?

Test factors of \(a\) and \(c\) to determine if the sum of the product of the factors for \(a\) and the factors for \(c\) are equal to \(b\). In this example, \(b = 7\).

By looking at the pairs of factors, you see that the sum of 5 and 2 equals 7, which is \(b\), so you may write the equation in factored form as

\[ x^2 + 7x + 10 = (x + 5)(x + 2). \]
Finding Zeros by Factoring

Suppose we take this example and use the multiplication tables to help us see the process in another way. Begin by listing the factors of 1 that must be considered for this problem.

1 and 1

List the factors of c that must be considered for this problem. Recall that c = 10.
10 and 1
5 and 2

Based on the factors and all the combinations, fill in the cells in the table to see what works.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x²</td>
<td>x</td>
</tr>
<tr>
<td>10</td>
<td>10x</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x²</td>
<td>2x</td>
</tr>
<tr>
<td>5</td>
<td>5x</td>
<td>10</td>
</tr>
</tbody>
</table>

As you can see, the sum of 10x + 1x = 11x, and 5x + 2x = 7x, so the factorization on the right is the correct choice. Therefore,

\[(x+5) (x+2) = x² + 7x + 10.\]

b. Factor: 6x² – 7x – 3

List all the factors of a. In this example a = 6, so the factors are
6 and 1
-6 and -1
2 and 3
-2 and -3

List all the factors of c. In this example c = -3, so the factors are
3 and -1
-3 and 1

We will consider only positive pairs of factors for a to minimize the number of possible pairs to be considered.
Finding Zeros by Factoring

For this problem, we will consider only the positive factors of $a$. If we did consider $-6$ and $-1$ or $-2$ and $-3$, these factors would end up paired with the other set of factors for $c$. For example, if we test $-6$ and $-1$ with $-3$ and $1$ to find that this set of factors works to give us $6x^2 - 7x - 3$, then $+6$ and $+1$ paired with $+3$ and $-1$ will also work.

Based on the factors that we will consider, and all the combinations, you can fill in the following cells in the multiplication tables and test which set of factors works to give $6x^2 - 7x - 3$.

<table>
<thead>
<tr>
<th></th>
<th>$x$</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6x$</td>
<td>$6x^2$</td>
<td>$-6x$</td>
</tr>
<tr>
<td>$3$</td>
<td>$3x$</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$x$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6x$</td>
<td>$6x^2$</td>
<td>$18x$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-x$</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$3x$</th>
<th>$-3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x$</td>
<td>$6x^2$</td>
<td>$-6x$</td>
</tr>
<tr>
<td>$1$</td>
<td>$3x$</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$3x$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x$</td>
<td>$6x^2$</td>
<td>$2x$</td>
</tr>
<tr>
<td>$-3$</td>
<td>$-9x$</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
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</table>

While grids offer one way to test for the correct factors, the distributive law can also be used. You will want to determine if the sum of the products is equal to $b$.

- $(6x - 3)(x + 1) = 6x^2 + 6x - 3x - 3 = 6x^2 + 3x - 3$
  By multiplying you see that the sum of the products does not give the middle term. In this first attempt, $b = 3$.
  Checking: $a = 6$, $c = -3$, $b = 3$

- $(x - 3)(6x + 1) =$
  Checking: $a = 6$, $c = -3$ and $b = -17$

- $(6x + 3)(x - 1) =$
  Checking: $a = 6$, $c = -3$ and $b = -3$

- $(6x - 1)(x + 3) =$
  Checking: $a = 6$, $c = -3$ and $b = 17$

- $(2x - 3)(3x + 1) =$
  Checking: $a = 6$, $c = -3$ and $b = -7$

Since the condition for finding $b$ has been met, the polynomial can be written in factored form as

$6x^2 - 7x - 3 = (2x - 3)(3x + 1)$
Finding Zeros by Factoring

c. Factor: \( x^2 - 16 \)

List all the factors of \( a \). In this example \( a = 1 \)

List all factors of \( c \). In this example \( c = -16 \) and the factors of \(-16\) are

- -16 and 1
- 16 and -1
- 4 and -4

Test the pairs of factors of \( a \) and \( c \) to determine if the sum of their products is equal to \( b \). In this example \( b = 0 \).

By looking at the pairs of factors, you see that the sum of 4 and -4 will equal \( b \), so you may write the equation in factored form as

\[ x^2 - 16 = (x + 4)(x - 4) \]

This factorization is referred to as **difference of squares** since the quadratic equation is the difference of two squared terms where \( b = 0 \).
Have students work on these individually. Review the results as a class. Students will have additional practice on factoring in the software.

### Finding Zeros by Factoring

8. Change the equation of the function written in standard form to factored form. If the form cannot be changed to factored form, simply write NF to mean “not factorable”.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>(x^2 + x - 2 = y) (y = (x + 2)(x - 1))</td>
</tr>
<tr>
<td>b.</td>
<td>(y = x^2 - 5x + 6) ((x - 3)(x - 2) = y)</td>
</tr>
<tr>
<td>c.</td>
<td>(x^2 - 4x + 4 = y) ((x - 2)(x - 2) = y)</td>
</tr>
<tr>
<td>d.</td>
<td>(f(x) = 7x^2 - 5x - 6) (NF)</td>
</tr>
<tr>
<td>e.</td>
<td>(x^2 - 12x = y) (x(x - 12) = y)</td>
</tr>
<tr>
<td>f.</td>
<td>(3x^2 - 7x - 20 = f(x)) (f(x) = (3x + 5)(x - 4))</td>
</tr>
<tr>
<td>g.</td>
<td>(f(x) = x^2 - 10x + 5) (NF)</td>
</tr>
<tr>
<td>h.</td>
<td>(5x^2 - 125 = y) (y = 5(x^2 - 25) = 5(x + 5)(x - 5))</td>
</tr>
<tr>
<td>i.</td>
<td>(-3x^2 - 5x + 6 = y) (NF)</td>
</tr>
<tr>
<td>j.</td>
<td>(y = 4x^2 - 36) (y = 4(x^2 - 9) = 4(x + 3)(x - 3))</td>
</tr>
<tr>
<td>k.</td>
<td>(x^2 - 5x = y) (x(x - 5) = y)</td>
</tr>
<tr>
<td>l.</td>
<td>(10x^2 - x - 3 = f(x)) ((2x + 1)(5x - 3) = f(x))</td>
</tr>
<tr>
<td>m.</td>
<td>(f(x) = 3x^2 + 21x + 30) (f(x) = 3(x^2 + 7x + 10) = 3(x + 5)(x + 2))</td>
</tr>
</tbody>
</table>
These problems tie the technique of factoring to finding the zeros of a function.

Tie finding the intercepts for a quadratic equation to finding the intercepts for a linear equation. Ask students to interpret and compare the interpretation of intercepts in the case of linear and quadratic equations.

<table>
<thead>
<tr>
<th>Finding Zeros by Factoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Choose any six of the factorable equations from problem 8. Graph the equations and indicate the x-intercepts. Write the intercepts as an ordered pair of the form (x,0).</td>
</tr>
</tbody>
</table>

*Graph choices will vary. See question 10 for the problem solution.*

<table>
<thead>
<tr>
<th>Finding Zeros by Factoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. Examine the intercept points from the graph and the equations in factored form. What do you observe? What conclusion can you draw?</td>
</tr>
</tbody>
</table>

*The x-intercept points on the graph correspond to the factors. If you substitute the value of x into each of the factors, one of the factors will be zero, so that the value of y will be zero. This means you can determine the x-intercepts by looking at the factors and finding the value of x when each factor is zero.*

<table>
<thead>
<tr>
<th>Finding Zeros by Factoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
</tr>
<tr>
<td>When finding the zeros or x-intercepts of a quadratic equation algebraically, the zeros represent the point where y = 0. To solve for x, you must set y equal to zero. For example, given</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  y &= x^2 + 4x + 3 \\
  \text{set } y &= 0 \\
  0 &= x^2 + 4x + 3. \\
  \text{after factoring } 0 &= (x + 3)(x + 1)
\end{align*}
\]

Since the product of (x + 3) and (x + 1) is zero, then one or both of these must be zero, so x = -3 and x = -1. This means the x-intercepts are (-3,0) and (-1,0).
Finding Zeros Using the Quadratic Formula

Factoring, while a useful technique for finding zeros, is not the only technique nor is it the most powerful one. The Quadratic Formula states, given any equation of the form

$$ax^2 + bx + c = 0 \text{ where } a \neq 0$$

all solutions can be found by solving

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula may be used to solve any quadratic equation.

Examples:

Use the quadratic formula to find the zeros of:

a. $$y = x^2 + 6x + 1$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

$$x = \frac{-6 \pm \sqrt{32}}{2}$$

$$x = \frac{-6 \pm 5.66}{2}$$

$$x = \frac{-6 + 5.66}{2} = 0.17 \quad \text{and} \quad x = \frac{-6 - 5.66}{2} = -5.83$$

The x-intercepts are (-0.17, 0) and (-5.83, 0).
Ask students "How many intercepts there would be if \( b^2 - 4ac = 0\)? If \( b^2 - 4ac < 0\)?"

Have them explain their answers.

### Finding Zeros Using the Quadratic Formula

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b.</strong> ( y = x^2 - 5x - 3 ) –  </td>
<td></td>
</tr>
</tbody>
</table>

\[
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-3)}}{2(1)}
\]

\[
x = \frac{-(-5) \pm \sqrt{25 + 12}}{2}
\]

\[
x = \frac{5 \pm \sqrt{37}}{2}
\]

\[
x = \frac{5 + 6.08}{2} = 5.54 \quad \text{and} \quad x = \frac{5 - 6.08}{2} = -0.54
\]

The x-intercepts are \((5.54, 0)\) and \((-0.54, 0)\).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **c.** \( y = 2x^2 - 3x + 1 \) &nbsp; &nbsp; |}

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)}
\]

\[
x = \frac{3 \pm \sqrt{9 - 8}}{4}
\]

\[
x = \frac{3 \pm \sqrt{1}}{4}
\]

\[
x = \frac{3 \pm 1}{4}
\]

\[
x = \frac{3 + 1}{4} = 1 \quad \text{and} \quad x = \frac{3 - 1}{4} = \frac{1}{2}
\]

The x-intercepts are \((1, 0)\) and \((\frac{1}{2}, 0)\).
Finding Zeros Using the Quadratic Formula

Find the roots or zeros of the equations using the Quadratic Formula. Check your results by graphing the equation using your graphing calculator. Sketch the graph.

1. $y = x^2 + 7x + 10$

   $$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(10)}}{2(1)}$$
   $$x = \frac{-7 \pm \sqrt{9}}{2}$$
   $$x = \frac{-7 \pm 3}{2}$$
   $$x = \frac{-7 + 3}{2} = -2$$
   $$x = \frac{-7 - 3}{2} = -5$$

2. $y = -3x^2 - 2x + 1$

   $$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-3)(1)}}{2(-3)}$$
   $$x = \frac{2 \pm \sqrt{4 + 12}}{-6}$$
   $$x = \frac{2 \pm \sqrt{16}}{-6}$$
   $$x = \frac{2 \pm 4}{-6}$$
   $$x = \frac{2 + 4}{-6} = -1$$
   $$x = \frac{2 - 4}{-6} = \frac{1}{3}$$
3. \( y = x^2 - 2x + 1 \)

\[
x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}
\]

\[
x = \frac{2 \pm \sqrt{0}}{2}
\]

\[
x = \frac{2}{2} = 1
\]

4. \( y = x^2 + 2x + 4 \)

\[
x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}
\]

\[
x = \frac{-2 \pm \sqrt{-16}}{2}
\]

No real roots
Finding Zeros Using the Quadratic Formula

Earlier in this unit, we defined the vertex of the parabola and the axis of symmetry. Recall that the vertex is the highest or lowest point on the graph of the quadratic function. The coordinates of the vertex are defined as:

\[ \left( \frac{-b}{2a}, f \left( \frac{-b}{2a} \right) \right) \]

This means \( x = \frac{-b}{2a} \) and \( y = f(x) = ax^2 + bx + c \) when \( x = \frac{-b}{2a} \).

For example, to find the vertex of the function \( y = x^2 + 2x + 1 \) compute the value of the x-coordinate. In this problem \( a = 1 \) and \( b = 2 \), so

\[ x = \frac{-2}{2} = -1 \]

Now evaluate the function when \( x = -1 \) to find the y-coordinate.

\[ y = (-1)^2 + 2(-1) + 1 \]
\[ y = 1 - 2 + 1 \]
\[ y = 0 \]

The vertex is \((-1, 0)\).

The axis of symmetry is, in general, the vertical line through the vertex and is defined as

\[ x = \frac{-b}{2a} \]

In the above example, the axis or line of symmetry is

\[ x = -1 \]
Finding Zeros Using the Quadratic Formula

In each of the following problems, find the:
   a. zeros
   b. vertex
   c. axis of symmetry.

Graph the equations on the grid below.

<table>
<thead>
<tr>
<th>5.  $x^2 - 10 = y$</th>
<th>(-3.16,0) and (3.16,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, -10)</td>
<td></td>
</tr>
<tr>
<td>$x = 0$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.  $4x^2 - 16 = y$</th>
<th>(-2,0) and (2,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, -16)</td>
<td></td>
</tr>
<tr>
<td>$x = 0$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7.  $x^2 + 10x + 25 = y$</th>
<th>(-5,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5,0)</td>
<td></td>
</tr>
<tr>
<td>$x = -5$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.  $3x^2 - 7x - 6 = y$</th>
<th>(3,0) and (-2/3,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.167, -10.1)</td>
<td></td>
</tr>
<tr>
<td>$x = 1.167$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9.  $-2x^2 + 4x + 5 = y$</th>
<th>(-0.871,0) and (2.871,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,7)</td>
<td></td>
</tr>
<tr>
<td>$x = 1$</td>
<td></td>
</tr>
</tbody>
</table>
Students will see other situations modeled by quadratic functions in the software.

The next three problems are all similar. As has been suggested when encountering similar problems, have one third of the groups do each of the problems. After presentations are made, spend class time comparing and contrasting the three problems so that all students become familiar with the intricacies of each of the problems.

As you have seen, many everyday situations can be modeled by linear functions. Take, for instance, the formulas such as distance as a function of your rate of speed times the length of time you travel, or force as a function of mass times acceleration that you saw in the section on Solving Equations Using Known Formulas. Given situations that involve these quantities, you can apply a known model or formula. This is also true for quadratic functions.

An example of where a known formula can be applied is in problems involving vertical motion. About five hundred years ago, Galileo discovered that vertical motion is the motion of an object that is thrown, hit, dropped, or shot straight up or down. He found that vertical motion can be modeled by the following equation:

\[ y = -\frac{1}{2} \cdot 32t^2 + vt + h \]

In this equation, \( y \) is the number of feet above the ground, \( t \) is the time in seconds from the moment the object starts moving, \( v \) is the initial upward/downward velocity in feet per second, and \( h \) is the initial height measured in feet.

The coefficient of \( t^2 \) deals with acceleration due to gravity. The acceleration due to gravity is actually \(-32 \text{ feet/second}^2\), and the coefficient of \( t^2 \) is \(-16\) or \(-16\). When working in meters, the equation is \( y = -\frac{1}{2} \cdot 9.8t^2 + vt + h \). Here the coefficient of \( t^2 \) will be half of \(-9.8\) meters/second\(^2\) (the metric equivalent of \(-32 \text{ feet/second}^2\)) or \(-4.9\).

Barry Bonds, a well-known baseball player and a heavy hitter, was playing in a critical game for his team. Everyone was hoping for a homerun, but instead he hit a foul ball out of the dirt, straight up, with an initial velocity of 160 feet per second. Represent this situation mathematically.

\[ h = -16t^2 + 160t \]
Modeling: Vertical Motion

1. Graph the equation representing the vertical motion of the ball after being hit.

Modeling: Vertical Motion

2. Using the graph, determine how high the ball is after:
   a. 1 second. *The height of the ball is 144 feet.*
   b. 2 seconds. *The height of the ball is 256 feet.*
   c. 10 seconds. *After 10 seconds the ball is on the ground, so the height is zero.*
   d. 15 seconds. *After 15 seconds the ball would be 1200 feet below the ground. This answer does not make sense in the situation.*

3. Verify your result by using the equation to determine the height of the ball.

\[
144 = -16(1)^2 + 160(1) \quad 0 = -16(10)^2 + 160(10)
\]

\[
256 = -16(2)^2 + 160(2) \quad -1200 = -16(15)^2 + 160(15)
\]
Modeling: Vertical Motion

4. Determine if the answers you found make sense based on the situation. Explain your reasoning.

   All the answers except (d) make sense based on the situation. A negative result does not make sense because the ball will not travel underground.

5. Do all the answers make sense if you look at them outside the context of the problem? Justify your answer.

   Outside the context of the problem, the answers all make sense since the solutions all represent points for time and height that are on the curve.

---

Modeling: Vertical Motion

6. When will the ball hit the ground? Answer in a complete sentence.

   The ball will hit the ground after 10 seconds.

7. When is the ball at its highest point? Answer in a complete sentence.

   The ball is at its highest point after 5 seconds. The height will be 400 feet.

8. When is the ball 384 feet off the ground? Justify your answer.

   The ball will reach a height of 384 feet when t = 4 and t = 6 seconds. On the graph, the y-value of 384 corresponds to an x-value of 4 seconds and an x-value of 6 seconds.
At Barry’s next time up at bat, he hit another foul ball straight up with an initial upward velocity of 160 feet per second, but this time his bat made contact with the ball 4 feet above the ground. Represent the situation mathematically.

\[ h = -16t^2 + 160t + 4 \]

Have students note the similarities and differences between the equations from the previous situation and this one and between the corresponding graphs.

9. Graph the equation representing the vertical motion of the ball after being hit.
10. Using the graph, determine how high the ball is after:
   a. 1 second. **The height of the ball will be 148 feet.**
   b. 2 seconds. **The height of the ball will be 260 feet.**
   c. 10 seconds. **The height of the ball will be 4 feet.**
   d. 15 seconds. **The height of the ball will be -1196 feet.** This answer does not make sense in this situation.

11. Verify your result by using the equation to determine the height of the ball.

\[
\begin{align*}
148 &= -16(1)^2 + 160(1) + 4 \\
260 &= -16(2)^2 + 160(2) + 4 \\
4 &= -16(10)^2 + 160(10) + 4 \\
-1196 &= -16(15)^2 + 160(15) + 4
\end{align*}
\]

12. Determine if the answers you found make sense based on the situation. Explain your reasoning.

*All the answers except (d) make sense since a height of -1196 does not make sense. The ball would have to be below ground.*

13. Do all the answers make sense if you look at them outside the context of the problem? Justify your answer.

*All the answers make sense out of the context of the problem. If you look on the graph, you will find all the points lie on the curve, which means they are solutions.*
14. When will the ball hit the ground? Answer in a complete sentence.

   The ball will hit the ground after 10.02 seconds.

15. When is the ball at its highest point? Answer in a complete sentence.

   The ball will reach its maximum height after 5 seconds. The height will be 404 feet.

16. When is the ball 384 feet off the ground? Justify your answer.

   The ball will be 384 feet off the ground when $t = 3.88$ and $t = 6.12$ seconds. Since the curve is a parabola and it is symmetric, for each height there will be two distinct values for the time.
Modeling: Vertical Motion

At Barry’s third time up at bat, he hit another foul ball straight up. This time, the initial upward velocity was 140 feet per second. His bat made contact with the ball 4 feet above the ground. Represent the situation mathematically.

\[ h = -16t^2 + 140t + 4 \]

17. Graph the equation representing the vertical motion of the ball after being hit.
18. Using the graph, determine how high the ball is after:
   a. 1 second. *The height of the ball will be 128 feet.*
   b. 2 seconds. *The height of the ball will be 220 feet.*
   c. 10 seconds. *The height of the ball will be -196 feet. This answer does not make sense in this situation.*
   d. 15 seconds. *The height of the ball will be -1496 feet. This answer does not make sense in this situation.*

19. Verify your result by using the equation to determine the height of the ball.

\[
\begin{align*}
128 &= -16(1)^2 + 140(1) + 4 \\
220 &= -16(2)^2 + 140(2) + 4 \\
-196 &= -16(10)^2 + 140(10) + 4 \\
-1496 &= -16(15)^2 + 140(15) + 4
\end{align*}
\]

20. Determine if the answers you found make sense based on the situation. Explain your reasoning.

*All the answers except c and d make sense based on the situation. A negative result does not make sense.*

21. Do all the answers make sense if you look at them outside the context of the problem? Justify your answer.

*Outside the context of the problem, the answers all make sense since the solutions all represent points for time and height that are on the curve.*
### Modeling: Vertical Motion

22. When will the ball hit the ground? Answer in a complete sentence.

*The ball will hit the ground after 8.78 seconds.*

23. When is the ball at its highest point? Answer in a complete sentence.

*The ball will be at its highest point when t = 4.375 seconds. The height will be 310.25 feet.*

24. When is the ball 384 feet off the ground? Justify your answer.

*The ball will never reach a height of 384 feet above the ground since the maximum height is 310.25 feet.*
Modeling: Vertical Motion

25. Compare and contrast the three scenarios by citing at least two similarities and two differences.

_The similarities are that the three parabolas all open downward. They each have a maximum value and two x-intercepts. The differences are y-intercepts and position of the curve along the axes. The maximum values differ as well._

26. How do the changes in the initial height affect the graphical representation of the function?

_The changes in the initial height affect the graphical representation by shifting the curve upward._

27. How do the changes in the initial velocity affect the graphical representation of the function?

_The initial velocity will shift the curve vertically and horizontally. Some students might say that the larger the h, the more the curve will shift to the right, and the larger the h, the higher the curve, as long as the initial height stays the same. (Note that this is only true when a is negative. If a is positive, an increase in h moves the curve downward and to the left.)_
Modeling: Vertical Motion

28. Graph each of the equations below. Compare these graphs to the ones you generated in the Barry Bonds problems. Does unit conversion change the graph of the parabola? Explain.

a. \( y = -4.9t^2 + 48.8t \)

b. \( y = -4.9t^2 + 48.8t + 1.2 \)

c. \( y = -4.9t^2 + 42.7t + 1.2 \)

The graphs of the curves are the same. The units for \( a, b, \) and \( c \) have been changed to metric units from English units. Since students may not account for conversion factors when creating scales and intervals, they will indicate that the equation written in terms of meters will fit under the curve written using feet.
Unit 7: Laws of Powers

Unit Objectives and Skills

At the completion of this unit, students will attain competency in identifying and applying the laws of powers for products and quotients.

Unit Overview

The focus of this unit is on multiplying and dividing variables raised to a power or powers. The unit is designed to introduce students to the laws formally and to the direct application of them in the simplification of mathematical expressions.

Unit Activities

Listed below are this unit’s activities along with the page numbers of the corresponding homework in the Assignments section.

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<th>Homework Assignments</th>
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<tr>
<td>Laws of Powers: Part 6</td>
<td>-</td>
</tr>
</tbody>
</table>

Suggestions for Classroom Implementation

Make up several problems relevant to a given law. Write each of the problems on a small piece of paper and put them in a hat to have students randomly draw their group’s problems. Have each group work on solving the problems and writing a summary on how the group arrived at the solutions. Ask each group to come up with a rule(s) that describes the process and to present it. Then look at the formal rule(s) and discuss the results.
Laws of Powers

**Unit Assessment**

*Laws of Powers: Part 1*

*Laws of Powers: Part 2*
Unit 7: Laws of Powers

Contents

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Laws of Powers
Laws of Powers

Powers or exponents are a notation that allows you to write repeated multiplication. For example:

\[ x \cdot x = x^2 \]
\[ x \cdot x \cdot x = x^3 \]
\[ x \cdot x \cdot x \cdot x = x^4 \]
\[ x \cdot x \cdot \ldots \cdot x = x^n \]

\( x \) is called the base and the number is called the exponent. The exponent or power represents the number of times you multiply a number by itself.

**Definition**

\[ A \cdot x^b = A \cdot x \cdot x \cdot \ldots \cdot x \]

\( b \) times

---

Laws of Powers: Part 1

1. Use the definition to expand each of the following expressions:
   a. \( 3x^4 = 3 \cdot x \cdot x \cdot x \cdot x \)
   
   b. \( 2x^5 = 2 \cdot x \cdot x \cdot x \cdot x \cdot x \)
   
   c. \( 5x^6 = 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \)
   
   d. \( 3y^6 = 3 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \)
## Laws of Powers: Part 1

2. Use the definition to simplify each of the following expressions.

   a. \( 3 \cdot x \cdot x \cdot x \cdot x = 3x^4 \)

   b. \( 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = 5x^6 \)

   c. \( 12 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = 12x^9 \)

   d. \( 2 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y = 2x^4y^4 \)

   e. \( 12 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c = 12a^3b^2c^4 \)

   f. \( 7 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z \cdot z = 7x^4y^2z^5 \)
<table>
<thead>
<tr>
<th>Laws of Powers: Part 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. For each of the following expressions, first expand each factor, regroup if necessary, and write the result using a single power when appropriate.</td>
</tr>
<tr>
<td>Example:</td>
</tr>
<tr>
<td>$5x^3 \cdot 2x^2 = 5 \cdot x \cdot x \cdot x \cdot 2 \cdot x \cdot x = 5 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x = 10x^5$</td>
</tr>
<tr>
<td>a. $x^3 \cdot x^2 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^5$</td>
</tr>
<tr>
<td>b. $x^5 \cdot x^7 = (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) = x^{15}$</td>
</tr>
<tr>
<td>c. $x^4 \cdot y^2 = x^4y^2$</td>
</tr>
<tr>
<td>d. $2^4 \cdot 3^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$</td>
</tr>
<tr>
<td>$16 \cdot 9 = 144$</td>
</tr>
</tbody>
</table>
4. Write each of the following expressions as a single power when appropriate. Try to find the resulting expressions without first expanding each factor.

a. \(2^4 \cdot 2^2 = 2^{4+2} = 2^6\)

b. \(3^x \cdot 2^x = 3 \cdot 2x^{4+2} = 6x^6\)

c. \(5x^3 \cdot (-2)^x = 5 \cdot (-2)x^{4+5} = -10x^9\)

d. \(8^x \cdot 9y^2 = 8 \cdot 9x^4y^2 = 72x^4y^2\)

e. \(2^x \cdot 3^x \cdot 7^x \cdot 3^y = 2 \cdot 3 \cdot 7 \cdot 3x^{4+2+5}y^3 = 126x^{11}y^3\)

f. \(-2^x \cdot 3^y \cdot 8^x \cdot 4y^x = -2 \cdot 3 \cdot 8 \cdot 4x^{4+5}y^{x+y} = -192x^9y^7\)

g. \(2x^3 + 3x^3 = 5x^3\) (Hint: Be careful here.)
You might want to have students rearrange like terms together to solidify their understanding of the concept of like terms as well as to ensure that the rules are applied properly.

<table>
<thead>
<tr>
<th>Laws of Powers: Part 1</th>
</tr>
</thead>
</table>
| h. \(2x^4 + 3x^4 - 7x^4 + 3y^2 =\)  
  \((2x^4 - 7x^4) + 3x^2 + 3y^2\)  
  \(-5x^4 + 3x^2 + 3y^2\) |
| i. \(2x^4 \cdot 3x^2 = 2x^2 \cdot 3x^3\) |
| j. \(2x^4 + 3y^2 - 7x^4 + 3y^2 =\)  
  \((2x^4 - 7x^4) + (3y^2 + 3y^2)\)  
  \(-5x^4 + 6y^2\) |
| k. \(2x^4 \cdot 3x^2 \cdot 7x^4 \cdot 3y^2 =\)  
  \(2 \cdot 3 \cdot 7 \cdot 3x^{4+2} + 4y^2\)  
  \(126x^{10}y^2\) |
| l. \(2x^4 \cdot 3x^2 = 2 \cdot 3x^{2+3} = 6x^5\) |
| m. \(2x^4 \cdot 3y^2 \cdot 7x^4 \cdot 3y^2 = 2 \cdot 3 \cdot 7 \cdot 3x^{4+4} y^{2+2} = 126x^8y^4\) |
| n. \(-4x^4 \cdot 4y^3 \cdot (-6x^3) \cdot 9y =\)  
  \(-4 \cdot 4 \cdot (-6) \cdot 9x^{5+3} y^{3+1}\)  
  \(864x^8y^4\) |
### Laws of Powers: Part 1

5. For the following expression, first expand each factor, regroup if necessary, and write the result using a single power when appropriate.

\[ \text{a times} \quad x^a \bullet x^b = (x \bullet x \bullet x \ldots) \bullet (x \bullet x \bullet x \ldots) = x^{(a + b)} \]

These examples generalize to what is often called the First Law of Powers.

**First Law of Powers**

*Multiplying Powers*

\[ Ax^a \bullet Bx^b = ABx^{a+b} \]
Students should discover the rule of subtracting exponents (the idea of a negative exponent should follow).

### Laws of Powers: Part 2

#### Input

Use the definition of a power to expand and then simplify each of the following expressions.

**Example:**

\[
\frac{x^4}{x^2} = \frac{1 \cdot 1 \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x \cdot x = x^2
\]

#### Process: Practice

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>( \frac{x^7}{x^4} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = x^3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>( \frac{2x^4}{4x} = \frac{2 \cdot x \cdot x \cdot x \cdot x}{4 \cdot x} = \frac{x^3}{2} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>( \frac{6x^3}{2x^4} = \frac{6 \cdot x \cdot x \cdot x}{2 \cdot x \cdot x \cdot x \cdot x} = \frac{3}{x} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>( \frac{12x^5}{15x^3} = \frac{12 \cdot x \cdot x \cdot x \cdot x \cdot x}{15 \cdot x \cdot x \cdot x} = \frac{4x^2}{5} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>( \frac{4y^5}{12x^3} = \frac{4 \cdot y \cdot y \cdot y \cdot y \cdot y}{12 \cdot x \cdot x \cdot x} = \frac{y^5}{3x^3} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>( \frac{14y^5}{20y^6} = \frac{14 \cdot y \cdot y \cdot y \cdot y \cdot y}{20 \cdot y \cdot y \cdot y \cdot y \cdot y} = \frac{7}{10y} = \frac{7}{10y} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>( \frac{3^5}{3^9} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3} = \frac{1}{3} )</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Remind students that when dividing a number by itself or canceling identical factors, the result is 1 not a blank space.
At this point, students may have noticed that the shorter method is to subtract the exponents.

### Laws of Powers: Part 2

9. Simplify each of the following expressions, using expansion only when you think that it would help.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{6x^5}{2x^3} = 3x^{5-3} = 3x^2 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{10x^6}{-2x^3} = -5x^{6-3} = -5x^3 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{6x^5}{3y^3} = \frac{2x^5}{y^3} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2x^4 \cdot 3x^2}{7x^3 \cdot 3x} = \frac{2x^{4+2-3-1}}{7} = \frac{2x^2}{7} ) or ( \frac{2x^6}{7x^4} = \frac{2x^{6-4}}{7} = \frac{2x^2}{7} )</td>
<td></td>
</tr>
<tr>
<td>( -\frac{2x^5 \cdot 3y^3}{8x^3 \cdot 4y^2} = -\frac{3x^{5-3}y^{3-2}}{16} = -\frac{3x^2y}{16} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2x^4}{-7x^4} = \frac{2}{-7} )</td>
<td></td>
</tr>
</tbody>
</table>
At this point, students should discover $x^{-n} = \frac{1}{x^n}$.

Students may need to revert to expanding each term and canceling like terms in the numerator and denominator.

### Laws of Powers: Part 2

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>i. $\frac{3x^2}{2x^3} = \frac{3 \cdot x \cdot x}{2 \cdot x \cdot x \cdot x} = \frac{3}{2x}$ $(\frac{3}{2}x^{-1})$</td>
<td></td>
</tr>
<tr>
<td>j. $\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^3}$ $(x^5 \cdot x^{-3})$</td>
<td></td>
</tr>
<tr>
<td>k. $\frac{10x^3}{-2x^6} = \frac{10 \cdot x \cdot x \cdot x}{-2 \cdot x \cdot x \cdot x \cdot x \cdot x} = -5x^{-3} \cdot 6 = -\frac{5}{x^3}$</td>
<td></td>
</tr>
<tr>
<td>l. $\frac{6x^3}{3y^5} = \frac{2x^3}{y^5}$</td>
<td></td>
</tr>
<tr>
<td>m. $\frac{2x \cdot 3x^2}{7x^3 \cdot 3x^4} = \frac{2x^3 \cdot 7}{7} = \frac{2x^4}{7x^4}$</td>
<td></td>
</tr>
<tr>
<td>n. $\frac{-12x^5 \cdot 3y}{8x^3 \cdot 4y^2} = \frac{-9x^5 \cdot 3y}{8} = \frac{-9x^2y^1}{8y} = -\frac{9x^2}{8y}$</td>
<td></td>
</tr>
<tr>
<td>o. $\frac{x^1}{x^2} = x^{1-2} = x^{-1}$</td>
<td></td>
</tr>
<tr>
<td>p. $\frac{x^2}{x^5} = x^{2-5} = x^{-3} = \frac{1}{x^3}$</td>
<td></td>
</tr>
</tbody>
</table>
Laws of Powers: Part 2

These examples generalize to the Second Law of Powers.

\[ \frac{A x^a}{B x^b} = \frac{A}{B} x^{a-b} \]

Laws of Powers: Part 3

Students should discover that to simplify a power raised to a power, the exponents are multiplied.

To simplify each of the following expressions, use the definition of exponents and expand twice.

Example:

\[ (x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6 \]
### Laws of Powers: Part 3

1. \((x^2)^4 = x^2 \cdot x^2 \cdot x^2 \cdot x^2 = x^{2+2+2+2} = x^8\)

2. \((x^3)^3 = x^3 \cdot x^3 \cdot x^3 = x^{3+3+3} = x^9\)

3. \((x^4)^5 = x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4 = x^{4+4+4+4+4} = x^{4 \cdot 5} = x^{20}\)

4. \((x^2)^5 = x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 = x^{2+2+2+2+2} = x^{2 \cdot 5} = x^{10}\)

5. \((x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{4+4+4} = x^{4 \cdot 3} = x^{12}\)

6. \((x^5)^3 = x^{5 \cdot 3} = x^{15}\)

7. \((x^3)^5 = x^3 \cdot 5 = x^{15}\)

8. \((x^4)^2 = (x^{4 \cdot 3})^2 = x^{12 \cdot 2} = x^{24}\)
<table>
<thead>
<tr>
<th></th>
<th>Laws of Powers: Part 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td>((x^{14})^6 = x^{14 \cdot 6} = x^{70})</td>
</tr>
<tr>
<td>10.</td>
<td>((x^5)^6 = x^{5 \cdot 6} = x^{30})</td>
</tr>
<tr>
<td>11.</td>
<td>((2^4)^3 = 2^{4 \cdot 3} = 2^{12} = 4096)</td>
</tr>
<tr>
<td>12.</td>
<td>((2^3)^3 = 2^{3 \cdot 3} = 2^6 = 64)</td>
</tr>
<tr>
<td>13.</td>
<td>((x^4)(y^3)^3 = x^{4 \cdot 3} y^{2 \cdot 3} = x^{12} y^6)</td>
</tr>
<tr>
<td>14.</td>
<td>(x^3(y^4)^3 = x^{1 \cdot 3} y^{4 \cdot 3} = x^3 y^{12})</td>
</tr>
<tr>
<td>15.</td>
<td>((2x)^3(y^4)^3 = 2^3 x^{1 \cdot 3} y^{4 \cdot 3} = 8 x^3 y^{12})</td>
</tr>
<tr>
<td>16.</td>
<td>((x^2)^3(y^3)^3 = x^{2 \cdot 3} y^{3 \cdot 3} = x^6 y^9)</td>
</tr>
</tbody>
</table>
Laws of Powers: Part 3

Third Law of Powers

Raising a Power to a Power

\[(x^a)^b = x^{ab}\]

Laws of Powers: Part 4

To simplify each of the following expressions, use the Fourth Law of Powers:

Fourth Law of Powers

Raising a Product to a Power

\[(Ax)^b = A^b x^a y^a\]

Example:

\[\left(3x^2y^3\right)^3 = 3x^6y^9 \cdot 3x^6y^9 \cdot 3x^6y^9\]

\[= (3 \cdot x \cdot x \cdot y \cdot y \cdot y) \cdot (3 \cdot x \cdot x \cdot y \cdot y \cdot y) \cdot (3 \cdot x \cdot x \cdot y \cdot y \cdot y)\]

\[= 3^3 \cdot x^6 \cdot x^6 \cdot x^6 \cdot y^9 \cdot y^9 \cdot y^9\]

\[= 27x^{18}y^9\]
Using the rule developed in Part 3, students should not have to expand through all steps shown in the example.

Be careful if students are using calculators. \(-7^2\) will display \(-49\) as the answer, not \(49 = (-7)^2\).

<table>
<thead>
<tr>
<th>Laws of Powers: Part 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((3x^2y^4)^2 = 3^2 \cdot x^{2\cdot2} \cdot y^{4\cdot2} = 9x^4y^8)</td>
</tr>
<tr>
<td>2. ((2x^3y^4)^2 = 2^3 \cdot x^{3\cdot2} \cdot y^{4\cdot3} = 8x^6y^{12})</td>
</tr>
<tr>
<td>3. ((5x^2y^2)^2 = 5^2 \cdot x^{2\cdot2} \cdot y^{2\cdot2} = 25x^4y^4)</td>
</tr>
<tr>
<td>4. ((x^2y)^2 = x^{2\cdot2} \cdot y^{1\cdot2} = x^4y^2)</td>
</tr>
<tr>
<td>5. ((10xy)^2 = 10^2 \cdot x^2 \cdot y^2 = 100x^2y^2)</td>
</tr>
<tr>
<td>6. ((-7x)^2 = (-7)^2 \cdot x^2 = 49x^4)</td>
</tr>
<tr>
<td>7. ((-9xy)^2 = (-9)^2 \cdot x^2 \cdot y^4 = 81x^2y^4)</td>
</tr>
<tr>
<td>8. ((11xy)^2 = 11^2 \cdot x^2 \cdot y^6 = 121x^2y^6)</td>
</tr>
<tr>
<td>Laws of Powers: Part 4</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>9. ((25xy^5)^2 = 25^2 x^2 y^{10} = 625x^2 y^{10})</td>
</tr>
<tr>
<td>10. ((-5x^3y^6)^2 = (-5)^2 x^9 y^{18} = -125x^9 y^{18})</td>
</tr>
<tr>
<td>11. ((-3xy^2)^3 = (-3)^3 x^3 y^6 = -27x^3 y^6)</td>
</tr>
<tr>
<td>12. ((2x^5y^3)^3 = 2^6 x^{30} y^{24} = 64x^{30} y^{24})</td>
</tr>
<tr>
<td>13. ((3x^2y^4)^2 \cdot (2x^2y^2)^2 = 9x^4 y^8 (4x^6 y^6) = 36x^{10} y^{14})</td>
</tr>
<tr>
<td>14. ((5x^2y^3)^2 \cdot (4x^3)^2 = 25x^4 y^2 (16x^6) = 400x^{10} y^2)</td>
</tr>
<tr>
<td>15. ((2x^2y^4)^2 \cdot (3xy^3)^2 \cdot (4y^2z^5)^2 = 4x^4 y^8 (27x^3 y^9)(64y^{12}z^{15}) = 6912x^7 y^{29} z^{15})</td>
</tr>
<tr>
<td>16. ((9xy)^2 \cdot (2x^7y)^2 \cdot (-3x^2y)^2 \cdot (5xz)^3 = 81x^2 y^2 (8x^6 y^3)(-27x^6 y^3)(125x^3 z^3) = -2,187,000x^{17} y^{8} z^{3})</td>
</tr>
</tbody>
</table>
### Laws of Powers: Part 5

1. Simplify each of the following expressions, using the appropriate expansions.

   a. \[
   \left(\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) = \frac{2^6}{3^6} = \frac{64}{729}
   \]

   b. \[
   \left(\frac{x}{y}\right)^6 = \left(\frac{x}{y}\right) \cdot \left(\frac{x}{y}\right) \cdot \left(\frac{x}{y}\right) \cdot \left(\frac{x}{y}\right) = \frac{x^6}{y^6}
   \]

   c. \[
   \left(\frac{2x}{y^5}\right)^2 = \left(\frac{2x}{y^5}\right) \cdot \left(\frac{2x}{y^5}\right) = \frac{4x^2}{y^{10}}
   \]

   d. \[
   \left(\frac{2x^2}{3y^3}\right)^2 = \frac{4x^4}{9y^6}
   \]

   e. \[
   \left(\frac{-2x}{5y^2}\right)^2 = \frac{4x^2}{25y^{10}}
   \]
Laws of Powers: Part 5

2. Simplify the following expressions, expanding only when you feel it is necessary.

   a. \[ \left( \frac{7x^3}{5y^2} \right)^2 = \frac{49x^6}{25y^4} \]

   b. \[ \left( \frac{11x^2}{y^3} \right)^2 = \frac{121x^4}{y^6} \]

   c. \[ \left( \frac{-5x^5}{15y^7} \right)^3 = \frac{-125x^{15}}{3375y^{21}} = \frac{-x^{15}}{27y^{18}} \]

   d. \[ \left( \frac{-2x^2y^3}{3xy^5} \right)^4 = \frac{16x^8y^{12}}{81x^4y^{20}} = \frac{16x^4}{81y^8} \]

   e. \[ \left( \frac{15xy^3}{5x^4y^2} \right)^3 = \left( \frac{3y}{x^3} \right)^3 = \frac{27y^3}{x^9} \]

   f. \[ \left( \frac{2x^3}{y^5} \right) \cdot \left( \frac{y^3}{x} \right)^2 = \frac{4x^2}{y^{10}} \cdot \frac{y^6}{x^2} = \frac{4}{y^4} \]

   g. \[ \left( \frac{x^4}{y^3} \right)^2 \cdot \left( \frac{y^4}{x^3} \right)^3 = \frac{x^8}{y^6} \cdot \frac{y^6}{x^8} = 1 \]
### Laws of Powers: Part 5

**Fifth Law of Powers**

*Raising a Quotient to a Power*

$$\left( \frac{x}{y} \right)^a = \frac{x^a}{y^a}$$

### Laws of Powers: Part 6

**Sixth Law of Powers**

*Definition of a Zero Power*

$$x^0 = 1$$

**Example:**

$$\frac{x^3}{x^2} = \frac{x \times x \times x}{x \times x} = \frac{x}{1} = 1$$

or

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

*Students may have derived this law previously when subtracting powers, but it is not presented formally until now.*
### Laws of Powers: Part 6

1. \( \frac{2^3}{2^3} = 1 \)

2. \( \frac{x^2}{x^2} = 1 \)

3. \( \frac{y^4}{y^1} = 1 \)

4. \( \frac{(2x)^3}{2x^2} = \frac{8x^3}{2x^2} = 4 \)

5. \( \frac{(xy)^2}{xy^2} = \frac{x^2y^2}{xy^2} = x \)

6. \( \frac{(x^3y^4)^2}{x^3y^4} = \frac{x^{12}y^4}{x^3y^4} = x^9 \)
In problem 9, watch that students do not cancel (reduce) before raising to the power:

\[
\frac{(x^3 y^2)^4}{(x^3 y^4)^2} = \frac{1}{y^4}
\]
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Assignment for $8 an Hour Problem

Name: _______________________

You are working as a math tutor at a learning center, making $8 an hour. Answer the following questions in complete sentences.

If you work 5 hours each day, how much money will you make in:

one week (5 days)?

**The amount earned for five hours per day for five days at $8 per hour is $200.**

one month (20 days)?

**The amount earned for five hours per day for twenty days at $8 per hour is $800.**

one year (240 days)?

**The amount earned for five hours per day for 240 days at $8 per hour is $9,600.**

How long will it take you to save enough money to buy:

a computer game machine costing $169?

**It will take about 21.1 hours or 4 to 5 days of work to buy a game machine costing $169.**

a computer costing $999?

**It will take about 125 hours or 25 days of work to buy a computer costing $999.**

a pair of pumps costing $119?

**It will take about 15 hours or 3 days of work to buy a pair of pumps costing $119.**

a television costing $249?

**It will take about 31.1 hours or 6 to 7 days of work to buy a television costing $249.**
If you got a raise of $0.20 per hour, how much more money would you make:

each week?

*Earnings would be $5 more per week.*

each month?

*Earnings would be $20 more per month.*

each year?

*Earnings would be $240 more per year.*
Assignment for Number Patterns

Name: ____________________________

For each problem, use complete sentences to answer three questions:

What do you know about the answer to the problem before you begin?
What is the solution to the problem? Show your work.
Does your solution make sense? Why or why not?

You have 45 pounds of dog food, and you must divide it among 60 dogs. How many pounds will each dog receive?

1. The given information is the total number of dogs and the total number of pounds of food. There are more dogs than pounds of food, so the amount will be less than a pound.
2. Each dog will receive \(\frac{3}{4}\) of a pound.

\[
\frac{45}{60} = .75 = \frac{3}{4}
\]

3. The answer makes sense but really depend on the size of the dogs.

Two out of every nine teachers are younger than 30. If there are 144 teachers in this building, how many are younger than 30?

1. Two out of every nine teachers are younger than 30 and there are 144 teachers, so about \(\frac{1}{3}\) of teachers should be younger than 30.
2. Thirty-two teachers of the 144 in the building are younger than 30.

\[
\frac{2}{9} = \frac{x}{4}
\]

\[x = 32\]

3. The answer makes sense.
Only four out of every five baseball players have batting averages over 0.200. If there are 750 baseball players, how many have batting averages less than 0.200?

1. Four out of every five players have batting averages over 0.200 and there are 750 players, so about 80 to 90% of the players will have a batting average of over 200.

2. Since 600 players have batting averages over 0.200, 150 players have averages less than 0.200.

\[
\frac{4}{5} = \frac{x}{750}
\]

\[x = 600\]

\[750 - 600 = 150\]

3. The answer makes sense but may not be accurate since it does not account for any players who have a batting average of exactly 0.200.
Assignment for Finding the 10\textsuperscript{th} Term

Name: ____________________________

For each problem, use complete sentences to answer three questions:

What do you know about the answer to the problem before you begin?
What is the solution to the problem? Show your work.
Does your solution make sense? Why or why not?

Six partners lost $540 in Las Vegas. If they divide their losses evenly, how much will it cost each partner?

1. Since 6 people lost money totaling $540, the amount of loss per person will be less than $100.
2. Each partner will lose $90.
\[
\frac{540}{6} = 90
\]
3. The answer makes sense.

There are 2,500,000,000 people in the world. If two-thirds of the people in the world live in poverty, how many people live in poverty?

1. If there are 2.5 billion people in the world and 2/3 live in poverty, the result should be about 2/3 of 25 or 16.5.
2. Approximately 1,650,000,000 people live in poverty.
\[
\left(\frac{2}{3}\right)\left(2,500,000,000\right) = 1,650,000,000
\]
3. The answer makes sense. (Students may make comments on the magnitude.)
Half of all the employees in a company have profit sharing. The total profits for the company are $250,000. If each employee with profit sharing receives an equal share of the profit and there are 1,250 employees in the company, how much will each receive?

1. There are 1,250 employees in the company and each shares $250,000 equally, so 625 people divide up the profits; therefore, the total should be about $400. Alternatively, if 1,250 employees have profit sharing then each receives $200.

2. Each employee’s share of the profit is $400.

\[
\frac{250,000}{625} = 400
\]

3. The answer makes sense.
Assignment for Soaps and Bath Beads

Name: ________________________________

1. On one particular Appletree & Elizabeth production line, the ratio of large sized boxes to medium sized boxes is 7 to 5. There are exactly 342 bath beads that must be placed in these boxes. How many of each type of gift box can be created? Show your work and explain your answer.

*There are 12 bath beads in each large box and there are 6 bath beads in each medium sized box.*

*For every 7 large boxes there are 5 medium boxes, which means that for every 84 (12*7) beads in the large boxes, there are 30 (6*5) beads in the medium boxes.*

*Thus, there are 114 (84+30) beads for every set of 7 large and 5 medium sized boxes.*

*If the total number of available beads is 342, then there must be 3 sets of the 7 large and 5 medium boxes. (342/114 = 3)*

*This means there are 21 (7*3) large boxes, and 15 (5*3) medium boxes.*

2. How many scented soaps are needed on the production line to fill the boxes? Make sure to show your work and explain your answer.

*There are 6 soaps in every large box. If there are 21 large boxes, there are 126 (21*6) soaps in the large boxes.*

*There are 4 soaps in every medium box. If there are 15 medium boxes, there are 60 (15*4) soaps in the large boxes.*

*This makes a total of 186 (126 + 60) soaps altogether in the 21 large boxes and 15 medium boxes.*
 Assignment for The Consultant Problem

Name: ________________________

A friend of your sister consults as a landscape designer and charges $67.50 per hour. Complete this table.

<table>
<thead>
<tr>
<th>Units</th>
<th>Hours Worked</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>hours</td>
<td></td>
<td>dollars</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>67.50</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>135.00</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>337.50</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>540.00</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>675.00</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>742.50</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>1,012.50</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>1,215.00</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>1,350.00</td>
</tr>
</tbody>
</table>

Would a line graph be useful to illustrate the information in this table? Why or why not?

Yes because it would provide a sense of the rate at which the amount earned increases with the number of hours worked. It would also indicate the amount earned for any number of hours based on the upper and lower bounds.
Construct a line graph on this grid. Include all the information from the table. Label the graph clearly. Your teacher will give your group an overlay, so that the whole class can see your graph.

Use your graph to answer the following questions. Write your answers in complete sentences.

How much will your sister’s friend make if he works:

10 hours?  **He will earn $675.00 in 10 hours.**

22 hours?  **He will earn $1485.00 in 22 hours.**

Write a sentence describing how you found these answers.

**Sample:**  *I found the number of hours worked on the horizontal axis. I traced this coordinate up to find the corresponding y-coordinate on the line. The y-coordinate tells the amount of money earned.*
How much will your sister’s friend make if he works:

- 3 hours 30 minutes? Your sister’s friend will make $236.25. (3.5*67.5)
- 5 hours 12 minutes? Your sister’s friend will make $351.00. (5.2*67.5)
- 7 hours 45 minutes? Your sister’s friend will earn $523.13.
- 8 hours 40 minutes? Your sister’s friend will earn $584.55.

Write a sentence describing how you found these answers.

First, the minutes must be converted to a part of an hour. Then, multiply the number of hours worked by the hourly rate, which is $67.50.

How many hours will he have to work to make:

- $1,000? Your sister’s friend will have to work 14.81 hours to make $1000.
- $500? He will have to work 7.41 hours to make $500.
- $33,000? He will have to work 488.89 hours to earn $33,000.

Write a sentence describing how you found these answers.

Divide the amount of money your sister’s friend will earn by the hourly wage of $67.50 to calculate the number of hours he will need to work.

Remember that mathematicians spend a great deal of time looking for number patterns. Use complete sentences to describe the number pattern in your table.

The pattern shows that for each hour your sister’s friend works, he earns $67.50 more. When the numbers increase by 1 in the hours column they will increase by 67.5 in the earnings column.

If your sister’s friend worked \( H \) hours, what is the expression for finding his total earnings?

\[ $67.50 \times H \]
Assignments
Assignment for U.S. Shirts

Name: ____________________________

You are going on vacation by car. You will be traveling at an average speed of 50 miles per hour.

Make a table showing how far you will travel in various time periods from 1 to 20 hours. Use this information to construct a line graph and find the algebraic expression.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Hours Driven</th>
<th>Total Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>hours</td>
<td>miles</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>550</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1000</td>
</tr>
</tbody>
</table>

Algebraic Expression: \(50T\)
Assignment for Hot Shirts

Name: __________________________

You are going on vacation by airplane. You will be traveling at an average speed of 325 miles per hour.

Make a table showing how far you will travel in various time periods from 1 to 20 hours. Use this information to construct a line graph and find the algebraic expression.

<table>
<thead>
<tr>
<th>Time in the Air (hours)</th>
<th>Distance Traveled (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 325t )</td>
</tr>
<tr>
<td>1</td>
<td>325</td>
</tr>
<tr>
<td>2</td>
<td>650</td>
</tr>
<tr>
<td>5</td>
<td>1625</td>
</tr>
<tr>
<td>11</td>
<td>3575</td>
</tr>
<tr>
<td>15</td>
<td>4875</td>
</tr>
<tr>
<td>18</td>
<td>5850</td>
</tr>
<tr>
<td>20</td>
<td>6500</td>
</tr>
</tbody>
</table>

Algebraic Expression: \( 325T \)
Assignment for Comparing U.S. and Hot Shirts

Name: ______________________

A friend is employed at a company where she makes $9 an hour.

Complete this table, including the algebraic expression. Draw a line graph displaying her total earnings and clearly label the axes.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Number of Hours</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Hours</td>
<td>Dollars</td>
</tr>
<tr>
<td>Expression</td>
<td>$H$</td>
<td>$9H$</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>162</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>180</td>
<td></td>
</tr>
</tbody>
</table>

Algebraic Expression: $9H$
Assignment for Finding the Nth Term

Name: ________________________________
For each problem, use complete sentences to answer three questions:

What do you know about the answer to the problem before you begin?

What is the solution to the problem? Show your work.

Does your solution make sense? Why or why not?

Five partners made a profit of $2,540 on their enterprise. If the profits are to be divided evenly, how much will each partner receive?

Before beginning the problem, we know there are five people dividing a total of $2,540, so that Jack will get about $500.

Each partner receives a total of $508. \[ \frac{2540}{5} = \$508. \]

The solutions do make sense because it is very close to the estimated answer.

There are 250 million people in the United States. If one out of every ten people has an annual income over $60,000, how many people make less than $60,000 per year?

If there are 250 million people and 1 out of every 10 has an income over $60,000, then 9 out of 10 have incomes under $60,000.

\[ \frac{9}{10} = \frac{x}{250,000,000} \]

\[ x = 225,000,000 \text{ people will incomes under } \$60,000. \]

The answer does make sense, but it includes all of the people who are too young to work, so it is inflated somewhat.

Three out of every five students are in a certain club, and each club member is to receive an equal share of the profits. If there are 1,000 students and the profits total $3,600, how much will each club member receive?

If three of every five are in a club and there are 1000 students total, then about 600 are in the club. If $3600 is divided among 600 people, each person will get a small amount.

\[ \frac{3600}{600} = \$6 \]

The answer makes sense for the many people who are in the club.
Assignment for Handshake Problem

Name: ____________________________

You have been asked to help schedule the games in your younger sister’s soccer league. There are 8 teams in the league. You have been asked to determine the total number of games that must be played so that every team will play every other team exactly once.

Someone suggests that to find the answer, you first try to figure out the total number of games for a league that has three teams, then four teams, and so on.

You must make a written presentation to the league officers. The report should include a description of how you arrived at your answer. Write the report below.

_Students should use the strategy they used in the Handshake Problem. They may also use simpler problems as suggested in the directions, using 3 teams in the league, then 4 teams and so on up to 8 teams._

_If n=8, then the number of games will be 7+6+5+4+3+2+1+0 = 28._

_Another strategy students might use is to list the games. For example:_

_team 1 plays team 2_

_team 1 plays 3_

_1 plays 4_

...

_Student might also create a table, grid or picture to connect the teams as they play each other._

_No matter the strategy used, the final written report should include the total number of games as 28. The final report should also include the description of whatever strategy the student used to deduce the answer of 28 games._
Assignment for Gauss’ Solution

Name: __________________________

For each problem, perform the following three tasks:

Before solving the problem, it is known that the number of students on the team will be less than 100 because only 1 out of 20 is on the team.

1000/20 = 50

There will be 50 people on the basketball team.

A recent advertisement offered a 25% discount on athletic shoes. If the regular price is $60, what is the sale price?

Before solving the problem, it is known that the price of the shoes is less than $60.

0.25 * 60 = 15

60 – 15 = 45

The shoes will cost $45 on sale.

A recent census reported that 10% of the people in the United States have incomes over $70,000 and 30% of the people have incomes less than $10,000. If there are 250 million people in the United States, how many are included in each of these two groups?

Before solving the problem, it is known that the number of people with incomes about $70,000 is less than the number of people with incomes under $10,000.

10% of 250 million = 25 million

30% of 250 million = 75 million

There are 25 million people with incomes above $70,000, and 75 million with incomes under $10,000.
Assignment for Left-Handed Learners

Name: __________________________

1. Even after performing this experiment in class, you are just not satisfied. That evening, you log onto the internet and find that approximately 12% of the population is left-handed. Assuming this is an accurate fact, how many left-handed people should there be in your math class? in your school? your city? state? How many left-handed people live in the United States? If the world population is approximately 6.1 billion, how many people are left-handed?

   Number of left-handed people = 12% of (Total Population of class, school, city or state)

   Worldwide = 732,000,000 people are left handed

2. Your teacher volunteers that when she was in high school, her class did the same type of survey and found that 17 of the 250 students in the graduating class were left-handed. The rest were right-handed. Using only the data from your teacher’s high school class and from your current class, has the number of left-handed students increased? decreased? remained the same? If a change had occurred, do you think this change is “significant”? Explain.

   \[ \frac{17}{250} = x\% \]

   \[ .068 = x\% \]

   \[ 6.8 = x \]

   The percent of left-handed people is almost double today; this statistic fits with the fact that innately left-handed individuals are not being forced to use their right hand. Since the percent is almost double, this seems to be a significant change.
Assignments
Assignment for Shadows and Proportions

Name: ________________

In the movie, *Honey, I Shrunk the Kids*, Rick Moranis is a hair-brained scientist who has created a shrinking machine. When the machine accidentally gets activated, his kids get shrunk.

In the movie, there are a number of shots of the kids in a variety of predicaments.

In one shot, a child appears to be \( \frac{1}{2} \) the length of the father’s nose, while the child climbs onto the father’s head.

In another shot, the same child’s arm appears to be about \( \frac{1}{6} \) the diameter of a Cheerio while the child is floating in a bowl with cereal and milk.

Are these two pictures consistent if the original height of the child is approximately 4 feet and the child’s arm is about 2 feet long?

Fully analyze this situation and justify any conclusions you make about the shrunken child.

*An adult nose is about 2 inches, which means the child is about 1 inch tall.*

*The diameter of a Cheerio is about \( \frac{1}{2} \) inch, so the child’s arm is about \( \frac{1}{12} \) inch.*

*These two pictures are not consistent. If these pictures were consistent, the child’s arm would be about \( \frac{1}{2} \) inch, not \( \frac{1}{12} \) inch.*

In the movie promotion, the kids were supposedly shrunken to only \( \frac{1}{4} \) inch tall. Does that agree with your calculations above?

*This does not agree with either of the two calculations in the above problem.*

Problem adapted from *Measurement in the Middle Grades*, NCTM, 1994
Assignments
Assignment for Making Punch

A typical male lion is about 2.1 m long and stands about 90 cm tall at the shoulder. He also weighs about 240 kg. The Sphinx, located near Cairo, Egypt, is a depiction of a man – lion combination. The Sphinx is about 73.2 meters long. This includes the paws, which are 15.3 meters long. If there existed a real lion that were the size of the Sphinx, how tall would you expect it to be? Show your work and explain any assumptions you made.

**Assuming the 2.1 meters as the length of a lion does not include the paws, then the Sphinx is 57.9 meters.**

\[
\frac{2.1 \text{ meters}}{0.9 \text{ meters}} = \frac{57.9 \text{ meters}}{x \text{ meters}}
\]

\[x = 24.8 \text{ meters}\]

**If the lion were the size of the Sphinx, it would be 24.8 meters tall**

If you make the assumption that the Sphinx’s paws are in proportion to the rest of its body, how long is the paw on an actual male lion?

\[15.3 \text{ meters is about } \frac{1}{5} \text{ of 73.2 meters, so the lion’s paw should be about } \frac{1}{5} \text{ of the size of its body.}\]

\[\frac{1}{5} \text{ of 2.1 meters} = 0.42 \text{ meters or 42 centimeters}\]

Challenge: If the Sphinx were a real lion, how much would you expect it to weigh? (Hint: Is weight dependent only upon height?)

**Weight is dependent upon all three dimensions of the lion, so the ratio of weights between the Sphinx and the lion is the cube of the ratio of lengths**

\[\frac{57.9}{2.1} = 27.57\]

\[27.57^3 = 20959.35, \text{ so the weight of the lion must be multiplied by 20959 to calculate the weight of the Sphinx.}\]

\[240 \times 20,959 \text{ is about 5 million pounds.}\]

Adapted from World’s Largest Math Event 4
Assignment for TV News Ratings

Name: ______________________

Two out of every three four-year-olds like crackers.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total 4-year-olds</th>
<th>4-year-olds Who Like Crackers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>people</td>
<td>people</td>
</tr>
<tr>
<td>Expression</td>
<td>$T$</td>
<td>$(2/3)T$</td>
</tr>
<tr>
<td>$3$</td>
<td>$2$</td>
<td></td>
</tr>
<tr>
<td>$30$</td>
<td>$20$</td>
<td></td>
</tr>
<tr>
<td>$300$</td>
<td>$200$</td>
<td></td>
</tr>
<tr>
<td>$3000$</td>
<td>$2000$</td>
<td></td>
</tr>
<tr>
<td>$30000$</td>
<td>$20000$</td>
<td></td>
</tr>
<tr>
<td>$300,000$</td>
<td>$200,000$</td>
<td></td>
</tr>
</tbody>
</table>

Total 4-Year-Olds (people) vs. Thousands

Draw a line graph illustrating the situation and clearly label the axes.
Assignment for Truck Life

Name: ______________________

Three out of four algebra students will receive an A, B, or C.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Algebra Students</th>
<th>Those Who Receive an A, B, or C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>People</td>
<td>People</td>
</tr>
<tr>
<td>Expression</td>
<td>P</td>
<td>3/4P</td>
</tr>
<tr>
<td>100</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>375</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>450</td>
<td></td>
</tr>
</tbody>
</table>

Total Algebra Students (students) vs. Algebra Students Who Receive an A, B, or C (students)
Assignments
Assignment for Women at a University

Name: _______________________

One out of every fifty drivers will have an automobile accident this year.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Drivers</th>
<th>Number of Drivers Who Have Had an Accident</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units Expression</td>
<td>People</td>
<td>People</td>
</tr>
<tr>
<td></td>
<td>$P$</td>
<td>$\frac{1}{50}P$</td>
</tr>
<tr>
<td>1,000,000</td>
<td>20,000</td>
<td></td>
</tr>
<tr>
<td>5,000,000</td>
<td>100,000</td>
<td></td>
</tr>
<tr>
<td>10,000,000</td>
<td>200,000</td>
<td></td>
</tr>
<tr>
<td>75,000,000</td>
<td>1,500,000</td>
<td></td>
</tr>
<tr>
<td>100,000,000</td>
<td>2,000,000</td>
<td></td>
</tr>
<tr>
<td>150,000,000</td>
<td>3,000,000</td>
<td></td>
</tr>
</tbody>
</table>

![Graph illustration](image-url)
Assignment for Plastic Containers

Name______________________________

The force of gravity on the surface of the Moon is about \( \frac{1}{6} \) of the force of gravity on the Earth’s surface.

1. How high can you jump vertically? (If you do not know, estimate by standing next to a wall and jumping straight up and touching the wall at the top of your jump.)
   
   *Answers will vary.*

2. If you were on the surface of the Moon, how high could you jump? Show your work and explain what you did to solve.
   
   *Answers will vary, but should be 6 times the answers from question number 1.*

3. How far can you long jump (with a running start and without a running start)? (If you don’t know, mark a starting point on the floor and jump as far as you can.)
   
   *Answers will vary from about 4 feet for a standing long jump to much farther for one with a running start.*

4. How far could you long jump on the surface of the moon – with and without a running start?
   
   *Answers will vary and should be 6 times as far as the previous two answers given.*

5. In track and field, hurdles are about 0.91 meters high for women and 1.07 meters. In order to make the event fair, how high would the hurdles have to be on the Moon?
   
   *The hurdles would have to be 6 times as high.*

   \((0.91)\times(6) = 5.46\) meters for women and
   \((1.07)\times(6) = 6.42\) meters high for men.

Adapted from World’s Largest Math Event 3
Assignment for Taxes Deducted from Your Paycheck

Name:__________________________

In Pennsylvania there is a 6% sales tax on the total price of items sold.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

Sample Answer:

<table>
<thead>
<tr>
<th>Total Price</th>
<th>Sales Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$0.06(C)$</td>
</tr>
<tr>
<td>10</td>
<td>0.60</td>
</tr>
<tr>
<td>15</td>
<td>0.90</td>
</tr>
<tr>
<td>25</td>
<td>1.50</td>
</tr>
<tr>
<td>37</td>
<td>2.22</td>
</tr>
<tr>
<td>42</td>
<td>2.52</td>
</tr>
<tr>
<td>86</td>
<td>5.16</td>
</tr>
<tr>
<td>120</td>
<td>7.20</td>
</tr>
</tbody>
</table>

![Graph showing sales tax vs. total price]
Assignment for Tipping in a Restaurant

Name: ______________________

Approximately 76% of ninth-graders in the Pittsburgh Public Schools will graduate from high school in four years.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

Sample Answer:

<table>
<thead>
<tr>
<th>Ninth-Graders</th>
<th>Those Who Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>People</td>
<td>People</td>
</tr>
<tr>
<td>$G$</td>
<td>$0.76G$</td>
</tr>
<tr>
<td>350</td>
<td>266</td>
</tr>
<tr>
<td>600</td>
<td>456</td>
</tr>
<tr>
<td>900</td>
<td>684</td>
</tr>
<tr>
<td>1500</td>
<td>1140</td>
</tr>
<tr>
<td>2500</td>
<td>1900</td>
</tr>
<tr>
<td>3000</td>
<td>2280</td>
</tr>
</tbody>
</table>

Ninth-Graders (people) vs. Those Who Graduated (people)
Assignments
Assignment #1 for Earning Sales Commission

Name: ______________________

A television salesperson is paid $150 per week, plus a commission of 5% on total sales.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

<table>
<thead>
<tr>
<th>Total Sales</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollars</td>
<td>Dollars</td>
</tr>
<tr>
<td>$S$</td>
<td>$150 + .05S$</td>
</tr>
<tr>
<td>500</td>
<td>175</td>
</tr>
<tr>
<td>800</td>
<td>190</td>
</tr>
<tr>
<td>1000</td>
<td>200</td>
</tr>
<tr>
<td>3000</td>
<td>300</td>
</tr>
<tr>
<td>7500</td>
<td>525</td>
</tr>
<tr>
<td>10000</td>
<td>650</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between total sales and total earnings]
Assignment #2 for Earning Sales Commission

Name: ______________________

A television salesperson is paid a commission of 5% on total weekly sales plus a $500 base salary per week.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

<table>
<thead>
<tr>
<th>Total Weekly Sales</th>
<th>Total Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5</td>
<td>500 + .05S</td>
</tr>
<tr>
<td>620</td>
<td>531</td>
</tr>
<tr>
<td>780</td>
<td>539</td>
</tr>
<tr>
<td>1000</td>
<td>550</td>
</tr>
<tr>
<td>1240</td>
<td>562</td>
</tr>
<tr>
<td>1460</td>
<td>573</td>
</tr>
<tr>
<td>2000</td>
<td>600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Weekly Sales (dollars)</th>
<th>Total Earnings (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>600</td>
<td>600</td>
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<td>700</td>
<td>700</td>
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<td>800</td>
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<td>900</td>
<td>900</td>
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<td>1000</td>
<td>1000</td>
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<td>1100</td>
<td>1100</td>
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<td>1200</td>
<td>1200</td>
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<td>2200</td>
<td>2200</td>
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<tr>
<td>2300</td>
<td>2300</td>
</tr>
<tr>
<td>2400</td>
<td>2400</td>
</tr>
<tr>
<td>2500</td>
<td>2500</td>
</tr>
</tbody>
</table>
Assignments
Assignment for Rent a Car from Go-Go Car Rentals

Name: _________________________

A computer rents for $2.50 per day, with a $25 fee for insurance.

1. Find the total cost of renting the computer for:
   a. 20 days
   
   **The total cost of renting the computer for 20 days is $75.00.**
   
   b. 33 days
   
   **The total cost of renting the computer for 33 days is $107.50.**
   
   Write a complete sentence describing how you found these answers.
   
   **Multiply the number of days by the cost per day and then add the insurance cost.**

2. Write an algebraic equation for the total cost of renting a computer.
   
   Total rent = \( 2.50x + 25 \) ____________

3. For how many days can you rent the computer for $67.50?
   a. Write an equation that states that the rental cost is $67.50.
      
      \( 2.50x + 25 = 67.50 \)
   
   b. Solve this equation, showing your work.
      
      \( 2.50x = 42.5 \)
      \( x = 17 \)
   
   c. Write your answer to question 3 in a complete sentence.
      
      **You can rent the computer for 17 days for a cost of $67.50.**
   
   d. Write a complete sentence describing how you solved the equation.
      
      **Subtract the insurance cost from the rental cost and then divide by the cost per day.**
4. For how many days can you rent the computer for $202.50?
   a. Write an equation that states that the rental cost is $202.50.
      \[ 2.50 \times x + 25 = 202.50 \]
   b. Solve this equation, showing your work.
      \[ 2.50 \times x = 177.5 \]
      \[ x = 71 \]
   c. Write your answer to question 4 in a complete sentence.
      You can rent the computer for 71 days for a cost of $202.50.
   d. Write a sentence describing how you solved the equation.
      Subtract the insurance cost from the total rental cost. Then divide by the cost per day.

5. For how many days can you rent the computer for $92.50?
   a. Write an equation that states that the rental cost is $92.50.
      \[ 2.50 \times x + 25 = 92.50 \]
   b. Solve this equation, showing your work.
      \[ 2.50 \times x = 67.5 \]
      \[ x = 27 \]
   c. Write your answer to question 5 in a complete sentence.
      You can rent the computer for 27 days for a cost of $92.50.
   d. Write a sentence describing how you solved the equation.
      Subtract the insurance cost from the rental cost. Divide the result by the cost per day.
6. Use the information from questions 1–5 to complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Total Rental Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>days</td>
<td>dollars</td>
</tr>
<tr>
<td>Expression</td>
<td>D</td>
<td>$2.5D + 25$</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>75.00</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>107.50</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>67.50</td>
</tr>
<tr>
<td></td>
<td>71</td>
<td>202.50</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>92.50</td>
</tr>
</tbody>
</table>

7. Graph the equation for this problem situation.
Assignment for Rent a Car from Wreckem Car Rentals

Name: ____________________________

A car rents for $23 per day, plus an insurance fee of $32.

1. What does it cost to rent this car for:
   a. 10 days?
      
      **It costs $262 to rent the car for 10 days.**
   b. 37 days?
      
      **The cost to rent the car for 37 days will be $883.**

   Write a sentence describing how you found these answers.
   
   **Multiply the number of days by the cost per day. Then, add the insurance cost to get the total.**

2. Write the algebraic rule for this problem situation.

   Car rental = **23 x + 32**

3. Write an equation you can use to find out the number of days for which you can rent this car for $331. Solve this equation, showing your work, and write your answer in a complete sentence. Write a complete sentence describing how you solved the equation.

   23 x + 32 = 331
   
   23 x = 299
   
   x = 13

   **For a cost of $331, you can rent a car for 13 days.**
4. Write an equation you can use to find out the number of days for which you can rent this car for $149. Solve this equation, showing your work, and write your answer in a complete sentence. Write a complete sentence describing how you solved the equation.

\[
23x = 32 = 149 \\
23x = 117 \\
x = 5.09
\]

You can rent the car for 5 days for a little under $149.

5. Write an equation you can use to find out the number of days for which you can rent this car for $279. Solve this equation, showing your work, and write your answer in a complete sentence. Write a complete sentence describing how you solved the equation.

\[
23x = 32 = 279 \\
23x = 247 \\
x = 10.74
\]

You can rent a car for 10 days for under $279.

6. Use the information from questions 1–5 to complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>days</td>
<td>dollars</td>
</tr>
<tr>
<td>Expression</td>
<td>t</td>
<td>(23t + 32)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>262</td>
</tr>
<tr>
<td></td>
<td>37</td>
<td>883</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>331</td>
</tr>
<tr>
<td></td>
<td>5.09</td>
<td>149</td>
</tr>
<tr>
<td></td>
<td>10.74</td>
<td>279</td>
</tr>
</tbody>
</table>
7. Graph the equation for this problem situation.
Assignment for Rent a Car from Good Rents Car Rentals

Name: ____________________________

A caterer charges $12.50 per person, plus a $150 fee for the hall rental.

1. Write the algebraic rule for this problem situation.

   Cost of the caterer = 12.5x + 50

2. What would it cost to cater a party for:
   a. 35 people?

   12.5(35) + 150 = 587.5  It will cost $587.50 to cater a party for 35 people.
   
   b. 97 people?

   12.5(97) + 150 = 687.5  It will cost $1362.50 to cater a party for 97 people.

3. Write an equation you can use to find out how many people you can invite to a party that costs $687.50. Solve this equation, showing your work, and write your answer in a complete sentence.

   12.5x = 150 = 687.50
   12.5x = 537.5
   x = 43

   For a cost of $687.50, you can invite 43 people.

4. Write an equation you can use to find out how many people you can invite to a party that costs $1287.50. Solve this equation, showing your work, and write your answer in a complete sentence.

   12.5x = 150 = 1286.5
   12.5x = 1137.5
   x = 91

   For a cost of $1287.50, you can cater a party for 91 people.
5. Write an equation you can use to find out how many people you can invite to a party that costs $2000. Solve this equation, showing your work, and write your answer in a complete sentence.

\[ 12.5x + 150 = 2000 \]
\[ 12.5x = 1850 \]
\[ x = 148 \]

You can invite 148 people for a cost of $2000.

6. Use the information from questions 1–5 to complete this table.

<table>
<thead>
<tr>
<th>Number of people</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( 12.5x + 150 )</td>
</tr>
<tr>
<td>35</td>
<td>587.50</td>
</tr>
<tr>
<td>97</td>
<td>1362.50</td>
</tr>
<tr>
<td>43</td>
<td>687.50</td>
</tr>
<tr>
<td>91</td>
<td>1287.50</td>
</tr>
<tr>
<td>148</td>
<td>2000.00</td>
</tr>
</tbody>
</table>

7. Graph the equation for this problem situation.
Assignment for Move a Sand Pile

Name: __________________________

You are riding in a hot-air balloon that is 525 feet in the air. It is descending at the rate of $10\frac{1}{2}$ feet per minute.

1. How many feet high will it be in 10 minutes?
   
   **In 10 minutes the balloon will be at a height of 420 feet.**
   
   $$525 - 10.5(10)$$

2. How high will it be in $20\frac{1}{2}$ minutes?
   
   **In 20.5 minutes, the balloon will be at a height of 309.75 feet.**
   
   $$525 - 10.5(20.5)$$

3. How high was it 9 minutes and 30 seconds ago?
   
   **The balloon was at a height of 624 feet and 9 inches, $9\frac{1}{2}$ minutes ago.**
   
   $$525 - 10.5(-9.5)$$

4. How high will it be in 34 minutes?
   
   **In 34 minutes, the balloon will be at a height of 168 feet.**
   
   $$525 - 10.5(34)$$

5. When will it land on the ground?
   
   **The balloon will land in 50 minutes.**
   
   
   $$0 = 525 - 10.5 x$$
   
   $$-525 = -10.5 x$$
   
   $$50 = x$$
6. Make a table of values, including an algebraic expression. Then construct a graph for this problem situation.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Height (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>$525 - 10.5x$</td>
</tr>
<tr>
<td>10</td>
<td>420</td>
</tr>
<tr>
<td>20.5</td>
<td>309.75</td>
</tr>
<tr>
<td>-9.5</td>
<td>624.75</td>
</tr>
<tr>
<td>34</td>
<td>168</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between time and height.](image)
Assignment for Engineer a Highway

Name: ______________________

Materials to construct birdhouses cost you $50. You plan to sell the birdhouses for $7 each.

Make a table of values, including the algebraic expression, and construct a graph for this problem situation.

Table of Values:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Birdhouses Sold</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>birdhouses</td>
<td>$</td>
</tr>
<tr>
<td>Expression</td>
<td>b</td>
<td>7b – 50</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-36</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-22</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>-8</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>34</td>
</tr>
</tbody>
</table>

Graph:
Assignment for Constructing Customized Rain Gutters

Name: ________________________

Temperatures tend to be similar when observed at the same place and time each year. In a town in Alaska, the average temperature for the fifth of January is -10° Celsius. Each day after that, the average temperature increases at a rate of one-fourth of a degree.

1. What would be the predicted temperature for 30 days after the 5th of January?

   The predicted temperature 30 days after January 5th would be -2.5° Celsius.

   \[-10 + (0.25)(30)\]

2. What would be the predicted temperature for 75 days after the 5th of January?

   The predicted temperature would be 8.75° Celsius, 75 days after January 5th.

   \[-10 + (0.25)(75)\]

3. What would be the predicted temperature for St. Valentine's Day? (Remember that there are 31 days in January.)

   The predicted temperature on Valentine’s Day is 0° Celsius.

   \[-10 + (0.25)(40)\]

4. What would be the predicted temperature for New Year’s Day?

   The predicted temperature on New Year’s Day (361 days) would be 80.25° Celsius.

   \[-10 + (0.25)(361)\]

5. What would the temperature have been on the previous Christmas Day?

   The temperature on the previous Christmas Day (11 days ago) would have been 12.75° Celsius.

   \[-10 + (0.25)(-11)\]
6. Make a table of values, including an algebraic expression. Then construct a graph for this problem situation.

<table>
<thead>
<tr>
<th>Days After January 30&lt;sup&gt;th&lt;/sup&gt;</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>days</td>
<td>degrees Celsius</td>
</tr>
<tr>
<td>x</td>
<td>-10 + .25x</td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>30</td>
<td>-2.5</td>
</tr>
<tr>
<td>75</td>
<td>8.75</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>361</td>
<td>80.25</td>
</tr>
<tr>
<td>-11</td>
<td>-12.75</td>
</tr>
</tbody>
</table>
Assignment for Functions

Name: _______________________

For each problem situation, use function notation to express the relationship between the variables symbolically, and state the domain and range of the function.

1. $8 an Hour Problem

In this problem situation, you were being paid $8 an hour to assemble furniture at Pat-E-Oh Furniture. You explored the relationship between the number of hours worked during the week and your total earnings for that week.

a. Designate a letter for the independent variable of the function, and designate a letter for the function name. Use function notation to express the relationship between the variables, using the letters you designated.

\[ x = \text{time in hours} \]
\[ f(x) = \text{total earnings} \]
\[ f(x) = 8x \]

b. Based on the number of hours you personally are able to work during any given week, state the domain for this function.

Answers will vary. In general, domain will be from 0 hours to the maximum number of hours per week.

c. Based on the domain from part b above, state the range for this function. In other words, what are the least and the most “total weekly earnings” that you personally could achieve in a week?

The range will be from 0 to 8 times the maximum number of hours from part (b).
2. U.S. Shirts

In this problem situation, you had a job at U.S. Shirts, calculating the cost of various T-shirt orders. For each order, there was a one-time setup charge of $15 and a charge of $8 per shirt. You explored the relationship between the number of shirts ordered and the total cost of the order.

a. Designate a letter for the independent variable of the function, and designate a letter for the function name. Use function notation to express the relationship between the variables, using the letters you designated.

\[ x = \text{shirts ordered} \]
\[ c(x) = \text{cost} \]
\[ c(x) = 8x + 15 \]

b. Suppose U.S. Shirts sets a maximum of 2000 T-shirts per order. State the domain for this function using complete sentences.

The domain is from 0 to 2000 shirts.

c. State the range for this function.

The range is $15 to $16,015.
Assignment #1 for Solving and Graphing Linear Inequalities

Name: __________________________

Draw the graph of each inequality on the number line.

1. \( x \geq 5 \)
   ![Graph of \( x \geq 5 \)]

2. \( x < -3 \)
   ![Graph of \( x < -3 \)]

3. \( x \leq -1 \)
   ![Graph of \( x \leq -1 \)]

4. \( x > -6 \)
   ![Graph of \( x > -6 \)]

5. \( x \leq -1 \text{ or } x > 4 \)
   ![Graph of \( x \leq -1 \text{ or } x > 4 \)]

6. \( x > -5 \text{ and } x < 4 \)
   ![Graph of \( x > -5 \text{ and } x < 4 \)]

7. \( x < 5 \text{ and } x > -2 \)
   ![Graph of \( x < 5 \text{ and } x > -2 \)]

8. \( x < 8 \text{ and } x \geq -6 \)
   ![Graph of \( x < 8 \text{ and } x \geq -6 \)]
9. \( x > -2 \) or \( x \leq -6 \)

10. \( x \geq 3 \) or \( x < -6 \)

Solve the following inequalities:

11. \(-4x + 2 < 7\)
   \[-4x + 2 - 2 < 7 - 2\]
   \[-4x < 5\]
   \[-4x > \frac{5}{-4}\]
   \[x > -\frac{5}{4}\]

12. \(5x + 8 \geq 2\)
   \[5x + 8 - 8 \geq 2 - 8\]
   \[5x \geq -6\]
   \[x \geq -\frac{6}{5}\]

13. \(\frac{x}{3} > 30\)
   \[\frac{x}{3} \cdot 3 < 30 \cdot 3\]
   \[x < -90\]
Assignment #2 for Solving and Graphing Linear Inequalities

Name: _________________________________

Part 1

To find the ordered pairs on the coordinate plane that satisfy a linear inequality, you first find those that would make the inequality an equation and then decide which of the other two sets of ordered pairs satisfies the inequality.

The procedure that is usually most efficient is to:

- Graph the inequality as if it were an equation
- Find any ordered pair not on this line
- Substitute the ordered pair for the variables in the inequality. If it satisfies the inequality then it is from the solution set. If it does not satisfy the inequality, then the other set must be from the solution set

When the points on the line do not satisfy the inequality, you indicate this by drawing the line as a dashed line and then shade in the area of the coordinate plane that does satisfy the inequality.

Use this procedure to graph the solution set for each of these inequalities.

1.  \( y > 3x + 4 \)
2.  \( y > 3x \)
3. \( y > x - 4 \)

4. \( y < 2x - 8 \)

5. \( y > 2x - 5 \)

6. \( y < x - 2 \)

7. \( y < 6 \)

8. \( y > -2x \)
9. \( y > 2 \)

10. \( y < -3x + 2 \)

11. \( y < -x + 8 \)

12. \( y > -8 \)
Part 2

Now you will solve inequalities using the other two inequality symbols: ≤ and ≥.

≤ means **less than or equal to**

≥ means **greater than or equal to**

The three symbols, <, >, and ≠, are called *strict inequalities* because equality is not included.

Since the two new symbols are ≤, **less than or equal to**, and ≥, **greater than or equal to**, ordered pairs that make the inequality equal are in the solution set. To graph these solution sets, you indicate that the line is included by drawing a *solid* line and then *shading in* the rest of the solution set.

Graph the solution sets for each of the following linear inequalities.

1. \( y \geq -x + 6 \)
2. \( y \leq x + 2 \)
3. \( y \geq 3x + 4 \) 

4. \( y \leq x - 5 \) 

5. \( y \geq x + 4 \) 

6. \( y \leq 3x + 3 \) 

7. \( y \leq -x - 2 \) 

8. \( y \geq -x - 2 \)
9. \( y \leq -2x - 2 \)

10. \( y \geq 4x - 9 \)

11. \( y \leq 3x + 4 \)

12. \( y \geq 5x + 6 \)
Assignment for Widgets

Name: ___________________________

Widgets cost $7 each, with a shipping charge of $11 per order.

1. Write an algebraic rule for the total cost of an order.
   
   Total cost = $7x + 11$

2. Find the total cost of an order for:
   
   a. 12 widgets $7(12) + 11 = $95$
   
   b. 257 widgets $7(257) + 11 = $1810$

3. How many widgets can you order for $88?
   
   a. Write an equation that states that the cost of the order is $88.
      
      $7x + 11 = 88$
   
   b. Solve this equation, showing your work.
      
      $7x = 77$
      
      $x = 11$
   
   c. Write your answer to question 3 in a complete sentence.
      
      For a cost of $88, 11 widgets can be ordered.

4. How many widgets can you order for $1488?
   
   a. Write an equation that states that the cost of the order is $1488.
      
      $7x + 11 = 1488$
   
   b. Solve this equation, showing your work.
      
      $7x = 1477$
      
      $x = 211$
   
   c. Write your answer to question 4 in a complete sentence.
      
      For a cost of $1488, 211 widgets can be ordered.
5. How many widgets can you order for $6472?

   a. Write an equation that states that the cost of the order is $6472.
      \[ 7x + 11 = 6472 \]

   b. Solve this equation, showing your work.
      \[ 7x = 6461 \]
      \[ x = 923 \]

   c. Write your answer to question 5 in a complete sentence.
      \textit{923 widgets can be ordered for a cost of $6472.}
Assignments for Dumbbells

Name ____________________________

Dumbbells are sold by the pound. Dumbbells Unlimited sells dumbbells for $0.47 per pound, plus $12 per order for shipping and handling.

1. Write an algebraic rule for the total cost of an order of dumbbells.
   
   Total cost = \(0.47x + 12\)

2. Find the total cost of ordering:
   
   a. 120 pounds of dumbbells
      
      \(0.47(120) + 12\)
      
      $68.40
   
   b. 300 pounds of dumbbells
      
      \(0.47(300) + 12\)
      
      $153.00

3. How many pounds of dumbbells can you order for $30.80?
   
   a. Write an equation that states that the cost of the order is $30.80.
      
      \(0.47x + 12 = 30.80\)
   
   b. Solve this equation, showing your work.
      
      \(0.47x + 12 = 18.8\)
      
      \(x = 40\)
   
   c. Write your answer to question 3 in a complete sentence.
      
      40 pounds of dumbbells can be ordered for a cost of $30.80.
4. How many pounds of dumbbells can you order for $471.66?
   a. Write an equation that states that the cost of the order is $471.66.
   \[0.47x + 12 = 471.66\]
   b. Solve this equation, showing your work.
   \[0.47x = 459.66\]
   \[x = 978\]
   c. Write your answer to question 4 in a complete sentence.
   *978 pounds of dumbbells can be ordered for a cost of $471.66.*

5. How many pounds of dumbbells can you order for $48.13?
   a. Write an equation that states that the cost of the order is $48.13.
   \[0.47x + 12 = 48.13\]
   b. Solve this equation, showing your work.
   \[0.47x = 36.13\]
   \[x = 76.87\]
   c. Write your answer to question 5 in a complete sentence.
   *76 pounds of dumbbells can be ordered for a cost of $48.13.*
Assignment for Computer Time and Dumpster

Name: ______________________

A company has total assets of $675,000. They estimate that these assets are increasing at the rate of $7,500 per week.

1. What will this company's total assets be in ten weeks?
   
   In 10 weeks, total assets will be $750,000.

2. What were their total assets one year ago?
   
   One year ago, total assets were $285,000.

3. How many weeks will it take for their assets to double?
   
   Total assets will double to 1,350,000 in 90 weeks.

4. How many weeks will it take for their assets to reach $750,000?
   
   Total assets will reach $750,000 in 10 weeks.

5. How long ago were their assets zero?
   
   Total assets were zero 90 weeks ago.
1. Make a table of values, including an algebraic expression. Then construct a graph for this problem situation.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Assets</td>
</tr>
<tr>
<td>weeks</td>
<td>dollars</td>
</tr>
<tr>
<td>x</td>
<td>675,000 + 7,500x</td>
</tr>
<tr>
<td>0</td>
<td>675,000</td>
</tr>
<tr>
<td>10</td>
<td>750,000</td>
</tr>
<tr>
<td>-52</td>
<td>285,000</td>
</tr>
<tr>
<td>90</td>
<td>1,350,000</td>
</tr>
<tr>
<td>10</td>
<td>750,000</td>
</tr>
<tr>
<td>-90</td>
<td>0</td>
</tr>
</tbody>
</table>

Graph: [Graph of the line with labeled axes]
Assignment for Decorating the Math Lab

Name: ____________________________

You have $20 and are paying out $2 per day.

1. Find an algebraic equation for this situation.
   
   Algebraic Equation: $y = 20 - 2x$
   
   Define each variable in your equation by writing a short phrase that describes what the variable represents.
   
   (first variable) $x = \text{time (days)}$
   
   (second variable) $y = \text{money left (dollars)}$

2. Make a table with at least 5 values:

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Money Left (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>
3. Graph the equation for this problem situation:

![Graph of the equation](image)

Use your graph, your equation, or your table to answer questions 4 – 8. Write your answers in complete sentences.

4. How would you describe your graph?
   
   The graph is decreasing.

5. Where does the graph intersect the horizontal axis? Why?
   
   The graph intersects the horizontal axis at x = 10 because after 10 days all of the money is gone.

6. Where does the graph intersect the vertical axis? Why?
   
   The graph intersects the vertical axis at y = 20 because you start with $20.
7. What is the value of the second variable when:
   a. the first variable is 7?
      \[ y = 20 - 2(7) = 20 - 14 = 6 \]
   b. the first variable is -9?
      \[ y = 20 - 2(-9) \]
      \[ y = 20 + 18 \]
      \[ y = 38 \]

8. What is the value of the first variable when:
   a. the second variable is 16?
      \[ 20 - 2x = 16 \]
      \[ -2x = -4 \]
      \[ x = 2 \]
   b. the second variable is -20?
      \[ 20 - 2x = -20 \]
      \[ -2x = -40 \]
      \[ x = 20 \]

9. Write an equation that states that the second variable equals -7.5. Solve this equation for the first variable.
   \[ 20 - 2x = 7.5 \]
   \[ -2x = -12.5 \]
   \[ x = 6.25 \]

10. Find the value of the second variable when the first variable equals 236.
    \[ y = 20 - 2(236) \]
    \[ y = 20 - 472 \]
    \[ y = -452 \]

11. Do problems 9 and 10 make sense in this problem situation? Why or why not?
    Problem 9 does not make sense because only whole number days are applicable.
    Problem 10 could make sense if -452 is interpreted as being $452 in debt.
Assignments
Assignment for Selling Balloons

Name: __________________________

For each equation, determine whether its graph increases or decreases and find the rate of increase or decrease. Then find the intercepts, make a table of values, and sketch a graph of each equation.

**Equation:** \( y = 4x + 4 \)

Will the graph of this equation increase (*go up*) or decrease (*go down*) from left to right?

*The graph will increase from left to right.*

What are the intercepts?

- **x-intercept =** -1
- **y-intercept =** 4

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

*If \( x \) increases by 1, \( y \) will increase by 4.*

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.

![Graph of the equation](image-url)
Assignments

**Equation:** \( y = 4x + 12 \)

Will the graph of this equation increase (go up) or decrease (go down) from left to right?

*The graph will increase from left to right.*

What are the intercepts?
- \( x \)-intercept = \(-3\)
- \( y \)-intercept = \(12\)

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

*If \( x \) increases by 1, \( y \) increases by 4.*

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
Equation: \( y = -x + 5 \)

Will the graph of this equation increase (go up) or decrease (go down) from left to right?

*The graph will decrease from left to right.*

What are the intercepts?

\[ x\text{-intercept} = 5 \]

\[ y\text{-intercept} = 5 \]

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

*If \( x \) increases by 1, \( y \) decreases by 1.*

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
**Equation:** $y = 9x - 5$

Will the graph of this equation increase (go up) or decrease (go down) from left to right?

*The graph will increase from left to right.*

What are the intercepts?

$x$-intercept $= \frac{5}{9}$

$y$-intercept $= -5$

Whenever $x$ increases by 1, how much does $y$ increase or decrease?

*If $x$ increases by 1, $y$ increases by 9.*

Make a table with at least 3 values for $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-14</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
Equation: $y = 10 + 4x$

Will the graph of this equation increase (go up) or decrease (go down) from left to right?

The graph will increase from left to right.

What are the intercepts?

$x$-intercept = -2.5

$y$-intercept = 10

Whenever $x$ increases by 1, how much does $y$ increase or decrease?

If $x$ increases by 1, $y$ increases by 4.

Make a table with at least 3 values for $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
Assignment for Equations, Rates, and Intercepts

Name: __________________________

For each equation, determine whether its graph increases or decreases and find the rate of increase or decrease. Then find the intercepts, make a table of values, and sketch a graph of each equation.

Equation: \( y = -5x + 4 \)

Will the graph of this equation increase (go up) or decrease (go down) from left to right?

*The graph will decrease from left to right.*

What are the intercepts?

\[ x\text{-intercept} = \frac{4}{5} \]

\[ y\text{-intercept} = 4 \]

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

Slope: \(-5\)

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
**Equation:** \( y = 3x - 9 \)

Will the graph of this equation increase *(go up)* or decrease *(go down)* from left to right?

*The graph will increase from left to right.*

What are the intercepts?

- \( x \)-intercept = 3
- \( y \)-intercept = -9

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

*When \( x \) increases by 1, \( y \) will increase by 3.*

Slope: 3

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-12</td>
</tr>
<tr>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
**Equation:** \( y = 12 + 2x \)

Will the graph of this equation increase (*go up*) or decrease (*go down*) from left to right?

*The graph will increase from left to right.*

What are the intercepts?

\( x \)-intercept = \(-6\)

\( y \)-intercept = \(12\)

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

*When \( x \) increases by 1, \( y \) increases by 2.*

Slope: \(2\)

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
Equation: \( y = 20x - 60 \)

Will the graph of this equation increase (go up) or decrease (go down) from left to right?

**The graph will increase from left to right.**

What are the intercepts?

- x-intercept = 3
- y-intercept = -60

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

**When \( x \) increases by 1, \( y \) increases by 20.**

Slope: 20

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-80</td>
</tr>
<tr>
<td>0</td>
<td>-60</td>
</tr>
<tr>
<td>1</td>
<td>-40</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
**Equation:** \( y = 1 - 2x \)

Will the graph of this equation increase (*go up*) or decrease (*go down*) from left to right?

*The graph will decrease from left to right.*

What are the intercepts?

\[
\text{x-intercept} = \frac{1}{2} \\
\text{y-intercept} = 1
\]

Whenever \( x \) increases by 1, how much does \( y \) increase or decrease?

*When \( x \) increases by 1, \( y \) decreases by 2.*

Slope: \(-2\)

Make a table with at least 3 values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Sketch a graph of the equation.
Assignment for Generating and Interpreting Linear Equations

Name: _______________________

For each equation, find the slope, the $y$-intercept, and the $x$-intercept. Then use the slope-intercept method to graph each line.

1. $y = 2x + 1$
   
   Slope: ______ 2 ______
   
   $y$-intercept: ______ 1 ______
   
   $x$-intercept: ______ \( \frac{1}{2} \) ______

2. $y = 3x$
   
   Slope: ______ 3 ______
   
   $y$-intercept: ______ 0 ______
   
   $x$-intercept: ______ 0 ______
3. \( y = x + 4 \)

Slope: \( \frac{1}{1} \)

\( y \)-intercept: 4

\( x \)-intercept: -4

4. \( y = 2x - 3 \)

Slope: \( \frac{2}{2} \)

\( y \)-intercept: -3

\( x \)-intercept: \( \frac{3}{2} \)

5. \( y = \frac{3}{5}x + 2 \)

Slope: \( \frac{3}{5} \)

\( y \)-intercept: 2

\( x \)-intercept: \( -\frac{3}{\frac{1}{3}} \)
6. \( y = 10x + 20 \)
   - Slope: \( 10 \)
   - \( y \)-intercept: \( 20 \)
   - \( x \)-intercept: \( -2 \)

7. \( y = 10x + 100 \)
   - Slope: \( 10 \)
   - \( y \)-intercept: \( 100 \)
   - \( x \)-intercept: \( -10 \)

8. \( y = 5x – 25 \)
   - Slope: \( 5 \)
   - \( y \)-intercept: \( -25 \)
   - \( x \)-intercept: \( 5 \)
Assignment #1 for Spending Money

Name: _________________________

In Antarctica, the temperature of the Ross Sea is \(-10^\circ\) Celsius at the beginning of October, and goes up about \(1^\circ\) Celsius per week.

1. Find an equation for this situation and write what each variable represents.
   \[ y = -10 + x, \text{ where } x \text{ is time in weeks and } y \text{ is temperature in degrees Celsius} \]

2. Use complete sentences to answer the following questions.
   a. What is the starting temperature?
      The starting temperature is \(-10\) degrees Celsius.
   b. How many degrees does the temperature go up each week?
      The temperature increases by 1 degree Celsius each week.
   c. In how many weeks will the temperature be \(0^\circ\) Celsius?
      The temperature will be 0 degrees in 10 weeks.
   d. What was the temperature five weeks before the beginning of October?
      Five weeks before the beginning of October, the temperature was \(-15\) degrees Celsius.
   e. What will the temperature be 10 weeks from the beginning of October?
      Ten weeks from the beginning of October, the temperature will be 0 degrees Celsius.

3. Use the equation to find the slope, the \(y\)-intercept, and the \(x\)-intercept.
   Slope = \(1\)  \(y\)-intercept = \(-10\)  \(x\)-intercept = \(10\)
   Does the graph increase or decrease from left to right?  The graph increases.

4. Use complete sentences to answer the following questions.
   a. What does the \(y\)-intercept represent in this problem situation?
      The \(y\)-intercept represents the temperature at the beginning of October.
   b. What does the slope represent in this situation?
      Slope represents how much the temperature increases each week.
   c. What does the \(x\)-intercept represent in this situation?
      The \(x\)-intercept represents when the temperature will be 0 degrees Celsius.
d. Explain why the graph of this equation increases (or decreases) in this problem situation.

*The graph will increase because the temperature is increasing.*

5. Graph your equation in three ways.
   a. First, draw the graph by plotting the y-intercept, the x-intercept, and the point where $x = 1$.

   ![Graph](image)

   b. Complete this table. Then use the values in the table to draw the graph on the second set of axes.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-12</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
</tr>
<tr>
<td>4</td>
<td>-6</td>
</tr>
</tbody>
</table>
c. On the third set of axes, draw the graph by plotting the $y$-intercept and using the slope.

6. Compare these 3 methods of drawing the graph. Which way was easiest for you? Why?

*Answers will vary. Focus on the students’ explanation of why the method chosen was easiest.*
Assignment #2 for Spending Money

Name: __________________________

Renting a trailer costs $25 per day, plus $5 per day for insurance and a one-time delivery and pickup fee of $30.

1. Find an equation for this situation that gives the cost for each day you rent the trailer, and write what each variable represents.

\[ y = 30x + 30, \text{ where } x \text{ is the time in days and } y \text{ is the cost in dollars} \]

2. Use complete sentences to answer the following questions.

a. What do you owe when the trailer is delivered?

You owe $30 when the trailer is delivered.

b. What does it cost you each day?

Each day, it costs $30 to rent the trailer.

c. How many days did you rent the trailer if your cost is $60?

If you rent the trailer for one day, your cost would be $60.

d. How many days did you rent the trailer if your cost is $135?

If your cost is $135, you rented the trailer for 3.5 days.

e. What is your cost if you rent it for 30 days?

Your cost to rent the trailer for 30 days is $930.

3. Use the equation to find the slope, the y-intercept, and the x-intercept.

Slope = \[ \frac{30}{1} \] y-intercept = \[ 30 \] x-intercept = -1

Does the graph increase or decrease from left to right? The graph increases.
4. Use complete sentences to answer the following questions.
   a. What does the y-intercept represent in this problem situation?
      \textit{The y-intercept represents the delivery and pickup fees.}
   
b. What does the slope represent in this situation?
      \textit{The slope represents the total cost per day.}
   
c. What does the x-intercept represent in this situation?
      \textit{The x-intercept has no meaning in the context of this problem.}
   
d. Explain why the graph of your equation increases (or decreases) in this problem situation.
      \textit{The graph will increase because the cost to rent the trailer increases as the number of days increases.}
5. Graph your equation.
Assignment for Comp-U-Us

Name: __________________________

Suppose you had sold the first 20 computers for $1,800 each and then sold each additional computer you assembled for $1,800.

Answer the following questions in complete sentences. Show all your work.

1. How much did you receive for the original 20 computers?

   You received $36,000 for the original 20 computers.

2. Define a variable for the number of additional computers you assembled.

   \( x \) = Additional computers assembled

3. Use your variable to write an expression for your total income, including what you already received for the original 20 computers.

   \[ 1800x + 36,000 \] = Total income

4. Use your expression to find your total income if you assemble and sell:

   a. 20 additional computers

      \[ y = 1800 \times 20 + 36,000 \]
      \[ y = $72,000 \]

   b. 50 additional computers

      \[ y = 1800 \times 50 + 36000 \]
      \[ y = $126,000 \]

   c. 100 additional computers

      \[ y = 1800 \times 100 + 36000 \]
      \[ y = $216,000 \]

   d. 200 additional computers

      \[ y = 1800 \times 200 + 36000 \]
      \[ y = $396,000 \]

5. How many additional computers must you assemble to earn a total income of $576,000?

   \[ 1800x + 36000 = 576000 \]
   \[ 1800x = 540,000 \]
   \[ x = 300 \text{ computers} \]
6. Complete this table:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Additional Computers</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>computers</td>
<td>dollars</td>
</tr>
<tr>
<td></td>
<td>$x$</td>
<td>$1800x+36,000$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>36,000</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>72,000</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>126,000</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>216,000</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>396,000</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>576,000</td>
</tr>
</tbody>
</table>

7. Construct a graph using the number of additional computers and the income for this situation.
Assignment #1 for Mowing Lawns

Name: ______________________

Five car salespeople are paid $500, split evenly 5 ways. In addition, each salesperson receives $250 for every car he or she sells.

Complete this table, including the algebraic expression. Draw a line graph illustrating this situation and clearly label the axes.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Money Received by Each Salesperson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cars Sold</td>
<td>dollars</td>
</tr>
<tr>
<td>cars</td>
<td>$250x + 100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expression</th>
<th>Units</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>cars</td>
<td>$250x + 100</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>2600</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>5100</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>7600</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>10100</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>12600</td>
</tr>
</tbody>
</table>

Number of cars sold (cars)

money received ($)
Assignment #2 for Mowing Lawns

Name: __________________________

You and your two friends have decided to split among yourselves both the cost of your purchases and your income. Your purchases totaled $750.

Use complete sentences to answer the following questions. Show all your work.

1. How much did each partner have to pay for the original purchases?
   
   *Each partner had to pay $250.*

2. Define a variable for the amount of income received.
   
   \( x = \text{Income received} \)

3. Use your variable to write an expression for each partner’s profit (that is, their income minus their share of the purchases).
   
   \[ \frac{1}{3} \times 250 = \text{Each partner’s profit} \]

4. How much will you each receive if your income is:
   
   a. $750?
      
      \[ y = (1/3) \times 750 - 250 = 0 \]
   
   b. $1,000?
      
      \[ y = (1/3) \times 1000 - 250 = 83.33 \]
   
   c. $2,000?
      
      \[ y = (1/3) \times 2000 - 250 = 416.67 \]
   
   d. $10,000?
      
      \[ y = (1/3) \times 10,000 - 250 = 3083.33 \]

5. If each person’s profit is $400, what was the total income?
   
   \[ \frac{1}{3}x - 250 = 400 \]
   
   \[ \frac{1}{3}x = 650 \]
   
   \[ x = 1950 \]
   
   *Each person’s profit is $1950.*
Complete this table:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Total Income</th>
<th>Each Partner's Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>dollars</td>
<td>dollars</td>
</tr>
<tr>
<td>Expression</td>
<td>$x$</td>
<td>$\frac{1}{3}x - 250$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-250</td>
</tr>
<tr>
<td>750</td>
<td>750</td>
<td>0</td>
</tr>
<tr>
<td>1,000</td>
<td>1,000</td>
<td>83.33</td>
</tr>
<tr>
<td>2,000</td>
<td>2,000</td>
<td>416.67</td>
</tr>
<tr>
<td>10,000</td>
<td>10,000</td>
<td>3083.33</td>
</tr>
<tr>
<td>1,950</td>
<td>1,950</td>
<td>400</td>
</tr>
</tbody>
</table>

Construct a graph using the total income and each partner's profit in this situation.
Assignment #1 for Distributive Property

Name: _____________________________

Simplify each expression.

4(x + 3) = 4x + 12  7(x + 3) = 7x + 21
2(3x + 4) = 6x + 8  (x - 100) • 10 = 10x - 1000
6(3x - 7) = 18x - 42  8(3x - 4) = 24x - 32
5(9 - 2x) = 45 - 10x  3(-2x + 5) = -6x + 15
4(-5x + 6) = -20x + 24  6(x + 12) = 6x + 72

\[
\frac{36 - 24x}{6} = 6 - 4x \\
\frac{24x - 46}{6} = 4x - \left(\frac{23}{3}\right)
\]

\[
\frac{56 - 7x}{7} = 8 - x \\
\frac{8x^2 + 40x}{8x} = x + 5
\]

Factor each expression:

3x + 6 = 3(x + 2)  5x + 80 = 5(x + 16)
7x - 28 = 7(x - 4)  4x + 18 = 2(2x + 9)
10x - 100 = 10(x - 10)  28x - 49 = 7(4x - 7)
-14x + 7 = 7(-2x + 1)  -5 - 15x = -5(1 + 3x)
4x + 7 = Cannot factor with whole numbers

-8x + 6 = -2(4x - 3) or 2(-4x + 3)
Assignment #2 for Distributive Property

Name: __________________________

Use the Distributive Property of Multiplication over Addition (Subtraction) to rewrite each expression.

Example: $5(x + 11) = 5x + 55$

$5(x + 7) = 5x + 35$  
$7(x + 7) = 7x + 49$  
$5(5x + 14) = 25x + 70$  
$(x - 10) \cdot 100 = 100x - 1000$  
$5(7 - 7x) = 35 - 35x$  
$7(x + 5) = 7x + 35$  
$4(x + 6) = 4x + 24$  
$6(x + 15) = 6x + 90$

Use the Distributive Property of Division over Addition (Subtraction) to rewrite each expression.

Example: $\frac{x + 15}{5} = \frac{x}{5} + 3$

$\frac{54 - 36x}{9} = 6 - 4x$  
$\frac{15x - 23}{3} = 5x - \frac{23}{3}$  
$\frac{16 - 6x}{2} = 8 - 3x$  
$\frac{9x^2 + 45x^2}{9x^2} = x + 5$  
$\frac{15x^2 - 15x}{3x} = 5x - 5$  
$\frac{400x - 700}{50} = 8x - 14$
Assignments

Factor each expression.

\[2x + 6 = 2(x + 3)\]
\[5x + 10 = 5(x + 2)\]
\[7x - 21 = 7(x - 3)\]
\[4x + 28 = 4(x + 7)\]
\[100x - 1000 = 100(x - 10)\]
\[35x - 49 = 7(5x - 7)\]

Simplify each expression.

\[11x - 6x + 18 - 3x = 2x + 18\]
\[18 - 3x + 7 = 25 - 3x\]
\[6(x - 4) + 3(8 - x) = 6x - 24 + 24 - 3x = 3x\]
\[18 - 6(x - 3) = 18 - 6x + 18 = 36 - 6x\]
\[-7(8x - 4) + 11 = -56x + 28 + 11 = 39 - 56x\]
\[8(3x + 6) - 3(8x + 6) = 24x + 48 - 24x - 18 = 30\]
Assignment for Solving Equations Using Known Formulas

1. Solve for $t$: $w = uv t$

$$t = \frac{w}{uv}$$

2. Solve for $b$: $A = \frac{1}{2}(2b + 2)h$

$$b = \frac{A}{h} - 1$$

3. Solve for $r$: $I = (p + 1000)rt$

$$r = \frac{I}{t(p+1000)}$$

4. Solve for $w$: $A = w(l - 10)$

$$w = \frac{A}{l - 10}$$
Assignment for Producing and Selling Markers and Making and Selling Shirts

Name ________________________________

1. In order to receive credit for you internship, you must write a report on what you learned about producing and selling pens. Write a detailed description of the situation. Include recommendations regarding how feasible it is for the company to make a profit producing this product.

   **Sample Response:**

   A company produces high color art markers. The cost to produce the markers is $2 each, but to start production there is a $100 set up cost. The company wants to make a profit and given the demand for these high color markers, it is very likely that the company will make a large profit. According to our analysis, once the company produces 125 markers their income is greater than their cost, so they are making a profit.

2. Using all the information you gathered in Making and Selling Shirts, write a paragraph describing the profitability of this situation in detail.

   **Sample Response:**

   The company charges $8.25 per shirt and the cost of producing the shirts is $7.50 per shirt plus a set up fee of $22.50 for each shirt design. For one shirt design, if the company sells 30 shirts it will break even. Since the company produces many popular designs on its shirts, it is likely that for any given shirt it will sell more than 30. When more than 30 shirts are sold the company’s income is greater than its production costs, so it makes a profit.
Assignment for Connecting Algebraic to Graphical Representations

Name ________________________________

1. Given the two equations
   \[ y = 3x - 5 \]
   \[ y = -2x + 10 \]

   predict whether there will be one solution, no solution or every point will be a solution. Explain.

   \textit{There will be one solution because the slopes are positive and negative so they must intersect.}

2. What method would you use to determine if your prediction is correct?

   \textit{Graphing the equations would be the best method.}

3. If there is a point of intersection, what are its coordinates?

   \textit{The coordinates are (3,4).}

4. If there is a point of intersection, verify your solution algebraically.

   \[
   \begin{align*}
   y &= 3x - 5 \\
   y &= -2x + 10 \\
   4 &= 3 \cdot 3 - 5 \\
   4 &= -2 \cdot 3 + 10 \\
   4 &= 4 \\
   4 &= 4
   \end{align*}
   \]
5. Using an appropriate method, determine if the point (1,4) is a solution to the system of equations.

\[ y = -2x + 6 \]
\[ y = 4x \]
\[ 4 = -2(1) + 6 \]
\[ 4 = 4 \]
\[ 4 = 4 \]

Yes, it is a solution to the system.

6. (1,2) is a solution to the system of equations.

\[ y = 2x - 2 \]
\[ y = 3 + 2x \]
\[ y = -2x + 6 \]
\[ 2 = 2 \cdot 1 - 2 \]
\[ 2 = -2(1) + 6 \]
\[ 2 \neq 0 \]
\[ 2 = 4 \]

No, (1,2) is not a solution to the system.

7. Write each equation in the slope–intercept form and graph the equations. Determine the point of intersection, if one exists.

a. \[ y = 3x - 5 \] and \[ y = -2x + 10 \]

Point of Intersection \((3,4)\)
\[ 4 = 3 \cdot 3 - 5 \]
\[ 4 = 9 - 5 \]
\[ 4 = 4 \]
\[ 4 = -2 \cdot 3 + 10 \]
\[ 4 = -6 + 10 \]
\[ 4 = 4 \]

b. \[ 2x + y = 3 \] and \[ 6x + 3y = 9 \]

Point of Intersection All points on the line.
They are the same line.
Assignment #1 for Solving Systems of Two Equations Algebraically

Name ________________________________

Solve each system algebraically. Then check your solution graphically, using the grid provided. Communicate your results by reporting the point of intersection of the lines.

1. System of equations: $4x - 5y = -27$
   $5x + 2y = 9$
   *(Solution processes will vary.)*

   Point of Intersection = $(-0.27, 5.18)$

2. System of equations: $2x - y = 1$
   $5x + 2y = 3$
   *(Solution processes will vary.)*

   Point of Intersection = $(0.5, 0.1)$
3. System of equations:
   \[ 7x + 9y = 2 \]
   \[ 3x + 4y = 0 \]

(Solution processes will vary.)

Point of Intersection = \((8, -6)\)
Assignment #2 for Solving Systems of Two Equations Algebraically

Name ________________________________

Solve each system algebraically.

1. System of equations:
   \[2x - y = 4\]
   \[5x + y = 3\]

   *(Solution processes will vary.)*

   Point of Intersection: \((1, -2)\)

2. System of equations:
   \[2x - y = 1\]
   \[5x + 2y = 3\]

   *(Solution processes will vary.)*

   Point of Intersection: \((0.5, 0.1)\)

3. System of equations:
   \[x + 2y = 2\]
   \[3x + 4y = 0\]

   *(Solution processes will vary.)*

   Point of Intersection: \((-4, 3)\)
4. System of equations: \(2x - y = 4\)  
\(-2x + 3y = 2\)

(Solution processes will vary.)

Point of Intersection: \((3.5, 3)\)

5. System of equations: \(2x - y = 3\)  
\(5x + 2y = 3\)

(Solution processes will vary.)

Point of Intersection: \((1, -1)\)

6. System of equations: \(2x + 5y = 2\)  
\(-3x + 4y = 20\)

(Solution processes will vary.)

Point of Intersection: \((-4, 2)\)
Assignment for Finding a Better Paying Job, World Oil: Supply and Demand, and Finding the Best Option

Name

As a member of the local teen community group, you and your friends took on the responsibility to analyze the most economical option for flooring for the new center. After all the bids were collected, the two most promising bids were for vinyl flooring and carpeting.

The vinyl flooring will cost $31,000 to install along with a monthly maintenance fee of $175 for cleaning and polishing. The carpeting will cost $22,500 to install along with a monthly maintenance fee of $325.

Your group must determine the best option over time. Consider in your analysis the fact that both products carry a 10-year warranty. When submitting your recommendation, make sure you justify it through a detailed mathematical analysis.

**Vinyl Flooring**

\[ y = 175x + 31,000 \]

\( x = \text{the time measured in months} \)

\( y = \text{the cost measured in dollars} \)

\[ 325x + 22,500 = 175x + 31,000 \]

\[ 150x = 8500 \]

\[ x = 56.67 \text{ months} \]

**Carpeting**

\[ y = 325x + 22,500 \]

\( x = \text{the time measured in} \)

\( y = \text{the cost measured in dollars} \)

\[ y = 175(56.67) + 31,000 \]

\[ y = 40,916.67 \]

*The cost will be equal at 56.67 months. At 56 months or less, the cost of carpeting is cheaper. At 57 months or more, the cost of vinyl flooring is cheaper.*
Assignment #1 for Stem and Leaf Plots

Name ________________________

You have a theory that foods that are higher in protein are lower in carbohydrates or vice versa. To test out this theory, you will need to gather data by looking at food labels. You will probably want to look at no more than 25 different types of foods in each category.

Using a stem and leaf plot or a split stem and leaf plot (where you organize the data on each side of the stem), compare the distributions and justify your conclusion.

Answers will vary but should reflect the following guidelines:

Students must examine 20 – 25 products to have a reasonable display in order to make the comparison.

In assessing students’ work, see if they find a relationship, direct or indirect, between protein and carbohydrate content. If there is no relationship, see if they indicate why—perhaps there are an insufficient number of data points.
Assignment #2 for Stem and Leaf Plots

Name ____________________

You have a theory that foods that are higher in fat are lower in sugar or vice versa. To test out this theory, you will need to gather data by looking at food labels. You will probably want to look at no more than 25 different types of foods in each category.

Using a stem and leaf plot or a split stem and leaf plot (where you organize the data on each side of the stem), compare the distributions and justify your conclusion.

**Answers will vary but should reflect the following guidelines:**

*As in Assignment #1, students should examine 20 – 25 products in order to have a reasonable display for the comparison.*

*In assessing students’ work, see if they find a relationship between sugar and fat content. If no relationship is found, see if they indicate why—perhaps there are an insufficient number of data points.*
Assignments
Assignment #1 for Histograms

Name _______________________

Using the example of the number line plot in your text, describe how you might represent the data using a histogram. Then draw the display.

Which is most effective for determining the distribution of heights in your class.

Sample Response:

*We can represent the data from the number line plot as a histogram by placing the heights on the x-axis and the frequency on the y-axis. We can group the heights by inches rather than half inches.*

[Histogram diagram showing height distribution]
Assignments
Assignment #2 for Histograms

Name __________________________

The table below indicates the number of US House Representatives by state.

<table>
<thead>
<tr>
<th>State</th>
<th>Number of Reps</th>
<th>State</th>
<th>Number of Reps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama</td>
<td>7</td>
<td>Montana</td>
<td>1</td>
</tr>
<tr>
<td>Alaska</td>
<td>1</td>
<td>Nebraska</td>
<td>3</td>
</tr>
<tr>
<td>Arizona</td>
<td>6</td>
<td>Nevada</td>
<td>2</td>
</tr>
<tr>
<td>Arkansas</td>
<td>4</td>
<td>New Hampshire</td>
<td>2</td>
</tr>
<tr>
<td>California</td>
<td>52</td>
<td>New Jersey</td>
<td>13</td>
</tr>
<tr>
<td>Colorado</td>
<td>6</td>
<td>New Mexico</td>
<td>3</td>
</tr>
<tr>
<td>Connecticut</td>
<td>6</td>
<td>New York</td>
<td>31</td>
</tr>
<tr>
<td>Delaware</td>
<td>1</td>
<td>North Carolina</td>
<td>12</td>
</tr>
<tr>
<td>Florida</td>
<td>23</td>
<td>North Dakota</td>
<td>1</td>
</tr>
<tr>
<td>Georgia</td>
<td>11</td>
<td>Ohio</td>
<td>19</td>
</tr>
<tr>
<td>Hawaii</td>
<td>2</td>
<td>Oklahoma</td>
<td>6</td>
</tr>
<tr>
<td>Idaho</td>
<td>2</td>
<td>Oregon</td>
<td>5</td>
</tr>
<tr>
<td>Illinois</td>
<td>10</td>
<td>Pennsylvania</td>
<td>21</td>
</tr>
<tr>
<td>Indiana</td>
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<td>Rhode Island</td>
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<tr>
<td>Iowa</td>
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<td>South Carolina</td>
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</tr>
<tr>
<td>Kansas</td>
<td>4</td>
<td>South Dakota</td>
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<td>Kentucky</td>
<td>6</td>
<td>Tennessee</td>
<td>9</td>
</tr>
<tr>
<td>Louisiana</td>
<td>7</td>
<td>Texas</td>
<td>30</td>
</tr>
<tr>
<td>Maine</td>
<td>2</td>
<td>Utah</td>
<td>3</td>
</tr>
<tr>
<td>Maryland</td>
<td>8</td>
<td>Vermont</td>
<td>1</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>10</td>
<td>Virginia</td>
<td>11</td>
</tr>
<tr>
<td>Michigan</td>
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<td>Minnesota</td>
<td>8</td>
<td>West Virginia</td>
<td>3</td>
</tr>
<tr>
<td>Mississippi</td>
<td>5</td>
<td>Wisconsin</td>
<td>9</td>
</tr>
<tr>
<td>Missouri</td>
<td>9</td>
<td>Wyoming</td>
<td>1</td>
</tr>
</tbody>
</table>

Represent the data using a histogram.

Describe the distribution.

*The highest frequency of representatives is between 0 and 14. There is one state with significantly more representatives than the others.*

What states have the highest frequency?

*California, Florida and New York are the states with the highest frequencies.*

Are there outliers?

*Yes, there are outliers.*

If so, which states?

*California is an outlier.*
Assignment for Measures of Central Tendency and Random Numbers

Name: ______________________________

Calculate the mean, the median, and the mode for each set of data.

1. Data: 15, 16, 19, 20, 20, 21, 22

   mean: $\bar{x} = \frac{\sum x}{n} = 19$   median = 20   mode = 20

   $mean = \frac{133}{7}$

2. Data: 111, 111, 313, 415, 612

   mean: $\bar{x} = \frac{\sum x}{n} = 312.4$   median = 313   mode = 111

   $mean = \frac{1562}{5}$

3. Data: 11, 54, 65, 98, 201

   mean: $\bar{x} = \frac{\sum x}{n} = 85.8$   median = 65   mode = no mode

   $mean = \frac{429}{5}$

4. Data: 24, 26, 28, 30, 30, 32

   mean: $\bar{x} = \frac{\sum x}{n} = 28.33$   median = 29   mode = 30

   $mean = \frac{170}{6}$   median is mean between 28 and 30
Assignment for Baseball History

Name _________________________

Give several examples of when the median value is a more appropriate statistic to use than the mean.

*Answers will vary. Some appropriate answers include home selling prices, salaries, expenditures, expenses and other situations that have outliers that would affect the mean.*

Give several examples of when the mean and the median provide the same information.

*Answers will vary. Test scores might be an example.*

Give an example of where the means are the same, but the standard deviations are different.

*Answers will vary.*
Assignments
Assignment for Breakfast Cereals and Airline Industry

Name _______________________

When the results of standardized tests are reported you will often find the scores represented in by quartile. This tells you your standing with respect to others who took the test.

Let’s look at the mean scores for males and females on the verbal and mathematics SAT’s from 1967 – 2000.

Average SAT® scores of entering college classes, 1967-2000

<table>
<thead>
<tr>
<th>Year</th>
<th>Male</th>
<th>Female</th>
<th>All</th>
</tr>
</thead>
<tbody>
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<td>Math</td>
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</tr>
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<td>Male Verbal</td>
<td>Male Math</td>
<td>Female Verbal</td>
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</tr>
<tr>
<td>2000</td>
<td>507</td>
<td>533</td>
<td>504</td>
</tr>
</tbody>
</table>

Source: www.collageboard.org

1. Represent the data using a visual display. Discuss why you chose the specific display you did.

   **Answers will vary.**

To answer the questions, you will need to define variables and variable type. To make comparisons, you may make observations, but you will want to back your observation with specific statistics. Name the statistics you think you will want to use to answer the questions below.

2. Compare the data on males and females on

   verbal portion of the SAT

   **Answers will vary but could include them mean, standard deviation and five number summary.**

   math portion of the SAT

   **Answers will vary but could include the mean, standard deviation and five number summary.**

   total means

   **Answers will vary but could include the mean, standard deviation and five number summary.**
3. Compare the scores of males between verbal and math.

   **Answers will vary but will probably state that males do better on the math section.**

4. Compare the scores of females between verbal and math

   **Answers will vary but will probably state that females do better on the verbal section.**

5. Discuss any trends you find in the data.

   **Answers will vary but should show that from 1967 to 2000, males’ scores on the verbal section have gone down. Math scores went down for about 30 years and are going up and almost equal to the 1967 score. For females, scores on the verbal section went downward with a slight upward trend; however, they never regained the high mean in 1967. Math scores have been fairly stable with only a slight dip in the 1970’s and 1980’s. The overall trends tend to match the trends of the females more than males for verbal and math.**

6. What are the standard deviations for scores for males on verbal, on math and total and for females on verbal, math and total?

   **Standard deviations:**

<table>
<thead>
<tr>
<th></th>
<th>Verbal</th>
<th>Math</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>11.34</td>
<td>5.90</td>
<td>15.53</td>
</tr>
<tr>
<td>Female</td>
<td>15.45</td>
<td>8.48</td>
<td>21.52</td>
</tr>
</tbody>
</table>

7. Is there anything you understand better about the relationship of these scores after computing the standard deviations? Do the standard deviations differ? How do they compare. If there are differences, in which data groups do you see these differences?

   **Answers will vary.**

   **Statistics from the data set that the students may use:**

<table>
<thead>
<tr>
<th></th>
<th>Verbal</th>
<th>Math</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
</tr>
<tr>
<td>MEAN</td>
<td>514.20</td>
<td>508.56</td>
<td>523.76</td>
</tr>
<tr>
<td>MINIMUM</td>
<td>501</td>
<td>495</td>
<td>515</td>
</tr>
<tr>
<td>Q1</td>
<td>507</td>
<td>498</td>
<td>520</td>
</tr>
<tr>
<td>MEDIAN</td>
<td>509.5</td>
<td>502.5</td>
<td>523</td>
</tr>
<tr>
<td>Q3</td>
<td>515</td>
<td>509</td>
<td>529</td>
</tr>
<tr>
<td>MAXIMUM</td>
<td>541</td>
<td>545</td>
<td>535</td>
</tr>
</tbody>
</table>
Assignment for Variance and Candy Sale

Name __________________________

Hikers around America are feeling short changed. There is a belief that there are not enough hiking trails and that the number of miles available is limited. Collect some data that either supports or negates this claim. Your analysis should include a data table and statistical evidence along with a summary of your findings. To help you in this process, you might try looking at the following website: www.TrailLink.com.

*Students should create a visual display (or displays) that will most appropriately show the number of trails and miles of trails available.*

*Students should also be deciding between stem and leaf plots and histograms as choices for their visual displays.*

*There should be a discussion regarding the distribution as a means of justifying the claim that there are not enough miles.*
Assignment for Mia’s “Growing Like a Weed”

Name __________________________

The typical gestation period (time from conception to birth) for a human baby is about 40 weeks (a bit longer than 9 months). Recently, the advanced development of ultrasound scanning of fetuses has begun to include the ability to make measurements of various parts of the baby’s body. By evaluating large groups of fetuses that later had normal birth weights, it’s been possible to create standard tables of fetal growth. The table below contains data about the length of the femur of a fetus at different times throughout the gestation period.

<table>
<thead>
<tr>
<th>Quantity Name</th>
<th>Gestation Time</th>
<th>Femur Length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>weeks</td>
<td>mm</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>12.5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>14</td>
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<td>20</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>18.8</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>
Create a scatter plot of this data, making sure to label the axes appropriately and making sure to use an appropriate scale.

1. Create a line that best approximates the data (your line of best fit).

2. Approximately how many millimeters is the femur growing every week? Explain how you know.

   Sample: The femur is growing about \( \frac{1}{2} \) of a millimeter each week. In 20 weeks the femur grows to 21 mm, so that is close to 0.5 mm each week. Also, the line of best fit comes close to (40, 20).

3. If the femur of the fetus continues to grow at this same rate, how long will it be when the baby is 3 months old?

   Assuming 3 months is 12 week, the additional growth will be 6 mm. If the baby was born with a 21 mm femur, it will be 27 mm long at 3 months of age.

4. Is it reasonable that the femur of the baby will continue to grow at the same rate as before the baby was born? Why or why not?

   It is not reasonable that this rate of growth would continue over a number of years. The growth rate would have to slow after a period of time.
Assignment for College Students and Retail Sales

Name ____________________________

You have a hypothesis that the cost of a box of cereal is a function of the grams of sugar per serving. You believe the more sugar, the higher the cost. To determine whether your hypothesis is correct, go to the nearest market and select 15 different cereals. Set up an appropriate analysis of the data and explain your findings. Be sure to note your sampling technique and to include a visual display of the data in addition to the table.

_Students should write the hypothesis in their own words. They should also indicate their sampling technique, which should be random._

_Students should choose a display that best determines if one factor is a function of the other. For example, students might choose a line graph or a number line plot._
Assignment for Stroop Test

Name: _________________________

On this grid, use the data from your table for Matching Lists to construct a graph with Length of List on the horizontal axis and Average Time on the vertical axis. Label the axes on your graph.

These graphs are based upon the sample data given in the Teacher Text. Your class’ graphs will vary based upon the actual data collected in your class. However, axes labels should be as indicated.
On this grid, use the data from your table for Non-Matching Lists to construct a graph with Length of List on the horizontal axis and Average Time on the vertical axis. Label the axes on your graph.

Use the equations of the line of best fit for Matching Lists and the line of best fit for Non-Matching Lists to answer the following questions.

1. How long would it take to read a list of:
   a. 100 words on a Matching List?
   b. 100 words on a Non-Matching List?
   c. 1000 words on a Matching List?
   d. 1000 words on a Non-Matching List?

For each, use the two regression equations found for the matching and non-matching lists. Substitute given independent variable values for $x$, and solve the dependent variable values.
Use the equations of the linear regression lines for *Matching Lists* and *Non-Matching Lists* to answer the following questions.

1. How many *matching* words could a person read in:
   a. 10 minutes?
   b. 1 hour?

2. How many *non-matching* words could a person read in:
   a. 10 minutes?
   b. 1 hour?

3. Do you think these answers are reasonable? Explain your answer.

   *Answers will vary.*

   *The further the given value is from the actual data collected, the less reliable it will be.*

   *Students may see the answers as reasonable for the requested number of words or length of list. However, it may be unreasonable to read a very long list or to continue reading for the requested amount of time.*

4. What do you think accounts for the differences in the times and the number of words for the two different lists?

   *Answers will vary.*

   *In general, times for matching lists should be less than comparable times for non-matching lists. Differences may be accounted for, in part, by the conflicting pieces of information being processed when a non-matching list is presented.*
Assignment for Hanford Nuclear Power Plant

Name: ____________________________

Use the equation of the linear regression line for the Hanford Nuclear Power Plant data to answer the following questions. \( y = 9.28x + 114.7 \)

1. What is the predicted cancer rate per 100,000 residents of a community where the exposure index is:
   a. 10.5? \( y = 9.28(10.5) + 114.7 \)
      \( y = 97.44 + 114.7 \)
      \( y = 212.14 \quad \text{The expected cancer rate is 212 people per 100,000 residents.} \)
   b. 21.7? \( y = 9.28(21.7) + 114.7 \)
      \( y = 201.376 + 114.7 \)
      \( y = 316.076 \quad \text{The cancer rate is 316 per 100,000 residents.} \)

2. What is the predicted exposure index of a community that has a cancer rate of:
   a. 1,000 per 100,000 residents? \( 1000 = 9.28x + 114.7 \)
      \( 885.3 = 9.28x \)
      \( x = 95.40 \quad \text{The predicted exposure index is 95.40.} \)
   b. 1,500 per 100,000 residents? \( 1500 = 9.28x + 114.7 \)
      \( 1385.3 = 9.28x \)
      \( x = 149.28 \quad \text{The predicted exposure index is 149.28.} \)
Assignment for Human Chain: Wrist Experiment

Name: __________________________

Use the data in this table to solve problem 1 on the next page.

<table>
<thead>
<tr>
<th>Rank</th>
<th>State</th>
<th>Expenditures per Student ($/student)</th>
<th>Rank</th>
<th>State</th>
<th>Expenditures per Student ($/student)</th>
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</thead>
<tbody>
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</tr>
<tr>
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<td>NJ</td>
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<td>26</td>
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<td>5,551</td>
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</tbody>
</table>

1. Construct a stem-and-leaf plot for the data on state expenditures in public education.

State Expenditures per Student in Thousands

2. Use the stem-and-leaf plot (and look at the data table) to describe the distribution of the data. Write a report that describes the distribution of the data in terms of clusters of values, overall pattern or trend to the data, and outliers (values that differ greatly from the others).

**Answers will vary, but should include the following observations:**

*The distribution is skewed to the right, because the majority of values are smaller.*

*The majority of values cluster between $4000 and $6000.*

*$3837 and $9455 are the extreme values, but are not technically outliers.*
Assignment for Human Chain: Shoulder Experiment

Name: ____________________________

1. Prepare a five-number summary, as well as calculating the mean for the data in the table below.

<table>
<thead>
<tr>
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<th>Number of Reps</th>
<th>Rank</th>
<th>State</th>
<th>Number of Reps</th>
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<td>26</td>
<td>CO</td>
<td>6</td>
<td>Total US</td>
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<td></td>
</tr>
</tbody>
</table>

Five-Number Summary

Low: 1

1st Quartile: 2

Median: 6

3rd Quartile: 10.5

High: 52

Mean: 8.53 based on 51 entities or 8.7 based on 50 states
2. Construct a box-and-whisker plot for the data.

3. Write a summary report about the data.

*Answers will vary but should include a discussion of outliers, clusters, distributions, etc.*
Assignment #1 for Exploring Quadratic Equations

Name: __________________________

Enter each equation into your graphing calculator and graph it.

- Fill in the table and sketch the graph
- Find the highest (or lowest) point on the graph
- Find the \( x \)-intercepts and the \( y \)-intercept

\[ y = x^2 \]

<table>
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</tr>
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<tr>
<td>-2</td>
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<tr>
<td>-1</td>
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<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Highest/lowest point

(0, 0)

\[ x \]-intercepts: (0, 0)

\[ y \]-intercept: (0, 0)
Assignments

\[ y = x^2 + 2 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
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<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

highest/lowest point
\( (0, 2) \)

\( x \)-intercepts: none
\( y \)-intercept: \((0, 2)\)
Assignment #2 for Exploring Quadratic Equations

Name: ____________________________

Enter each equation into your graphing calculator and graph it.

- Fill in the table and sketch the graph
- Find the highest (or lowest) point on the graph
- Find the x-intercepts and the y-intercept

\[ y = x^2 + 3 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
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<td>-1</td>
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<td>1</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>7</td>
</tr>
</tbody>
</table>

highest/lowest point
(0, 3)

x-intercepts: none
y-intercept: (0,3)
Assignments

\[ y = x^2 - 5 \]

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-2 & -2 \\
-1 & -4 \\
0 & -5 \\
1 & -4 \\
2 & -1 \\
\hline
\end{array}
\]

highest/lowest point

\( (0, -5) \)

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
\end{array}
\]

x-intercepts: \((\sqrt{5}, 0); (-\sqrt{5}, 0)\)

y-intercept: \((0, -5)\)
Assignment #1 for Finding Zeros by Factoring

Name: __________________________

1. Complete each of the following multiplication tables to factor the quadratic function and to find its x-intercepts.

   a. \( y = x^2 + x - 12 = (x + 4)(x - 3) \)
      x-intercepts: \((-4,0)\) and \((3,0)\)

      \[
      \begin{array}{c|c|c}
        \cdot & x & 4 \\
        \hline
        x & x^2 & 4x \\
        -3 & -3x & -1 \\
      \end{array}
      \]

   b. \( y = x^2 - 8x + 7 = (x - 7)(x - 1) \)
      x-intercepts: \((7,0)\) and \((1,0)\)

      \[
      \begin{array}{c|c|c}
        \cdot & x & 7 \\
        \hline
        x & x^2 & 7 \\
        -1 & -1 & 7 \\
        1 & 1 & 7 \\
      \end{array}
      \]

   c. \( y = x^2 + 9x - 90 = (x + 15)(x - 6) \)
      x-intercepts: \((-15,0)\) and \((6,0)\)

      \[
      \begin{array}{c|c|c}
        \cdot & x & 15 \\
        \hline
        x & x^2 & 15x \\
        -6 & -6 & -90 \\
      \end{array}
      \]
2. Find the $x$-intercepts and the factors of each of the following quadratic functions.

a. $y = x^2 - 10x + 24 = (x - 6)(x - 4)$

$x$-intercepts: $(6,0)$ and $(4,0)$

b. $y = x^2 - 2x - 63 = (x - 9)(x + 7)$

$x$-intercepts: $(9,0)$ and $(-7,0)$

c. $y = x^2 + 2x - 3 = (x + 3)(x - 1)$

$x$-intercepts: $(-3,0)$ and $(1,0)$
3. Find the roots (solutions) of each of the quadratic equations.
   a. $x^2 - 10x - 56 = 0$
      roots: \textbf{14 and -4}
   
   b. $x^2 + 18x + 32 = 0$
      roots: \textbf{-16 and -2}
   
   c. $x^2 + 11x - 60 = 0$
      roots: \textbf{-15 and 4}
Assignment #2 for Finding Zeros by Factoring

Name: ________________________________

Factor each of the following quadratic functions and find their \(x\)-intercepts.

1. \(y = x^2 + 5x + 6 = (x + 2)(x + 3)\) \(x\)-intercepts: \((-2,0)\) \((-3,0)\)

2. \(y = x^2 - 5x + 6 = (x - 2)(x - 3)\) \(x\)-intercepts: \((2,0)\) \((3,0)\)

3. \(y = x^2 + x - 6 = (x + 3)(x - 2)\) \(x\)-intercepts: \((-3,0)\) \((2,0)\)

4. \(y = x^2 - 5x - 6 = (x - 6)(x + 1)\) \(x\)-intercepts: \((6,0)\) \((-1,0)\)

5. \(y = x^2 - 7x + 6 = (x - 6)(x - 1)\) \(x\)-intercepts: \((6,0)\) \((1,0)\)

6. \(y = x^2 - 7x + 6 = (x + 6)(x + 1)\) \(x\)-intercepts: \((-6,0)\) \((-1,0)\)

7. \(y = x^2 + 5x - 6 = (x + 6)(x - 1)\) \(x\)-intercepts: \((-6,0)\) \((1,0)\)

8. \(y = x^2 - 5x - 6 = (x - 6)(x + 1)\) \(x\)-intercepts: \((6,0)\) \((-1,0)\)

9. \(y = x^2 + 5x + 4 = (x + 4)(x + 1)\) \(x\)-intercepts: \((-4,0)\) \((-1,0)\)

10. \(y = x^2 - 5x - 24 = (x - 8)(x + 3)\) \(x\)-intercepts: \((8,0)\) \((-3,0)\)

11. \(y = x^2 + 5x - 36 = (x + 9)(x - 4)\) \(x\)-intercepts: \((-9,0)\) \((4,0)\)

12. \(y = x^2 + 4x - 12 = (x + 6)(x - 2)\) \(x\)-intercepts: \((-6,0)\) \((2,0)\)

13. \(y = x^2 - 3x + 2 = (x - 2)(x - 1)\) \(x\)-intercepts: \((2,1)\) \((1,0)\)
Assignments

14. \( y = x^2 - x - 56 = (x - 8)(x + 7) \)  
   \text{x-intercepts: (8,0) (-7,0)}

15. \( y = x^2 - x - 42 = (x - 7)(x + 6) \)  
   \text{x-intercepts: (7,0) (-6,0)}

16. \( y = x^2 + x - 20 = (x + 5)(x - 4) \)  
   \text{x-intercepts: (-5,0) (4,0)}

17. \( y = x^2 - 25 = (x + 5)(x - 5) \)  
   \text{x-intercepts: (-5,0) (5,0)}

18. \( y = x^2 - 6x = (x)(x - 6) \)  
   \text{x-intercepts: (0,0) (6,0)}

19. \( y = x^2 - 16 = (x + 4)(x - 4) \)  
   \text{x-intercepts: (-4,0) (4,0)}

20. \( y = x^2 - 7x - 18 = (x - 9)(x + 2) \)  
   \text{x-intercepts: (9,0) (-2,0)}

21. \( y = x^2 - 4x - 12 = (x - 6)(x + 2) \)  
   \text{x-intercepts: (6,0) (-2,0)}

22. \( y = x^2 - 6x + 5 = (x - 5)(x - 1) \)  
   \text{x-intercepts: (5,0) (1,0)}

23. \( y = x^2 - 2x + 1 = (x - 1)(x - 1) \)  
   \text{x-intercepts: (1,0)}

24. \( y = x^2 - 8x + 7 = (x - 7)(x - 1) \)  
   \text{x-intercepts: (7,0) (1,0)}

25. \( y = x^2 - 4x - 165 = (x - 15)(x + 11) \)  
   \text{x-intercepts: (15,0) (-11,0)}

26. \( y = x^2 + 18x - 63 = (x + 21)(x - 3) \)  
   \text{x-intercepts: (-21,0) (3,0)}

27. \( y = x^2 - 11x + 10 = (x - 10)(x - 1) \)  
   \text{x-intercepts: (10,0) (1,0)}

28. \( y = x^2 + 6x - 55 = (x + 11)(x - 5) \)  
   \text{x-intercepts: (-11,0) (5,0)}
29. \( y = x^2 - 9x + 8 = (x - 8)(x - 1) \)  
\( \text{x-intercepts: (8,0) (1,0)} \)

30. \( y = x^2 - 13x + 12 = (x - 12)(x - 1) \)  
\( \text{x-intercepts: (12,0) (1,0)} \)

31. \( y = x^2 - 13x = (x)(x - 13) \)  
\( \text{x-intercepts: (0,0) (13,0)} \)

32. \( y = x^2 - 12x + 36 = (x - 6)(x - 6) \)  
\( \text{x-intercepts: (6,0)} \)

33. \( y = x^2 - 14x + 49 = (x - 7)(x - 7) \)  
\( \text{x-intercepts: (7,0)} \)

34. \( y = x^2 - 625 = (x + 25)(x - 25) \)  
\( \text{x-intercepts: (-25,0) (25,0)} \)

35. \( y = x^2 - 196 = (x - 14)(x + 14) \)  
\( \text{x-intercepts: (14,0) (-14,0)} \)

36. \( y = x^2 - 16x + 64 = (x - 8)(x - 8) \)  
\( \text{x-intercepts: (8,0)} \)

37. \( y = x^2 - 5x = (x)(x - 5) \)  
\( \text{x-intercepts: (0,0) (5,0)} \)

38. \( y = x^2 - 1x = (x)(x - 1) \)  
\( \text{x-intercepts: (0,0) (1,0)} \)

39. \( y = x^2 + 18x + 81 = (x + 9)(x + 9) \)  
\( \text{x-intercepts: (-9,0)} \)

40. \( y = x^2 + 20x + 100 = (x + 10)(x + 10) \)  
\( \text{x-intercepts: (-10,0)} \)
Assignment #1 for Finding Zeros using the Quadratic Formula

Name: ______________________________

General form of the quadratic equation:

\[ y = Ax^2 + Bx + C \]

In this equation, \( A, B, \) and \( C \) are real numbers, but \( A \) is not zero.

Quadratic formula:

\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

For each quadratic equation:

- Write the values of \( A, B, \) and \( C \) from the general form
- Use the quadratic formula to solve the equation for the roots (and the \( x \)-intercepts)
- Use the roots (or the \( x \)-intercepts) to write the equation in factored form

\[ y = x^2 - 5x - 14 = (x - 7)(x + 2) \]

\( A = 1 \) \( B = -5 \) \( C = -14 \)

Solve the equation:

\[ x = \frac{5 \pm \sqrt{25 - 4(1)(-14)}}{2(1)} \]

\[ x = \frac{5 \pm \sqrt{81}}{2} \]

\[ x = \frac{5 \pm 9}{2} \]

\( x = 7 \) or \( x = -2 \)

\( x \)-intercepts: \((7,0)\) \((-2,0)\)
Assignments

\[ y = x^2 - 12x - 13 = (x - 13)(x + 1) \]

\[ A = 1 \quad B = -12 \quad C = -13 \]

Solve the equation:

\[ x = \frac{12 \pm \sqrt{144 - 4(1)(-13)}}{2(1)} \]
\[ x = \frac{12 \pm \sqrt{196}}{2} \]
\[ x = \frac{12 \pm 14}{2} \]
\[ x = 13 \text{ or } x = -1 \]

x-intercepts: (13,0) (-1,0)

\[ y = x^2 + 8x - 33 = (x + 11)(x - 3) \]

\[ A = 1 \quad B = 8 \quad C = -33 \]

Solve the equation:

\[ x = \frac{-8 \pm \sqrt{64 - 4(1)(-33)}}{2(1)} \]
\[ x = \frac{-8 \pm \sqrt{196}}{2} \]
\[ x = \frac{-8 \pm 14}{2} \]
\[ x = 3 \text{ or } x = -11 \]

x-intercepts: (3,0) (-11,0)
Assignment #2 for Finding Zeros using the Quadratic Formula

Name: ____________________________

General form of the quadratic equation:
\[ y = Ax^2 + Bx + C \]
In this equation, \( A \), \( B \), and \( C \) are real numbers, but \( A \) is not zero.

Quadratic formula:
\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

For each quadratic equation:
- Write the values of \( A \), \( B \), and \( C \) from the general form
- Use the quadratic formula to solve the equation for the roots (and the \( x \)-intercepts)
- Find the \( y \)-intercept
- Find the vertex
- Use the average of the \( x \)-intercepts to find the equation of the axis of symmetry
- Calculate the sum of the roots and the product of the roots
- Use the roots (or the \( x \)-intercepts) to write the equation in factored form
- Sketch the graph

\[ y = x^2 - 6x + 8 = (x - 4)(x - 2) \]

\( A = 1 \quad B = -6 \quad C = 8 \)

Solve the equation:
\[ x = \frac{6 \pm \sqrt{36 - 4(1)(8)}}{2(1)} \quad x = \frac{6 \pm 2}{2} \]
\[ x = \frac{6 \pm \sqrt{4}}{2} \quad x = 4 \text{ or } x = 2 \]

\( x \)-intercepts: \((4,0) \quad (2,0)\)

\( y \)-intercept: \((0,8)\)

Axis of symmetry: \( x = 3 \)

Vertex: \((3, -1)\)

Sum of roots = 6

Product of roots = 8
\[ y = x^2 - 8x + 12 = (x - 6)(x - 2) \]

\[ A = 1 \quad B = -8 \quad C = 12 \]

Solve the equation:

\[ x = \frac{8 \pm \sqrt{64 - 4(1)(12)}}{2(1)} \quad x = \frac{8 \pm 4}{2} \]

\[ x = \frac{8 \pm \sqrt{16}}{2} \quad x = 6 \text{ or } x = 2 \]

x-intercepts: \((6,0)\) \((2,0)\)

y-intercept: \((0,12)\)

Axis of symmetry: \(x = 4\)

Vertex: \((4, -4)\)

Sum of roots = 8

Product of roots = 12
Assignment for Modeling: Vertical Motion

Name: ______________________

Mark McGwire hits a foul ball straight up from a height of four feet off the ground. The following equation models the vertical motion of the ball:

\[ y = -16t^2 + vt + h \]

In this equation, \( y \) is the number of feet the ball is above the ground, \( t \) is the time in seconds since the ball was hit, \( v \) is the initial velocity (or speed) of the ball in feet per second, and \( h \) is the height at which the ball was originally hit.

1. If the initial velocity of the foul ball is 130 feet per second, write the equation for the foul ball.

   Equation: \(-16t^2 + 130t + 4\)

   Use the equation from problem 1 to answer the following questions.

2. How high is the ball after:

   1 second after it is hit?
   \[ y = -16(1)^2 + 130(1) + 4 = -16 + 130 + 4 = 118 \]
   The ball is 118 feet above the ground 1 second after it is hit.

   2 seconds?
   \[ y = -16(2)^2 + 130(2) + 4 = -64 + 260 + 4 = 200 \]
   The ball is 200 feet above the ground 2 seconds after it is hit.

   10 seconds?
   \[ y = -16(10)^2 + 130(10) + 4 = -1600 + 1300 + 4 = -296 \]
   The ball is -296 feet above the ground 10 seconds after it is hit.

3. Do all of the answers to question 2 make sense in this problem situation?

   No. The height after 10 seconds does not make sense, because the ball cannot go below ground by 296 feet.

4. Use the quadratic formula to find when the ball is on the ground. How do you know?

   \[ x = \frac{-130 \pm \sqrt{130^2 - 4(-16)(4)}}{2(-16)} \]
   \[ x = \frac{-130 \pm 130.98}{-32} \]
   \[ x = \frac{-130 \pm \sqrt{17156}}{-32} \]
   \[ x = -0.03 \text{ or } x = 8.16 \]

   The ball will hit the ground after about 8.16 seconds because that is when the height is 0 feet.
5. When is the ball at its highest point? How do you know? What is its height at this point?

*The ball will be at a maximum height of 286.06 feet after about 4.06 seconds because the vertex of the curve is at (4.06, 286.06).*

6. Use the quadratic formula to find when the ball will be back at the level from which it was hit.

\[
4 = -16t^2 + 130t + 4, \text{ solving for } 0, \quad 0 = -16t^2 + 130t
\]

\[
x = \frac{-130 \pm \sqrt{130^2 - 4(-16)(0)}}{2(-16)} \quad x = \frac{-130 \pm 130}{-32}
\]

\[
x = \frac{-130 \pm \sqrt{16900}}{-32} \quad x = 0 \quad \text{or} \quad x = 8.125
\]

*The ball will be back at 4 feet above the ground after 8.125 seconds.*

7. When is the ball 200 feet off the ground?

(Hint: Think about whether there is more than one answer.)

*Using an answer from question 2, it is already known that the ball is 200 feet off the ground after 2 seconds. The parabola is symmetric, so there is a second time when the ball is 200 feet off the ground. The vertex is the highest point, which is reached after about 4.06 seconds. Therefore, the ball will be at 200 feet 6.12 seconds after it is hit. (2 is 2.06 to the left of 4.06 and 6.12 is 2.06 to the right of 4.06)*

8. Using the information you found for this problem situation, complete the following chart.

<table>
<thead>
<tr>
<th>Time</th>
<th>Height Above Ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 second</td>
<td>118 feet</td>
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<td>2 seconds</td>
<td>200 feet</td>
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<tr>
<td>10 seconds</td>
<td>-296 feet</td>
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<tr>
<td>4.06 sec</td>
<td>268.06 feet</td>
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<tr>
<td>-0.03 sec</td>
<td>8.16 sec</td>
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<tr>
<td>0 sec</td>
<td>8.125 sec</td>
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<tr>
<td>2 sec</td>
<td>6.12 sec</td>
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</tbody>
</table>
9. Graph the equation for this problem situation.
Assignment #1 for Powers

Name: ___________________________

1. Use the definition of a power to expand and then simplify each of the following expressions.
   a. \( \frac{x^6}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{x \cdot x \cdot x}{1} = x^3 \)
   b. \( \frac{3x^4}{6x} = \frac{3 \cdot x \cdot x \cdot x}{6 \cdot x} = \frac{x \cdot x}{2} = \frac{x^2}{2} \)
   c. \( \frac{12x}{15x^3} = \frac{12 \cdot x}{15 \cdot x \cdot x \cdot x} = \frac{4}{5 \cdot x \cdot x} = \frac{4}{5x^2} \)

2. Write each of the following expressions as a single power when appropriate. Try finding each result without using expansion.
   a. \( \frac{8x^6}{2x^3} = 4x^3 \)
   b. \( \frac{10x}{-12x^4} = -\frac{5}{6x^3} \)
   c. \( \frac{-16x^6}{12y^3} = -\frac{4x^6}{3y^3} \)
   d. \( \frac{14x^4 \cdot -2x^3}{-7x^3 \cdot -3x^3} = \frac{-28x^7}{21x^9} = -\frac{4}{3} \)
   e. \( \frac{-12x \cdot -3y^3}{-18x^2 \cdot -2y^4} = \frac{36xy^3}{36x^2y^4} = \frac{y^3}{x} \)
Assignment #2 for Powers

Name: ____________________________

1. To simplify each of the following expressions, use the definition of exponents and expand twice.
   a. \((x^3)^3 = x^2 \cdot x^2 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6\)
   b. \((x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{12}\)
   c. \((x^5)^4 = x^2 \cdot x^2 \cdot x^2 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^8\)

2. Simplify each of the following expressions, expanding whenever you feel that is necessary.
   a. \((x^5)^5 = x^{15}\)
   b. \((x^3)^3 = x^{15}\)
   c. \((x^4)^5 = x^{15}\)
   d. \((x^3)^2 = x^{24}\)
   e. \((x^{10})^4 = x^{40}\)
   f. \((2x^5)^3 = 8x^{15}\)
 Assignment #3 for Powers

Name: ____________________________

1. To simplify each of the following expressions, use the definition of exponents, expand twice, regroup, and write the result as a single expression.
   a. \((-2x^3y)^3 = -2x^9y^3\)
      \(= -2 \cdot x \cdot x \cdot x \cdot y \cdot -2 \cdot x \cdot x \cdot x \cdot y \)
      \(= -2 \cdot -2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \)
      \(= 4x^6y^2\)

   b. \((5x^2y^3)^3 = 5x^6y^9\)
      \(= 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \)  
      \(= 5 \cdot 5 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \)
      \(= 125x^9y^9\)

2. Simplify each of the following expressions, using expansion only when necessary.
   a. \((-6x^3)^3 = -216x^9\)
   b. \((-8xy^3)^2 = 64x^2y^6\)
   c. \((11xy)^2 = 121x^2y^2\)
   d. \((20xy)^3 = 8000x^3y^9\)
   e. \((6x^3y^2)^3 = 216x^9y^6\)
   f. \((-3x^4y^2)^4 = 81x^8y^8\)
   g. \((-2x^3y^5)^5 = -32x^{15}y^{10}\)
   h. \((-2x^3y^6)^6 = 64x^{18}y^{30}\)
Assignment #4 for Powers

Name: ______________________________

1. Simplify each of the following expressions, using the appropriate expansions.
   a. \( \left( \frac{3}{2} \right)^5 = \left( \frac{3}{2} \right) \left( \frac{3}{2} \right) \left( \frac{3}{2} \right) \left( \frac{3}{2} \right) = \frac{243}{32} \)

   b. \( \left( \frac{2x}{3y} \right)^3 = \left( \frac{2x}{3y} \right) \left( \frac{2x}{3y} \right) \left( \frac{2x}{3y} \right) = \frac{8x^3}{27y^3} \)

   c. \( \left( \frac{-2x^3}{3y^2} \right)^2 = \left( \frac{-2x^3}{3y^2} \right) \left( \frac{-2x^3}{3y^2} \right) = \frac{4x^6}{9y^4} \)

2. Simplify the following expressions, expanding only when you feel it is necessary.
   a. \( \left( \frac{3x^2}{4y^3} \right)^2 = \frac{9x^4}{16y^6} \)

   b. \( \left( \frac{11x^3}{-7y^2} \right)^2 = \frac{121x^6}{49y^4} \)

   c. \( \left( \frac{-15x^5}{5y^6} \right)^3 = \frac{-3375x^{15}}{125y^{18}} = \frac{-27x^5}{y^6} \)

   d. \( \left( \frac{-15xy^2}{20x^2y} \right)^3 = \frac{-3375x^3y^6}{8000x^6y^9} = \frac{-27}{64x^3y^3} \)

   e. \( \left( \frac{-2x^2}{y^3} \right)^3 \cdot \left( \frac{10y^5}{6x} \right)^3 = \frac{4x^2}{y^3} \cdot \frac{1000y^5}{216x^3} = \frac{4000x^2y^8}{216y^3x^3} = \frac{1000}{54y^x} = \frac{500}{27xy^2} \)
Assignments
Assignment #5 for Powers

Name: __________________________

Simplify the following expressions, expanding only when you feel it is necessary.

1. \( \frac{(3x)^2}{2x^2} = \frac{9x^2}{2x^2} = \frac{9}{2} \)

2. \( \frac{(xy)^3}{xy^3} = \frac{x^3y^3}{xy^3} = x^2 \)

3. \( \frac{(x^3y)^3}{x^9y^2} = \frac{x^9y^3}{x^9y^2} = y \)

4. \( \frac{(-2x^3)^2}{-2x^6} = \frac{4x^6}{2x^6} = -2 \)

5. \( \frac{(-3x^2y)^3}{(-2x^3y^2)^2} = \frac{-27x^6y^3}{4x^6y^4} = -\frac{27}{4y} \)

6. \( \frac{(-2x^2y)^3}{(3x^4y^3)^2} = \frac{-8x^6y^3}{9x^8y^6} = -\frac{8x^6}{9y^3} \)
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Number Patterns Test

Name: ______________________

Task 1

For each sequence, find the next 3 terms and describe the pattern.

1. 3, 8, 13, 18, 23,           ,           ,           ,...

2. 1, 2, 3, 5, 8, 13,           ,           ,           ,...

3. 1, 4, 9, 16,           ,           ,           ,...

4. 1, 100, 2, 99, 3, 98,           ,           ,           ,...

5. 3, -9, 27, -81,           ,           ,           ,...

Task 2

For each sequence, find the 10th term and describe the pattern.

1. 

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>5</td>
<td>15</td>
<td>25</td>
<td>35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>2</td>
<td>-4</td>
<td>8</td>
<td>-16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Task 3

For each sequence, find an expression for the $N^{th}$ term and describe the pattern.

1. **Term Number** | 1 | 2 | 3 | 4 | ... | $N$
---|---|---|---|---|---|---
**Sequence** | 2 | 7 | 12 | 17 | ... |

2. **Term Number** | 1 | 2 | 3 | 4 | ... | $N$
---|---|---|---|---|---|---
**Sequence** | 5 | 25 | 125 | 625 | ... |

3. **Term Number** | 1 | 2 | 3 | 4 | ... | $N$
---|---|---|---|---|---|---
**Sequence** | 10 | 18 | 26 | 34 | ... |
Toothpick Test

Name: __________________

The following toothpick diagrams show a single square constructed from 4 toothpicks and two squares constructed from 7 toothpicks.

<table>
<thead>
<tr>
<th>One Square</th>
<th>Two Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="One Square Diagram" /></td>
<td><img src="image2.png" alt="Two Squares Diagram" /></td>
</tr>
</tbody>
</table>

1. Draw toothpick diagrams that show how to construct three squares, four squares, and five squares in the same manner.
   a. Three squares:
   ![Three Squares Diagram](image3.png)

   b. Four squares:
   ![Four Squares Diagram](image4.png)

   c. Five squares:
   ![Five Squares Diagram](image5.png)
2. Complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Number of Squares</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
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<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

3. Use the information in the table and your diagrams to describe the pattern produced by constructing the toothpick squares. Write your answer in complete sentences.

4. Find an expression that gives the number of toothpicks required to construct $N$ squares.
Earning Money

Name: ____________________

A friend of yours is employed at a company where he makes $5.25 an hour.

Make a table that shows his earnings for various numbers of hours worked (between 1 hour and 20 hours). Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vacation Flights

Name: ____________________

You are going on vacation by airplane. The average speed of the plane is 325 miles per hour.

Make a table that shows how far you will travel for various times between 1 hour and 20 hours. Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Time in the Air</th>
<th>Distance Traveled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Labels

<table>
<thead>
<tr>
<th>Units</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph
Students Who Don’t Take Algebra

Name: ____________________

One out of every four high school students never takes algebra.

Make a table that shows how many students don’t take algebra at schools of various sizes. Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Expression</th>
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</thead>
<tbody>
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</tr>
</tbody>
</table>

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Paying Taxes

Name: ____________________

The average family in the United States pays about 38% of their income in taxes of all kinds.

Make a table that shows the amount of taxes paid by families with various incomes. Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
Stocking the Shelves

Name: _______________________

You are a volunteer for the school store. One of the most popular items is cool ranch flavored tortilla chips. There are two local vendors that will deliver the chips to the school store. Each makes their deliveries on the first of every month. Chips-R-Us charges 39 cents per bag with a monthly delivery fee of $25. It’s In The Bag charges 18 cents per bag with a monthly delivery fee of $65.

Write an algebraic equation to model the monthly cost of purchasing chips from Chips-R-Us.

1. Write an algebraic equation to model the monthly cost of purchasing chips from Chips-R-Us.

2. Write an algebraic equation to model the monthly cost of purchasing chips from It’s In The Bag.

3. Complete the following table summarizing the cost of purchasing chips from each vendor based upon the actual sales by month for the last school year.
<table>
<thead>
<tr>
<th>Label</th>
<th>Month</th>
<th>Number of bags of chips purchased</th>
<th>Cost of purchasing from Chips-R-Us</th>
<th>Cost of purchasing from It’s In The Bag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>September</td>
<td></td>
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</tr>
<tr>
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<td>October</td>
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<td>November</td>
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<td>March</td>
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<td>April</td>
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<tr>
<td></td>
<td>May</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>June</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Generate a graph displaying the cost of purchasing from both Chips-R-Us and It’s In The Bag.
5. How many bags of chips can you purchase with each vendor if you have the following budgets? Explain how you determined each and write your answer in a complete sentence.
   a. $50
   b. $125
   c. $200

6. Compare the price to purchase from each of the two vendors. Prepare a written analysis indicating which vendor will cost the least and when. Be sure to include when the costs will be the same.

7. If you had to choose one vendor for the entire school year, which would you choose. Would your choice change if you could switch vendors each month? Explain your choices using complete sentences.
Integers and Solving Equations: Form A

Name: _____________________

Find the answer to each problem without using your calculator.

1. 7 + (-14) =  
2. -19 + 110 =  

3. -2 • 10 =  
4. \( \frac{210}{-10} = \)  

5. 13 – 25 =  
6. -10(-6) =  

7. -23 + (-14) =  
8. \( \frac{-41 + 33}{-2} = \)  

9. \( \frac{10 + (-4) - 4}{2} = \)  
10. 20(-3) – 10 =  

Solve each equation. Show all your work.

11. \( e + 5 = 8 \) \hspace{1cm} 12. \( f + 9 = 13 \)

13. \( -2g = -20 \) \hspace{1cm} 14. \( 3h = 18 \)

15. \( 20x = -10 \) \hspace{1cm} 16. \( 5j = -25 \)

17. \( 7 = \frac{k}{-12} \) \hspace{1cm} 18. \( -2 – L = 18 \)
Integers and Solving Equations: Form B

Name: _____________________

Find the answer to each problem without using your calculator.

1. 17 + (-14) =

2. -190 + 110 =

3. -21 \cdot 10 =

4. \frac{-50}{-10} =

5. -13 – 5 =

6. -10(-16) =

7. -213 + (-124) =

8. \frac{-1 – 3}{-2} =

9. \frac{10 – 4 – 4}{-2} =

10. 2(-3) – 10 =

Solve each equation. Show all your work.

11. 2E + 6 = 10

12. 4x + 8 = 32

13. -2G + 26 = 50

14. 3h – 18 = 18

15. 22x + 11 = -11

16. 3J + (-6) = -15

17. 12 = 12 + \frac{k}{-7}

18. -4 – 4L = 16
At the Driving Range

Name: _______________________

Inspired by the saying “practice makes perfect”, you decide to perfect your golf swing by going to the local driving range. Unfortunately, you do not have your own set of clubs yet so you must rent from the range. Each bucket of 100 golf balls costs $7.50 and it costs $3.00 to rent clubs for 2 hours.

1. Write an algebraic equation for the total cost at the driving range dependent on the number of buckets of balls you buy. Write your equation using functional notation with \( C(x) \) representing the cost.

2. Show all work and answer in complete sentence
   a. What is the total cost to hit five buckets worth of balls?

   b. How many buckets can you buy with $20?

   c. Find \( x \) if \( C(x) = 28 \)

   d. Find \( C(10) \)

3. Generate a graph displaying the total cost of practicing at the driving range
4. The owner of the driving range is looking at the receipts for the day. Are the following costs correct? Explain why or why not in a complete sentence.
   a. $48 for 6 buckets and club rental

   b. $70 for 9 buckets and club rental

5. What would be the domain based on the problem situation?

6. Based on your answer to question 5, what would be the range based on the problem situation?

7. Set up an inequality to solve each. Graph the solution to each.
   a. Determine the number of buckets of balls you could hit if you had to spend at most $50 but at least $25.

   b. \( C(x) > 55 \) or \( C(x) \leq 40 \)
Balloon Problem

Name: __________________

Here you will examine a problem situation about buying and selling helium balloons. Show all your work and write your answers in complete sentences.

You have the opportunity to run the helium balloon concession at a local fair. In order to sell balloons, you must pay a $25 fee to the fair sponsors and buy $75 worth of balloons and helium. You can sell each balloon for $0.80. You need to calculate your net profit for the fair. Remember, profit is the money you bring in minus the expenses you have to pay out.

1. How much profit will you have if you sell:
   a. 100 balloons?

   b. 200 balloons?

   c. 500 balloons?

   d. 50 balloons?

   e. 80 balloons?

   Write a complete sentence describing how you found these answers.

2. Do your answers in 1d and 1e make sense? Why or why not?

3. Algebraic Expression or Equation for Profit _____________________
4. How many balloons must you sell to make a profit of:
   a. $40?
   b. $100?

   Write a complete sentence describing how you found these answers.

5. How many balloons must you sell to post the following losses? (Note: A loss is a negative number. For example, a loss of $20 is -$20.)
   a. $20
   b. $50
   c. $100

   Write a complete sentence describing how you found these answers.
6. Use the information you have found so far to complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Balloons Sold</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td></td>
<td>500</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Expressions</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>120</td>
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<tr>
<td></td>
<td>150</td>
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<tr>
<td></td>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>

7. Use the information from the table to construct a graph that illustrates this problem situation.
Assessments

8. How many balloons do you have to sell in order to *break even*?

Justify your answer, and write a complete sentence describing how you found this answer.
Televisions Problem and Climber Problem

Name: ______________________

Task 1

Use this graph to answer the questions about televisions.
Assessments

1. What information does this graph show?

2. How much does it cost to produce:
   a. thirty-five 19-inch televisions?
   b. twenty-five 19-inch televisions?
   c. thirteen 19-inch televisions?

3. How much does it cost to produce:
   a. forty-five 21-inch televisions?
   b. fifteen 21-inch televisions?
   c. eight 21-inch televisions?

4. How many of each type of television must be produced for the costs to be the same? How do you know?

5. Use the graph to predict the cost of producing:
   a. sixty 19-inch televisions.
   b. sixty 21-inch televisions.
   c. one hundred 21-inch televisions.
   d. one hundred 19-inch televisions.
Extra credit:

Find an algebraic expression or equation for the cost to produce 19-inch televisions. Find another algebraic expression or equation for the cost to produce 21-inch televisions.

Rule for producing 19-inch televisions: __________________________

Rule for producing 21-inch televisions: __________________________

Task 2

A mountain climber is currently 77 feet above ground level on the side of a cliff. She is climbing at the rate of about 7 feet per minute.

Answer the following questions in complete sentences.

1. How many feet above ground level will she be in:
   a. 10 minutes?
   b. 25 minutes?

2. Currently, how many feet above ground level is she?

3. How many feet above ground level was she:
   a. 5 minutes ago?
   b. 10 minutes ago?
   c. 15 minutes ago?

4. How many minutes ago did she start her climb?
Assessments

5. When will she be:
   a. 154 feet above ground level?
   b. 175 feet above ground level?
   c. 287 feet above ground level?

6. The top of the cliff is 450 feet above ground level. When will she reach the top of the cliff?

7. Write an algebraic rule for the climber’s height above ground level.
8. Fill in this table and construct a graph for this problem.

<table>
<thead>
<tr>
<th>Labels</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Units</td>
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</table>

Graph
Widgets-R-Us

Name: ____________________

Widgets cost $4 each, plus $10 per order for shipping and handling.

For this problem situation, find an appropriate algebraic equation, define each variable, make a table of values, construct a graph, and use this information to answer the questions.

1. Find an algebraic equation for this situation.
   
   Equation: ____________________

   Define each variable in your equation by writing a short phrase that describes what the variable represents.

   \( x = \) ____________________

   \( y = \) ____________________

2. Make a table with at least 5 values of \( x \) and \( y \).

   \[
   \begin{array}{|c|c|}
   \hline
   x & y \\
   \hline
   -2 & \\
   \hline
   -5 & \\
   0 & \\
   \hline
   \end{array}
   \]
Assessments

3. Graph the equation for this problem situation.

Use your graph, your equation, or your table to answer the following questions. Write your answers in complete sentences.

4. How would you describe your graph?

5. Where does the graph intersect the x-axis? Why?

6. Where does the graph intersect the y-axis? Why?
7. What is the cost if you order:
   a. 7 widgets?
   b. -9 widgets?

8. How many widgets did you order if the total cost is:
   a. $66?
   b. -$10?

9. Write an equation that states: the cost of an order of widgets is $212. Solve this equation to find how many widgets you ordered.

10. Find the cost if you order -23,456 widgets.

11. Do problems 9 and 10 make sense in this problem situation? Why or why not?
Computer Rental Problem

Name: ______________________

For a college computer class in which you are enrolled, you must rent a computer. Two rental companies carry the particular model you need.

Computronics rents the computer you need for $2.50 a day plus a one-time registration fee of $27. A second company, Computers for Students, rents the same computer at a rate of $4.75 per day.

You know you will need the computer for 30 days or less, but you don’t know exactly how many days. From whom should you rent? In order to answer this question, write the problem situation for each company, construct a table of values, and find the algebraic expression. Then draw graphs for both companies on a single grid.

Task 1

Write the problem situation for Computronics. Construct a table of values and find the expression.

Problem Situation:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Expression</th>
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</table>
**Task 2**

Write the problem situation for Computers for Students. Construct a table of values and find the expression.

Problem Situation:

<table>
<thead>
<tr>
<th>Labels</th>
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<tbody>
<tr>
<td>Units</td>
<td></td>
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<tr>
<td>Expression</td>
<td></td>
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</tbody>
</table>

**Task 3**

Draw graphs that show the information from each company.

**Task 4**

Use the information you have gathered and analyzed in Tasks 1–3 to write a paragraph explaining which company you will rent from and why. Write your answer in complete sentences.
Systems: Part 1

Name: ____________________________

For each system of equations:

• Find the slope and y-intercept for each equation
• Graph both lines on the same coordinate system
• Find the point of intersection of the lines

1. System of equations:
   \[ y = 3x - 5 \]
   \[ y = -2x + 5 \]
   \[ y = 3x - 5 \]
   Slope: ____________
   y-intercept: _________
   \[ y = -2x + 5 \]
   Slope: ____________
   y-intercept: _________

Point of intersection: _______

2. System of equations:
   \[ y = -x + 2 \]
   \[ y = x + 4 \]
   \[ y = -x + 2 \]
   Slope: ____________
   y-intercept: _________
   \[ y = x + 4 \]
   Slope: ____________
   y-intercept: _________

Point of intersection: _______
Systems: Part 2

Name: ______________________

For each system of equations:

• Find the slope and y-intercept for each equation
• Graph both lines on the same coordinate system
• Find the point of intersection of the lines

1. System of equations:
   1. \( y = 3x + 5 \)
   2. \( y = -2x + 5 \)

   \( y = 3x + 5 \)
   Slope: ____________
   y-intercept: _________

   \( y = -2x + 5 \)
   Slope: ____________
   y-intercept: _________

Point of intersection: _______

2. System of equations:
   1. \( y = -x + 3 \)
   2. \( y = 2x + 6 \)

   \( y = -x + 3 \)
   Slope: ____________
   y-intercept: _________

   \( y = 2x + 6 \)
   Slope: ____________
   y-intercept: _________

Point of intersection: _______
Systems: Part 3

Name: ____________________

For each system of equations:

• Find the slope and $y$-intercept for each equation
• Graph both lines on the same coordinate system
• Find the point of intersection of the lines
• Check your answer on the calculator

1. System of equations:
   $y = -3x + 4$
   $y = 3x - 2$

   $y = -3x + 4$
   \[\text{Slope: } \hspace{1cm} \text{y-intercept: }\]
   $y = 3x - 2$
   \[\text{Slope: } \hspace{1cm} \text{y-intercept: }\]

   Point of intersection: ________
2. System of equations:  
   \( y = -2x - 7 \)  
   \( y = 2x + 5 \)

\( y = -2x - 7 \)
Slope: ____________
y-intercept: ________

\( y = 2x + 5 \)
Slope: ____________
y-intercept: ________

Point of intersection: ________
## Baseball

**Name:** ________________________

The data set below represents the top 40-homerun hitters in the American and National Baseball Leagues for 2001.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1) Alex Rodriguez, Tex 52</td>
<td>1) Barry Bonds, SF 73</td>
</tr>
<tr>
<td>2) Jim Thome, Cle 49</td>
<td>2) Sammy Sosa, ChC 64</td>
</tr>
<tr>
<td>3) R. Palmeiro, Tex 47</td>
<td>3) Luis Gonzalez, Ari 57</td>
</tr>
<tr>
<td>4) Manny Ramirez, Bos 41</td>
<td>4) Shawn Green, LA 49</td>
</tr>
<tr>
<td>5) Troy Glaus, Ana 41</td>
<td>5) Todd Helton, Col 49</td>
</tr>
<tr>
<td>6) Carlos Delgado, Tor 39</td>
<td>6) Richie Sexson, Mil 45</td>
</tr>
<tr>
<td>7) Jason Giambi, Oak 38</td>
<td>7) Phil Nevin, SD 41</td>
</tr>
<tr>
<td>8) Bret Boone, Sea 37</td>
<td>8) Jeff Bagwell, Hou 39</td>
</tr>
<tr>
<td>9) Juan Gonzalez, Cle 35</td>
<td>9) Chipper Jones, Atl 38</td>
</tr>
<tr>
<td>10) Jose Cruz, Tor 34</td>
<td>10) Larry Walker, Col 38</td>
</tr>
<tr>
<td>11) Tino Martinez, NY 34</td>
<td>11) Rich Aurilia, SF 37</td>
</tr>
<tr>
<td>12) Paul Konerko, CWS 32</td>
<td>12) Brian Giles, Pit 37</td>
</tr>
<tr>
<td>13) Eric Chavez, Oak 32</td>
<td>13) Albert Pujols, Stl 37</td>
</tr>
<tr>
<td>14) Miguel Tejada, Oak 31</td>
<td>14) Mike Piazza, NYM 36</td>
</tr>
<tr>
<td>15) M. Ordonez, CWS 31</td>
<td>15) Gary Sheffield, LA 36</td>
</tr>
<tr>
<td>16) Mike Sweeney, KC 29</td>
<td>16) Jeromy Burnitz, Mil 34</td>
</tr>
<tr>
<td>17) Jose Valentín, CWS 28</td>
<td>17) Andruw Jones, Atl 34</td>
</tr>
<tr>
<td>18) G. Anderson, Ana 28</td>
<td>18) V. Guerrero, Mon 34</td>
</tr>
<tr>
<td>19) Ellis Burks, Cle 28</td>
<td>19) Aramis Ramirez, Pit 34</td>
</tr>
<tr>
<td>20) Raul Mondesi, Tor 27</td>
<td>20) Lance Berkman, Hou 34</td>
</tr>
<tr>
<td>21) Trot Nixon, Bos 27</td>
<td>21) Reggie Sanders, Ari 33</td>
</tr>
<tr>
<td>22) Torii Hunter, Min 27</td>
<td>22) Cliff Floyd, Fla 31</td>
</tr>
<tr>
<td>23) Jermaine Dye, KC/Oak 26</td>
<td>23) Bobby Abreu, Phi 31</td>
</tr>
<tr>
<td>24) Corey Koskie, Min 26</td>
<td>24) Ryan Klesko, SD 30</td>
</tr>
<tr>
<td>25) B. Williams, NY 26</td>
<td>25) Jim Edmonds, Stl 30</td>
</tr>
<tr>
<td>26) Mike Cameron, Sea 25</td>
<td>26) Mark McGwire, Stl 29</td>
</tr>
<tr>
<td>27) Tony Batista, Bal/Tor 25</td>
<td>27) J.D. Drew, Stl 27</td>
</tr>
<tr>
<td>28) Ivan Rodriguez, Tex 25</td>
<td>28) Pat Burrell, Phi 27</td>
</tr>
<tr>
<td>29) Carlos Beltran, KC 24</td>
<td>29) Moises Alou, Hou 27</td>
</tr>
<tr>
<td>30) Carlos Lee, CWS 24</td>
<td>30) Jose Hernandez, Mil 25</td>
</tr>
<tr>
<td>32) Ruben Sierra, Tex 23</td>
<td>32) Scott Rolen, Phi 25</td>
</tr>
<tr>
<td>33) Edgar Martinez, Sea 23</td>
<td>33) Bubba Trammell, SD 25</td>
</tr>
<tr>
<td>34) Jorge Posada, NY 22</td>
<td>34) Paul Lo Duca, LA 25</td>
</tr>
<tr>
<td>35) Brian Daubach, Bos 22</td>
<td>35) Lee Stevens, Mon 25</td>
</tr>
<tr>
<td>36) Derek Jeter, NYY 21</td>
<td>36) Vinny Castilla, Hou 23</td>
</tr>
<tr>
<td>37) Paul O’Neill, NYY 21</td>
<td>37) Preston Wilson, Fla 23</td>
</tr>
<tr>
<td>38) John Olerud, Sea 21</td>
<td>38) Jeff Kent, SF 22</td>
</tr>
<tr>
<td>39) Ray Durham, CWS 20</td>
<td>39) K. Griffey Jr., Cin 22</td>
</tr>
<tr>
<td>40) Marty Cordova, Cle 20</td>
<td>40) Dmitri Young, Cin 21</td>
</tr>
</tbody>
</table>

Source: [www.espn.com](http://www.espn.com)
Assessments

1. To compare the two leagues construct
   a. Stem and Leaf Plot
   b. Histogram
2. What type of information can you gather from each type of plot?

3. Based on the stem and leaf plots what can you say about the performance of the two leagues?

4. Based on the histogram what can you say about the performance of the two leagues?

5. Do the two displays provide the same information? Explain your reasoning.

6. Which display is more informative? Why?

7. From which distribution is it easier to determine the mean number of home runs?
Assessments
Data

Name:_________________________

1. Use the data table to graph the points. Then find the slope, the y-intercept, and the equation that fits the data points.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>51</td>
</tr>
<tr>
<td>11</td>
<td>63</td>
</tr>
</tbody>
</table>

Slope:______________
y-intercept:__________
Equation: y = __________

For each set of data, calculate the mean, the median, and the mode.

2. Data: 14, 23, 24, 19, 15, 18, 19

mean: \( \bar{x} = \) __________ median = ________ mode = _______

3. Data: 1111, 1415, 1612, 1111, 1313

mean: \( \bar{x} = \) __________ median = ________ mode = _______
Running the Mile

Name: ___________________

This table shows the fastest time in seconds that a human ran the mile from 1945 to 1985. Find the slope, the \( y \)-intercept, and the equation of the linear regression line for number of years since 1945 versus time in seconds. Then answer the questions.

<table>
<thead>
<tr>
<th>Year</th>
<th>Time in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1945</td>
<td>241.4</td>
</tr>
<tr>
<td>1954</td>
<td>238.0</td>
</tr>
<tr>
<td>1957</td>
<td>237.2</td>
</tr>
<tr>
<td>1962</td>
<td>234.4</td>
</tr>
<tr>
<td>1964</td>
<td>234.1</td>
</tr>
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<td>1965</td>
<td>233.6</td>
</tr>
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<td>1966</td>
<td>231.3</td>
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<td>1967</td>
<td>231.1</td>
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<tr>
<td>1975</td>
<td>229.4</td>
</tr>
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<td>1979</td>
<td>229.0</td>
</tr>
<tr>
<td>1980</td>
<td>228.8</td>
</tr>
<tr>
<td>1981</td>
<td>228.4</td>
</tr>
<tr>
<td>1981</td>
<td>227.3</td>
</tr>
<tr>
<td>1985</td>
<td>226.3</td>
</tr>
</tbody>
</table>

Slope: \( b = \) _______________  \( y \)-intercept: \( a = \) _______________

Equation: \( y = \) _______________  Regression coefficient: \( r = \) _______________

Write your answers in complete sentences.

1. Use your model (equation) to predict how fast a runner in 1990 should be able to run the mile.
Assessments

2. Use your model (equation) to predict how fast a runner in 1993 should be able to run the mile.

3. In what year does your model (equation) predict that the fastest time a runner ran the mile was 237.5 seconds?

4. In what year does your model (equation) predict that 200 seconds is the fastest time a runner will run the mile?

5. How good a fit is your model? Explain your thinking.
Squares: Part 1

Name: ____________________

For each equation:

• Enter the equation into your graphing calculator and graph it
• Fill in at least five x and y values in the table
• Sketch the graph
• Find the x-intercepts and the y-intercept
• Find the equation for the axis of symmetry
• Find the coordinates of the vertex

1. \( y = x^2 + 5x + 4 = ( \quad ) ( \quad ) \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
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</tbody>
</table>

x-intercepts: ______________
y-intercept: ______________
Axis of symmetry: \( x = \) ______
Vertex: (____, _____)
2. \( y = x^2 - 5x + 6 = ( \quad ) ( \quad ) \)

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
& \\
& \\
& \\
\hline
\end{array}
\]

- x-intercepts: ________________
- y-intercept: ________________
- Axis of symmetry: \( x = \) ______
- Vertex: (____, ____)

\[
\text{Grid}
\]
Squares: Part 2

Name: ______________________

For each equation:

• Enter the equation into your graphing calculator and graph it
• Fill in at least five $x$ and $y$ values in the table
• Sketch the graph
• Find the $x$-intercepts and the $y$-intercept
• Find the equation for the axis of symmetry
• Find the coordinates of the vertex

1. $y = x^2 + 7x + 6 = ( ) ( )$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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</thead>
<tbody>
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</tbody>
</table>

$x$-intercepts: ________________

$y$-intercept: ________________

Axis of symmetry: $x = ______$

Vertex: (____, ____)

Cognitive Tutor® Algebra I © 2002 | 55
2. \( y = x^2 - 6x + 8 = (\quad)(\quad) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

\( x \)-intercepts: ________________
\( y \)-intercept: ________________
Axis of symmetry: \( x = \) ______
Vertex: (____, _____)
Quadratic Formula

Name: ______________________

General form of the quadratic equation:

\[ y = Ax^2 + Bx + C \]

Quadratic formula:

\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

For each quadratic equation:

- Write the values of \( A \), \( B \), and \( C \) from the general form
- Use the quadratic formula to solve the equation for the \( x \)-intercepts
- Use the \( x \)-intercepts to write the equation in factored form

1. \( y = x^2 - 9x - 10 = ( \quad ) ( \quad ) \)
   \[ A = \quad \quad \quad B = \quad \quad \quad C = \quad \quad \]
   \( x \)-intercepts: ______________________________

2. \( y = 2x^2 - 16x - 128 = ( \quad ) ( \quad ) \)
   \[ A = \quad \quad \quad B = \quad \quad \quad C = \quad \quad \]
   \( x \)-intercepts: ______________________________

3. \( y = 3x^2 + 8x - 10 = ( \quad ) ( \quad ) \)
   \[ A = \quad \quad \quad B = \quad \quad \quad C = \quad \quad \]
   \( x \)-intercepts: ______________________________
Vertical Motion

Name: ____________________

A cannonball is shot straight up into the air from the side of a mountain. The flight of the cannonball can be modeled by this equation for vertical motion:

\[ y = -16x^2 + vx + h \]

In this equation, \( y \) is the number of feet above the valley, \( x \) is the time in seconds from the moment the ball is shot, \( v \) is the initial velocity (or speed) in feet per second, and \( h \) is the initial height in feet of the cannonball above the valley floor.

The initial velocity of the cannonball is 600 feet per second, and the cannon is 100 feet above the valley floor.

1. Write the equation for flight of the cannonball.

Use complete sentences to answer the following questions.

2. How high is the cannonball:
   a. 1 second after it is shot?
   b. 20 seconds after?
   c. 50 seconds after?
   d. 5 seconds before it is shot?
   e. 50 seconds before it is shot?

3. Do all the answers to question 2 make sense in this problem situation?
4. Use the quadratic formula to find when the cannonball is on the ground. How do you know?

5. When is the cannonball at its highest point? How high does it get?

6. Use the quadratic formula to find when the cannonball will be back at its original level. How do you know?

7. When is the cannonball 3000 feet above the valley floor?
8. Use the information you have found so far about the problem situation to fill in this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Units</th>
<th>Expression</th>
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</tbody>
</table>

9. Graph your equation on the following grid.
Law of Powers: Part 1

Name: ______________________

1. Write each of the following expressions as a single power when appropriate.
   a. \(2x^4 \cdot 3x^3 = \)

   b. \(5x^3 \cdot 2y^7 \cdot x^4 = \)

   c. \(4x^2 + 3x^3 - 6y^6 = \)

   d. \(-4x^5 \cdot 5xy \cdot 8y^3 \cdot 3x^6y = \)

   e. \(2x \cdot 4y \cdot 5x^4 = \)

2. Simplify each of the following expressions, if necessary.
   a. \(\frac{3x^5}{x^3} = \)

   b. \(\frac{42x^4}{6y^2} = \)

   c. \(\frac{121z^{14}}{11z^{10}} = \)
Assessments

d. \( \frac{744x^6}{3x^4} = \)

e. \( \frac{10y^4}{20y^2} = \)

3. Simplify each of the following expressions.
   a. \( (x^4)^2 = \)

   b. \( (y^3)^5 = \)

   c. \( (x^6)^2 = \)

   d. \( [(x^7)^3]^2 = \)

   e. \( (y^4)^7 = \)
Law of Powers: Part 2

Name: ______________________

1. Simplify each of the following expressions.
   a. \((3x^2y^3)^3 = \)
   b. \((2x^2y^3z^2)^2 = \)
   c. \((-4x^3y^3)^3 = \)
   d. \((2x^3y^3)^3 \cdot (-2xy)^2 = \)
   e. \((2xy)^2 \cdot (5x^2y^3)^3 = \)

2. Simplify each the following expressions.
   a. \(\frac{(xy)^3}{xy^3} = \)
   b. \(\frac{(2x^2y^3)^2}{4x^{10}y^5} = \)
   c. \(\frac{(2x^5y^3)^2}{2^{11}x^2y^2} = \)
   d. \(\frac{(x^2y^3)^2 \cdot (x^3y^2)}{(x^2y^3)^3} = \)
   e. \(\frac{(y^2z^4)^2 \cdot (yz)^3}{y^2z^1} = \)
Assessments
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Number Patterns Test

Name: _____________________

Task 1

For each sequence, find the next 3 terms and describe the pattern.

1. 3, 8, 13, 18, 23, 28, 33, 38.
   
   Add five to each term. OR Multiply the term number by 5 and subtract 2.

2. 1, 2, 3, 5, 8, 13, 21, 34, 55.
   
   Add two previous terms to get the next term.

3. 1, 4, 9, 16, 25, 36, 49...
   
   Each term is the term number squared. OR Add the next odd number.

4. 1, 100, 2, 99, 3, 98, 4, 97, 5....
   
   Odd numbered terms increase by 1. Even numbered terms decrease by 1.

5. 3, -9, 27, -81, 243, -729, 2187...
   
   Multiply the previous term by $-3$ each time. OR $(-1)^{n-1}(2)^n$

Task 2

For each sequence, find the 10th term and describe the pattern.

1. | Term Number | 1 | 2 | 3 | 4 | ... | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>5</td>
<td>15</td>
<td>25</td>
<td>35</td>
<td>...</td>
<td>95</td>
</tr>
</tbody>
</table>

   The function is $10n - 5$, so multiply the term number by 10 and subtract 5.

2. | Term Number | 1 | 2 | 3 | 4 | ... | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>2</td>
<td>-4</td>
<td>8</td>
<td>-16</td>
<td>...</td>
<td>-1024</td>
</tr>
</tbody>
</table>

   $Z'$, even terms negative, odd terms positive OR $(-1)^{n-1}(2)^n$ OR Multiply the previous term by 2.
Odd term N length string of 1’s, even terms N/2 length string of 5’s

Task 3

For each sequence, find an expression for the \(N\)th term and describe the pattern.

1. Term Number | 1 | 2 | 3 | 4 | ... | N
Sequence | 2 | 7 | 12 | 17 | ... | 5n-3

Multiply the term number by 5 and subtract 3.

2. Term Number | 1 | 2 | 3 | 4 | ... | N
Sequence | 5 | 25 | 125 | 625 | ... | 5^n

Raise five to the power of the term number.

3. Term Number | 1 | 2 | 3 | 4 | ... | N
Sequence | 10 | 18 | 26 | 34 | ... | 8n + 2

Multiply each term number by 8 and add 2 to find each term.
Toothpick Test

Name: ______________________

The following toothpick diagrams show a single square constructed from 4 toothpicks and two squares constructed from 7 toothpicks.

1. Draw toothpick diagrams that show how to construct three squares, four squares, and five squares in the same manner.
   a. Three squares:

   

   b. Four squares:

   

   c. Five squares:

   

2. Complete this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Number of Squares</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Squares</td>
<td>Tooth Picks</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>151</td>
</tr>
</tbody>
</table>

3. Use the information in the table and your diagrams to describe the pattern produced by constructing the toothpick squares. Write your answer in complete sentences.

Multiply the number of desired squares by 3 and add 1.

4. Find an expression that gives the number of toothpicks required to construct $N$ squares.

$3n + 1$
Earning Money

Name: __________________________

A friend of yours is employed at a company where he makes $5.25 an hour.

Make a table that shows his earnings for various numbers of hours worked (between 1 hour and 20 hours). Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>hours</td>
<td>dollars</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>5.25x</td>
</tr>
<tr>
<td>1</td>
<td>5.25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10.50</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15.75</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>26.25</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>52.50</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>78.75</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>105</td>
<td></td>
</tr>
</tbody>
</table>
Vacation Flights

Name: ______________________

You are going on vacation by airplane. The average speed of the plane is 325 miles per hour.

Make a table that shows how far you will travel for various times between 1 hour and 20 hours. Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Time in the Air</th>
<th>Distance Traveled</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hours</strong></td>
<td><strong>Miles</strong></td>
</tr>
<tr>
<td>1</td>
<td>325</td>
</tr>
<tr>
<td>2</td>
<td>650</td>
</tr>
<tr>
<td>3</td>
<td>975</td>
</tr>
<tr>
<td>5</td>
<td>1625</td>
</tr>
<tr>
<td>10</td>
<td>3250</td>
</tr>
<tr>
<td>20</td>
<td>6500</td>
</tr>
</tbody>
</table>

Labels
- Time in the Air (x)
- Distance Traveled (325x)

Expression: $x$ miles per hour times $t$ hours, or $325x$ miles.

Graph:
- X-axis: Time in the Air (hours)
- Y-axis: Distance Traveled (miles)
- Line graph showing the relationship between time in the air and distance traveled.
Assessments
Students Who Don’t Take Algebra

Name: ____________________

One out of every four high school students never takes algebra.

Make a table that shows how many students don’t take algebra at schools of various sizes. Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Labels</th>
<th>High School Students who do not take Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Students</td>
</tr>
<tr>
<td>Expression</td>
<td>s</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>12.5</td>
</tr>
<tr>
<td>100</td>
<td>25</td>
</tr>
<tr>
<td>150</td>
<td>37.5</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>250</td>
<td>62.5</td>
</tr>
<tr>
<td>300</td>
<td>75</td>
</tr>
</tbody>
</table>

![Graph showing students who do not take Algebra vs. total students]
Paying Taxes

Name: ________________________

The average family in the United States pays about 38% of their income in taxes of all kinds.

Make a table that shows the amount of taxes paid by families with various incomes. Include the algebraic expression in your table. Use this information to construct a line graph.

<table>
<thead>
<tr>
<th>Income (thousands of $)</th>
<th>Taxes Paid (thousands of $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td>0</td>
</tr>
<tr>
<td>5000</td>
<td>1900</td>
</tr>
<tr>
<td>10,000</td>
<td>3800</td>
</tr>
<tr>
<td>15,000</td>
<td>5700</td>
</tr>
<tr>
<td>20,000</td>
<td>7600</td>
</tr>
<tr>
<td>25,000</td>
<td>9500</td>
</tr>
<tr>
<td>30,000</td>
<td>11,400</td>
</tr>
</tbody>
</table>

Labels
- Income
- Taxes Paid

Units
- Income
- Taxes Paid

Expression
- $0.38x$
Assessments
Stocking the Shelves

Name: ____________________

You are a volunteer for the school store. One of the most popular items is cool ranch flavored tortilla chips. There are two local vendors that will deliver the chips to the school store. Each makes their deliveries on the first of every month. Chips-R-Us charges 39 cents per bag with a monthly delivery fee of $25. It’s In The Bag charges 18 cents per bag with a monthly delivery fee of $65.

1. Write an algebraic equation to model the monthly cost of purchasing chips from Chips-R-Us.

\[ Y = 0.39x + 25 \]

\[ X = \# \text{ of bags of chips purchased} \]

\[ Y = \text{Cost of purchasing from Chips-R-Us} \]

2. Write an algebraic equation to model the monthly cost of purchasing chips from It’s In The Bag.

\[ Y = 0.18x + 65 \]

\[ X = \# \text{ of bags of chips purchased} \]

\[ Y = \text{Cost of purchasing from It’s In The Bag} \]

3. Complete the following table summarizing the cost of purchasing chips from each vendor based upon the actual sales by month for the last school year.
<table>
<thead>
<tr>
<th>Label</th>
<th>Month</th>
<th>Number of bags of chips purchased</th>
<th>Cost of purchasing from Chips-R-Us</th>
<th>Cost of purchasing from It’s In The Bag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td>Bags</td>
<td>Dollars</td>
<td></td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>187</td>
<td>97.93</td>
<td>98.66</td>
</tr>
<tr>
<td></td>
<td>October</td>
<td>229</td>
<td>114.31</td>
<td>106.22</td>
</tr>
<tr>
<td></td>
<td>November</td>
<td>162</td>
<td>88.18</td>
<td>94.16</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>137</td>
<td>78.43</td>
<td>89.66</td>
</tr>
<tr>
<td></td>
<td>January</td>
<td>171</td>
<td>91.69</td>
<td>95.78</td>
</tr>
<tr>
<td></td>
<td>February</td>
<td>201</td>
<td>103.39</td>
<td>101.18</td>
</tr>
<tr>
<td></td>
<td>March</td>
<td>192</td>
<td>99.88</td>
<td>99.56</td>
</tr>
<tr>
<td></td>
<td>April</td>
<td>258</td>
<td>125.62</td>
<td>111.44</td>
</tr>
<tr>
<td></td>
<td>May</td>
<td>210</td>
<td>106.90</td>
<td>102.80</td>
</tr>
<tr>
<td></td>
<td>June</td>
<td>79</td>
<td>55.81</td>
<td>79.22</td>
</tr>
<tr>
<td>Expression</td>
<td>X</td>
<td>0.39x+25</td>
<td>0.18x+65</td>
<td></td>
</tr>
</tbody>
</table>

4. Generate a graph displaying the cost of purchasing from both Chips-R-Us and It's In The Bag.
5. How many bags of chips can you purchase with each vendor if you have the following budgets? Explain how you determined each and write your answer in a complete sentence.
   a. $50
      
      *Chips-R-Us* 64 bags
      *It’s In The Bag* 0 bags
   b. $125
      
      *Chips-R-Us* 256 bags
      *It’s In The Bag* 333 bags
   c. $200
      
      *Chips-R-Us* 448 bags
      *It’s In The Bag* 750 bags

6. Compare the price to purchase from each of the two vendors. Prepare a written analysis indicating which vendor will cost the least and when. Be sure to include when the costs will be the same.

   *Equal cost of $99.29 when approximately 190 bags are purchased.*
   *Less than 190 bags, Chips-R-Us is cheaper.*  *More than 190 bags, It’s In The Bag is cheaper.*

7. If you had to choose one vendor for the entire school year, which would you choose? Would your choice change if you could switch vendors each month? Explain your choices using complete sentences.

   *Answers Will Vary*
Integers and Solving Equations: Form A

Name: ________________________

Find the answer to each problem without using your calculator.

1. \(7 + (-14) = \boxed{-7}\)
2. \(-19 + 110 = \boxed{91}\)

3. \(-2 \cdot 10 = \boxed{20}\)
4. \(\frac{210}{-10} = \boxed{-21}\)

5. \(13 - 25 = \boxed{-12}\)
6. \(-10(-6) = \boxed{60}\)

7. \(-23 + (-14) = \boxed{-37}\)
8. \(\frac{-41 + 33}{-2} = \boxed{4}\)

9. \(\frac{10 + (-4) - 4}{2} = \boxed{1}\)
10. \(20(-3) - 10 = \boxed{-70}\)

Solve each equation. Show all your work.

11. \(e + 5 = 8\) \(e = 8 - 5 = \boxed{3}\)
12. \(f + 9 = 13\) \(f = 13 - 9 = \boxed{4}\)

13. \(-2g = -20\) \(g = \frac{-20}{-2} = \boxed{10}\)
14. \(3h = 18\) \(h = \frac{18}{3} = \boxed{6}\)

15. \(20x = -10\) \(x = \frac{-10}{20} = \boxed{-1/2}\)
16. \(5j = -25\) \(j = \frac{-25}{5} = \boxed{-5}\)

17. \(7 = \frac{k}{-12}\) \(k = 7(-12) = \boxed{-84}\)
18. \(-2 - L = 18\) \(L = \boxed{-2-18} = \boxed{-20}\)
Integers and Solving Equations: Form B

Name: ________________________

Find the answer to each problem without using your calculator.

1. 17 + (-14) = 3
2. -190 + 110 = -80

3. -21 • 10 = -210
4. \( \frac{-50}{-10} = 5 \)

5. -13 – 5 = -18
6. -10(-16) = 160

7. -213 + (-124) = -337
8. \( \frac{-1 - 3}{-2} = 2 \)

9. \( \frac{10 - 4 - 4}{-2} = 1 \)
10. 2(-3) – 10 = -16

Solve each equation. Show all your work.

11. \( 2E + 6 = 10 \)
    \( 2E = 4 \)
    \( E = 4/2 = 2 \)
12. \( 4x + 8 = 32 \);
    \( 4x = 24 \)
    \( x = 24/4 = 6 \)

13. \( -2G + 26 = 50 \)
    \( -2G = 24 \)
    \( G = 24/-2 = -12 \)
14. \( 3h - 18 = 18 \)
    \( 3h = 36 \)
    \( h = 36/3 = 12 \)

15. \( 22x + 11 = -11 \)
    \( 22x = -22 \)
    \( x = -22/22 = -1 \)
16. \( 3j + (-6) = -15 \)
    \( 3j = -9 \)
    \( j = -9/3 = -3 \)

17. \( 12 = 12 + \frac{k}{-7} \)
    \( 0 = \frac{k}{-7} \)
    \( k = 0 \)
18. \( -4 - 4L = 16 \)
    \( -4L = 20 \)
    \( L = -5 \)

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Assessments
At the Driving Range

Name: _______________________

Inspired by the saying “practice makes perfect”, you decide to perfect your golf swing by going to the local driving range. Unfortunately, you do not have your own set of clubs yet so you must rent from the range. Each bucket of 100 golf balls costs $7.50 and it costs $3.00 to rent clubs for 2 hours.

1. Write an algebraic equation for the total cost at the driving range dependent on the number of buckets of balls you buy. Write your equation using functional notation with C(x) representing the cost.

   \[ C(x) = 7.50x + 3 \]
   \[ X = \text{# of buckets of balls} \]
   \[ C(x) = \text{Total cost} \]

2. Show all work and answer in complete sentence
   a. What is the total cost to hit five buckets worth of balls?
      \[ \$40.50 \]
   b. How many buckets can you buy with $20?
      \[ \text{2 buckets} \]
   c. Find x if C(x) = 28
      \[ 3.33 \]
   d. Find C(10)
      \[ 78 \]

3. Generate a graph displaying the total cost of practicing at the driving range
4. The owner of the driving range is looking at the receipts for the day. Are the following costs correct? Explain why or why not in a complete sentence.
   a. $48 for 6 buckets and club rental
      Yes, based on the cost of a bucket of balls.
   b. $70 for 9 buckets and club rental
      No, since the cost would be $72 based on the cost of a bucket of balls.

5. What would be the domain based on the problem situation?
   Since club rental is only for two hours domain should consist of 0 buckets to the maximum number of buckets you could hit in two hours.

6. Based on your answer to question 5, what would be the range based on the problem situation?
   Range could both start at 0 or 3 and end with \( C(x) \) for the maximum value in the domain.

7. Set up an inequality to solve each. Graph the solution to each.
   a. Determine the number of buckets of balls you could hit if you had to spend at most $50 but at least $25.
      You could hit between 2 and 6 buckets
      \[ 2 \leq x \leq 6 \]
   b. \( C(x) > 55 \) or \( C(x) \leq 40 \)
      \[ x > 6.93 \text{ OR } x \leq 4.93 \]
Balloon Problem

Name: _____________________

Here you will examine a problem situation about buying and selling helium balloons. Show all your work and write your answers in complete sentences.

You have the opportunity to run the helium balloon concession at a local fair. In order to sell balloons, you must pay a $25 fee to the fair sponsors and buy $75 worth of balloons and helium. You can sell each balloon for $0.80. You need to calculate your net profit for the fair. Remember, profit is the money you bring in minus the expenses you have to pay out.

1. How much profit will you have if you sell:
   a. 100 balloons? $20
    
   b. 200 balloons? $60
    
   c. 500 balloons? $300
    
   d. 50 balloons? $60
    
   e. 80 balloons? $36

   Write a complete sentence describing how you found these answers.
   
   Multiply the number of balloons by 0.80 and subtract 100.

2. Do your answers in 1d and 1e make sense? Why or why not?
   Yes, these answers make sense because negative numbers for profit indicate that your expenses are greater than your income.

3. Algebraic Expression or Equation for Profit $0.80x - 100$
4. How many balloons must you sell to make a profit of:
   a. $40?  
      175 balloons
   b. $100?  
      250 balloons

   Write a complete sentence describing how you found these answers.
   *Add 100 and then divide by 0.80.*

5. How many balloons must you sell to post the following losses? (Note: A loss is a negative number. For example, a loss of $20 is -$20.)
   a. $20  
      100 balloons
   b. $50  
      62.5 balloons
   c. $100  
      0 balloons

   Write a complete sentence describing how you found these answers.
   *Add 100 and then divide by 0.80.*
6. Use the information you have found so far to complete this table.

<table>
<thead>
<tr>
<th>Balloons Sold</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>60</td>
</tr>
<tr>
<td>500</td>
<td>300</td>
</tr>
<tr>
<td>50</td>
<td>-60</td>
</tr>
<tr>
<td>225</td>
<td>80</td>
</tr>
<tr>
<td>175</td>
<td>40</td>
</tr>
<tr>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>100</td>
<td>-20</td>
</tr>
<tr>
<td>10</td>
<td>-92</td>
</tr>
<tr>
<td>0</td>
<td>-100</td>
</tr>
<tr>
<td>300</td>
<td>140</td>
</tr>
<tr>
<td>400</td>
<td>220</td>
</tr>
<tr>
<td>275</td>
<td>120</td>
</tr>
<tr>
<td>312.5</td>
<td>150</td>
</tr>
<tr>
<td>120</td>
<td>-4</td>
</tr>
</tbody>
</table>

7. Use the information from the table to construct a graph that illustrates this problem situation.
8. How many balloons do you have to sell in order to break even?

Justify your answer, and write a complete sentence describing how you found this answer.

*You need to sell 125 balloons to break even. This can be seen by looking at the graph. The x-intercept is 125. When you sell 125 balloons, you will have covered all your investments and are ready to begin to make money.*
Televisions Problem and Climber Problem

Name: ____________________

Task 1

Use this graph to answer the questions about televisions.
1. What information does this graph show?

*The cost to produce 19 in. and 21 in. TVs.*

2. How much does it cost to produce:
   a. thirty-five 19-inch televisions?
      \[\text{\$1800}\]
   b. twenty-five 19-inch televisions?
      \[\text{\$1350}\]
   c. thirteen 19-inch televisions?
      \[\text{\$800}\]

3. How much does it cost to produce:
   a. forty-five 21-inch televisions?
      \[\text{\$1950}\]
   b. fifteen 21-inch televisions?
      \[\text{\$1000}\]
   c. eight 21-inch televisions?
      \[\text{\$750}\]

4. How many of each type of television must be produced for the costs to be the same? How do you know?

   \[\text{\$750}\]

5. Use the graph to predict the cost of producing:
   a. sixty 19-inch televisions.
      \[\text{\$2900}\]
   b. sixty 21-inch televisions.
      \[\text{\$2420}\]
   c. one hundred 21-inch televisions.
      \[\text{\$3700}\]
   d. one hundred 19-inch televisions.
      \[\text{\$4700}\]
Extra credit:

Find an algebraic expression or equation for the cost to produce 19-inch televisions. Find another algebraic expression or equation for the cost to produce 21-inch televisions.

Rule for producing 19-inch televisions: \(45x+200\)

Rule for producing 21-inch televisions: \(32x+500\)

Task 2

A mountain climber is currently 77 feet above ground level on the side of a cliff. She is climbing at the rate of about 7 feet per minute.

Answer the following questions in complete sentences.

1. How many feet above ground level will she be in:
   a. 10 minutes? \(147\) ft
   
   b. 25 minutes? \(252\) ft

2. Currently, how many feet above ground level is she? \(77\) ft

3. How many feet above ground level was she:
   a. 5 minutes ago? \(40\) ft
   
   b. 10 minutes ago? \(5\) ft

   c. 15 minutes ago? \(-30\) ft

4. How many minutes ago did she start her climb? \(11\) minutes
5. When will she be:
   a. 154 feet above ground level?
      \[11 \text{ minutes}\]
   b. 175 feet above ground level?
      \[14 \text{ minutes}\]
   c. 287 feet above ground level?
      \[14 \text{ minutes}\]

6. The top of the cliff is 450 feet above ground level. When will she reach the top of the cliff?
   \[53.29 \text{ minutes}\]

7. Write an algebraic rule for the climber’s height above ground level.
   \[y = 7x + 77\]
   \[x = \text{time in minute}\]
   \[y = \text{height in feet}\]
8. Fill in this table and construct a graph for this problem.

<table>
<thead>
<tr>
<th>Time</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes</td>
<td>Feet</td>
</tr>
<tr>
<td>$x$</td>
<td>$7x + 77$</td>
</tr>
<tr>
<td>10</td>
<td>147</td>
</tr>
<tr>
<td>25</td>
<td>252</td>
</tr>
<tr>
<td>0</td>
<td>77</td>
</tr>
<tr>
<td>-5</td>
<td>40</td>
</tr>
<tr>
<td>-10</td>
<td>5</td>
</tr>
<tr>
<td>-11</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>154</td>
</tr>
<tr>
<td>14</td>
<td>175</td>
</tr>
</tbody>
</table>

Graph:
- X-axis: time (minutes)
- Y-axis: height (feet)
Widgets-R-Us

Name: ______________________

Widgets cost $4 each, plus $10 per order for shipping and handling.

For this problem situation, find an appropriate algebraic equation, define each variable, make a table of values, construct a graph, and use this information to answer the questions.

1. Find an algebraic equation for this situation.

   Equation:  \( y = 4x + 10 \)  

   Define each variable in your equation by writing a short phrase that describes what the variable represents.

   \( x = \text{number of widgets} \)  

   \( y = \text{total cost} \)  

2. Make a table with at least 5 values of \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-5</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>
3. Graph the equation for this problem situation.

Use your graph, your equation, or your table to answer the following questions. Write your answers in complete sentences.

4. How would you describe your graph?
   
   *Answers will vary.*

5. Where does the graph intersect the x-axis? Why?
   
   *The graph intersects the x-axis at x = -2.5 because when y = 0, x = 2.5.*

6. Where does the graph intersect the y-axis? Why?
   
   *The graph intersects the y-axis at y = 10 because when x = 0, y = 10.*
7. What is the cost if you order:
   a. 7 widgets?  $38
   b. -9 widgets?  -$26

8. How many widgets did you order if the total cost is:
   a. $66?  14 widgets
   b. -$10?  -5 widgets

9. Write an equation that states: the cost of an order of widgets is $212. Solve this equation to find how many widgets you ordered.

   $4x + 10 = 212$
   $4x = 202$
   $x = 50.5$ widgets

10. Find the cost if you order -23,456 widgets.

    -$93,814

11. Do problems 9 and 10 make sense in this problem situation? Why or why not?

    No, you must order a whole number of widgets and the cost and number of widgets must be positive.
Computer Rental Problem

Name: ______________________

For a college computer class in which you are enrolled, you must rent a computer. Two rental companies carry the particular model you need.

Computronics rents the computer you need for $2.50 a day plus a one-time registration fee of $27. A second company, Computers for Students, rents the same computer at a rate of $4.75 per day.

You know you will need the computer for 30 days or less, but you don’t know exactly how many days. From whom should you rent? In order to answer this question, write the problem situation for each company, construct a table of values, and find the algebraic expression. Then draw graphs for both companies on a single grid.

Task 1

Write the problem situation for Computronics. Construct a table of values and find the expression.

Problem Situation:

<table>
<thead>
<tr>
<th>Time (Days)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27.00</td>
</tr>
<tr>
<td>5</td>
<td>39.50</td>
</tr>
<tr>
<td>10</td>
<td>52.00</td>
</tr>
<tr>
<td>20</td>
<td>77.00</td>
</tr>
<tr>
<td>30</td>
<td>102.00</td>
</tr>
</tbody>
</table>
Task 2

Write the problem situation for Computers for Students. Construct a table of values and find the expression.

Problem Situation:

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Days</td>
<td>$</td>
</tr>
<tr>
<td>Expressions</td>
<td>t</td>
<td>4.75t</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>23.75</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>47.50</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>95.00</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>142.50</td>
<td></td>
</tr>
</tbody>
</table>

Task 3

Draw graphs that show the information from each company.

Task 4

Use the information you have gathered and analyzed in Tasks 1–3 to write a paragraph explaining which company you will rent from and why. Write your answer in complete sentences.

*If you are renting for less than 12 days you should rent from Computers For Students. If you are renting for more than 12 days, you should rent from Computronics. If you are renting for exactly 12 days the cost is equal.*
Systems: Part 1

Name: ______________________

For each system of equations:

- Find the slope and y-intercept for each equation
- Graph both lines on the same coordinate system
- Find the point of intersection of the lines

1. System of equations: \( y = 3x - 5 \) and \( y = -2x + 5 \)
   
   \( y = 3x - 5 \)
   
   Slope: \( 3 \)
   
   y-intercept: \((0, -5)\)
   
   \( y = -2x + 5 \)
   
   Slope: \(-2\)
   
   y-intercept: \((0, 5)\)

   Point of intersection: \((2, 1)\)

2. System of equations: \( y = -x + 2 \) and \( y = x + 4 \)
   
   \( y = -x + 2 \)
   
   Slope: \(-1\)
   
   y-intercept: \((0, 2)\)
   
   \( y = x + 4 \)
   
   Slope: \(1\)
   
   y-intercept: \((0, 4)\)

   Point of intersection: \((-1, 3)\)
Systems: Part 2

Name: ____________________________

For each system of equations:

- Find the slope and $y$-intercept for each equation
- Graph both lines on the same coordinate system
- Find the point of intersection of the lines

1. System of equations:
   $y = 3x + 5$
   $y = -2x + 5$
   $y = 3x + 5$
   Slope: \(3\)
   $y$-intercept: \((0, 5)\)
   $y = -2x + 5$
   Slope: \(-2\)
   $y$-intercept: \((0, 5)\)
   Point of intersection: \((0, 5)\)

2. System of equations:
   $y = -x + 3$
   $y = 2x + 6$
   $y = -x + 3$
   Slope: \(-1\)
   $y$-intercept: \((0, 3)\)
   $y = 2x + 6$
   Slope: \(2\)
   $y$-intercept: \((0, 6)\)
   Point of intersection: \((-1, 4)\)
Systems: Part 3

Name: ______________________

For each system of equations:

- Find the slope and y-intercept for each equation
- Graph both lines on the same coordinate system
- Find the point of intersection of the lines
- Check your answer on the calculator

1. System of equations:
   \[ y = -3x + 4 \]
   \[ y = 3x - 2 \]

\[
\begin{align*}
-3x + 4 &= 3x - 2 \\
6 &= 6x \\
1 &= x
\end{align*}
\]

\[
\begin{align*}
3(1) - 2 &= y \\
3 - 2 &= y \\
1 &= y
\end{align*}
\]

\[
\begin{align*}
-3(1) + 4 &= y \\
-3 + 4 &= y \\
1 &= y
\end{align*}
\]

\[
\begin{align*}
y &= -3x + 4 \\
\text{Slope:} &\quad -3 \\
\text{y-intercept:} &\quad (0, 4)
\end{align*}
\]

\[
\begin{align*}
y &= 3x - 2 \\
\text{Slope:} &\quad 3 \\
\text{y-intercept:} &\quad (0, -2)
\end{align*}
\]

Point of intersection: \((1, 1)\)
2. System of equations:  \[ y = -2x - 7 \]
                    \[ y = 2x + 5 \]

\[
\begin{align*}
-2x - 7 &= 2x + 5 \\
-12 &= 4x \\
-3 &= x \\
y &= -2x - 7 \\
\text{Slope:} & \quad -2 \\
y-\text{intercept:} & \quad (0, -7) \\
y &= 2x + 5 \\
\text{Slope:} & \quad 2 \\
y-\text{intercept:} & \quad (0, 5)
\end{align*}
\]

Point of intersection: \((-3, -1)\)
## Baseball

**Name: ______________________________**

The data set below represents the top 40 homerun hitters in the American and National Baseball Leagues for 2001.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Alex Rodriguez, Tex 52</td>
<td>1) Barry Bonds, SF 73</td>
</tr>
<tr>
<td>2) Jim Thome, Cle 49</td>
<td>2) Sammy Sosa, ChC 64</td>
</tr>
<tr>
<td>3) R. Palmeiro, Tex 47</td>
<td>3) Luis Gonzalez, Ari 57</td>
</tr>
<tr>
<td>4) Manny Ramirez, Bos 41</td>
<td>4) Shawn Green, LA 49</td>
</tr>
<tr>
<td>5) Troy Glaus, Ana 41</td>
<td>5) Todd Helton, Col 49</td>
</tr>
<tr>
<td>6) Carlos Delgado, Tor 39</td>
<td>6) Richie Sexson, Mil 45</td>
</tr>
<tr>
<td>7) Jason Giambi, Oak 38</td>
<td>7) Phil Nevin, SD 41</td>
</tr>
<tr>
<td>8) Bret Boone, Sea 37</td>
<td>8) Jeff Bagwell, Hou 39</td>
</tr>
<tr>
<td>9) Juan Gonzalez, Cle 35</td>
<td>9) Chipper Jones, Atl 38</td>
</tr>
<tr>
<td>10) Jose Cruz, Tor 34</td>
<td>10) Larry Walker, Col 38</td>
</tr>
<tr>
<td>11) Tino Martinez, NY 34</td>
<td>11) Rich Aurilia, SF 37</td>
</tr>
<tr>
<td>12) Paul Konerko, CWS 32</td>
<td>12) Brian Giles, Pit 37</td>
</tr>
<tr>
<td>13) Eric Chavez, Oak 32</td>
<td>13) Albert Pujols, Stl 37</td>
</tr>
<tr>
<td>14) Miguel Tejada, Oak 31</td>
<td>14) Mike Piazza, NYM 36</td>
</tr>
<tr>
<td>15) M. Ordonez, CWS 31</td>
<td>15) Gary Sheffield, LA 36</td>
</tr>
<tr>
<td>16) Mike Sweeney, KC 29</td>
<td>16) Jeromy Burnitz, Mil 34</td>
</tr>
<tr>
<td>17) Jose Valentin, CWS 28</td>
<td>17) Andruw Jones, Atl 34</td>
</tr>
<tr>
<td>18) G. Anderson, Ana 28</td>
<td>18) V. Guerrero, Mon 34</td>
</tr>
<tr>
<td>19) Ellis Burks, Cle 28</td>
<td>19) Aramis Ramirez, Pit 34</td>
</tr>
<tr>
<td>20) Raul Mondesi, Tor 27</td>
<td>20) Lance Berkman, Hou 34</td>
</tr>
<tr>
<td>21) Trot Nixon, Bos 27</td>
<td>21) Reggie Sanders, Ari 33</td>
</tr>
<tr>
<td>22) Torii Hunter, Min 27</td>
<td>22) Cliff Floyd, Fl 31</td>
</tr>
<tr>
<td>23) Jermaine Dye, KC/Oak 26</td>
<td>23) Bobby Abreu, Phi 31</td>
</tr>
<tr>
<td>24) Corey Koskie, Min 26</td>
<td>24) Ryan Klesko, SD 30</td>
</tr>
<tr>
<td>25) B. Williams, NYY 26</td>
<td>25) Jim Edmonds, Stl 30</td>
</tr>
<tr>
<td>26) Mike Cameron, Sea 25</td>
<td>26) Mark McGwire, Stl 29</td>
</tr>
<tr>
<td>27) Tony Batista, Bal/Tor 25</td>
<td>27) J.D. Drew, Stl 27</td>
</tr>
<tr>
<td>28) Ivan Rodriguez, Tex 25</td>
<td>28) Pat Burrell, Phi 27</td>
</tr>
<tr>
<td>29) Carlos Beltran, KC 24</td>
<td>29) Moises Alou, Hou 27</td>
</tr>
<tr>
<td>30) Carlos Lee, CWS 24</td>
<td>30) Jose Hernandez, Mil 25</td>
</tr>
<tr>
<td>32) Ruben Sierra, Tex 23</td>
<td>32) Scott Rolen, Phi 25</td>
</tr>
<tr>
<td>33) Edgar Martinez, Sea 23</td>
<td>33) Bubba Trammell, SD 25</td>
</tr>
<tr>
<td>34) Jorge Posada, NYY 22</td>
<td>34) Paul Lo Duca, LA 25</td>
</tr>
<tr>
<td>35) Brian Daubach, Bos 22</td>
<td>35) Lee Stevens, Mon 25</td>
</tr>
<tr>
<td>36) Derek Jeter, NYY 21</td>
<td>36) Vinny Castilla, Hou 23</td>
</tr>
<tr>
<td>37) Paul O’Neill, NYY 21</td>
<td>37) Preston Wilson, Fl 23</td>
</tr>
<tr>
<td>38) John Olerud, Sea 21</td>
<td>38) Jeff Kent, SF 22</td>
</tr>
<tr>
<td>39) Ray Durham, CWS 20</td>
<td>39) K. Griffey Jr., Cin 22</td>
</tr>
<tr>
<td>40) Marty Cordova, Cle 20</td>
<td>40) Dmitri Young, Cin 21</td>
</tr>
</tbody>
</table>

Source: www.espn.com
1. To compare the two leagues construct
   a. Stem and Leaf Plot
   b. Histogram

**National League Stem and Leaf Plot**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 2 3 3 5 5 5 5 5 5 7 7 7 9</td>
</tr>
<tr>
<td>3</td>
<td>0 0 1 1 3 4 4 4 4 4 6 6 7 7 7 8 8 9</td>
</tr>
<tr>
<td>4</td>
<td>1 5 9 9</td>
</tr>
<tr>
<td>5</td>
<td>6 6 4</td>
</tr>
<tr>
<td>6</td>
<td>3 3 9 9</td>
</tr>
<tr>
<td>7</td>
<td>6 6 4</td>
</tr>
</tbody>
</table>

*stem width = 10*

**America League Stem and Leaf Plot**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 0 1 1 1 2 2 3 3 4 4 4 5 5 5 6 6 6 7 7 7 8 8 8 9</td>
</tr>
<tr>
<td>3</td>
<td>1 1 2 2 4 4 5 7 8 9</td>
</tr>
<tr>
<td>4</td>
<td>1 1 7 9</td>
</tr>
<tr>
<td>5</td>
<td>2 2 2 2</td>
</tr>
<tr>
<td>6</td>
<td>1 1 7 9</td>
</tr>
<tr>
<td>7</td>
<td>1 1 7 9</td>
</tr>
</tbody>
</table>

*stem width = 10*

**National League Histogram**

```
<table>
<thead>
<tr>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>
```

**American League Histogram**

```
<table>
<thead>
<tr>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>
```
2. What type of information can you gather from each type of plot?

*From the stem and leaf plot you can determine the distribution of the data and therefore which league hit more homeruns. You can also see specifically the number of homeruns hit per person in the data set. From the histogram you can also see a distribution, but it will be the frequency of a homeruns hits within an interval.*

3. Based on the stem and leaf plots what can you say about the performance of the two leagues?

*Based on the stem and leaf plot, the distribution is skewed to the right, meaning the majority of the 40 top homerun hitters hits between 20 and 40 homeruns in each league. The national league had more people hit homeruns between 50 and 80. The American League had most of its top 40 hitters hit between 20 and 30 homeruns.*

4. Based on the histogram what can you say about the performance of the two leagues?

*Based on the histogram, you can see that the American League had the highest frequency of homeruns hit in the interval 20-24 and 25-29. The performance is shown to be almost equivalent, but the National League has a higher frequency of homeruns in the intervals from 50 to 70.*

5. Do the two displays provide the same information? Explain your reasoning.

*Students may say yes, since it is the same data set, however you want the answer to reflect the fact that the stem and leaf allows you to see actual numbers while the histogram only provides the frequency of individuals who hit homeruns within an interval.*

6. Which display is more informative? Why?

*Answers will vary, but students should focus on stating the original purpose for generating a display, such as wanting a one-for-one view of the number of homeruns hit by the top forty players. Then the stem and leaf would be the appropriate answer.*

7. From which distribution is it easier to determine the mean number of home runs?

*Since the stem and leaf give exact values it will be easier to use to determine the mean.*
Data

Name: __________________________

1. Use the data table to graph the points. Then find the slope, the y-intercept, and the equation that fits the data points.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>51</td>
</tr>
<tr>
<td>11</td>
<td>63</td>
</tr>
</tbody>
</table>

Slope: 5.22

y-intercept: (0, 7.61)

Equation: \( y = 5.22x + 7.61 \)

For each set of data, calculate the mean, the median, and the mode.

2. Data: 14, 23, 24, 19, 15, 18, 19

\[
\text{mean: } \bar{x} = \frac{132}{7} = 18 \frac{6}{7} = 18.86 \\
\text{median: } 19 \\
\text{mode: } 19
\]

3. Data: 1111, 1415, 1612, 1111, 1313

\[
\text{mean: } \bar{x} = \frac{6562}{5} = 1312 \frac{2}{5} = 1312.4 \\
\text{median: } 1313 \\
\text{mode: } 1111
\]
Running the Mile

Name: _____________________

This table shows the fastest time in seconds that a human ran the mile from 1945 to 1985. Find the slope, the \( y \)-intercept, and the equation of the linear regression line for \textit{number of years since 1945} versus \textit{time in seconds}. Then answer the questions.

<table>
<thead>
<tr>
<th>Year</th>
<th>Time in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1945</td>
<td>241.4</td>
</tr>
<tr>
<td>1954</td>
<td>238.0</td>
</tr>
<tr>
<td>1957</td>
<td>237.2</td>
</tr>
<tr>
<td>1962</td>
<td>234.4</td>
</tr>
<tr>
<td>1964</td>
<td>234.1</td>
</tr>
<tr>
<td>1965</td>
<td>233.6</td>
</tr>
<tr>
<td>1966</td>
<td>231.3</td>
</tr>
<tr>
<td>1967</td>
<td>231.1</td>
</tr>
<tr>
<td>1975</td>
<td>229.4</td>
</tr>
<tr>
<td>1979</td>
<td>229.0</td>
</tr>
<tr>
<td>1980</td>
<td>228.8</td>
</tr>
<tr>
<td>1981</td>
<td>228.4</td>
</tr>
<tr>
<td>1981</td>
<td>227.3</td>
</tr>
<tr>
<td>1985</td>
<td>226.3</td>
</tr>
</tbody>
</table>

Slope: \( b = \mathbf{-0.3674} \)  
y-intercept: \( a = (0, 240.8495) \)

Equation: \( y = \mathbf{-0.3674x + 240.8495} \)  
Regression coefficient: \( r = \mathbf{-0.9818} \)

Write your answers in complete sentences.

1. Use your model (equation) to predict how fast a runner in 1990 should be able to run the mile.

   \[ 45 \text{ years} \times \mathbf{-0.3674} = \mathbf{-16.533} \]
   \[ -16.533 + 240.8495 = 224.3165 \]

   \textit{Answer: 224.3165 seconds}
2. Use your model (equation) to predict how fast a runner in 1993 should be able to run the mile.

\[ 48 \text{ years} \cdot -0.3674 = -17.6352 \]

\[-17.6352 + 240.8495 = 223.2143 \]

Answer: 223.2143 seconds

3. In what year does your model (equation) predict that the fastest time a runner ran the mile was 237.5 seconds?

\[-0.3674x + 240.8495 = 237.5 \]

1945

\[-0.3674x = -3.3495 \]

9

\[ x = 9.1168 \quad \text{Answer:} \quad 1954 \]

\[ x = 9 \text{ years} \]

4. In what year does your model (equation) predict that 200 seconds is the fastest time a runner will run the mile?

\[-0.3674x + 240.8495 = 200 \]

1945

\[-0.3674x = -40.8495 \]

111

\[ x = 111.1854 \quad \text{Answer:} \quad 2056 \]

\[ x = 111 \text{ years} \]

5. How good a fit is your model? Explain your thinking.

The model is a good fit because the regression coefficient value is close to –1.
Squares: Part 1

Name: ____________________

For each equation:

- Enter the equation into your graphing calculator and graph it
- Fill in at least five \( x \) and \( y \) values in the table
- Sketch the graph
- Find the \( x \)-intercepts and the \( y \)-intercept
- Find the equation for the axis of symmetry
- Find the coordinates of the vertex

1. \( y = x^2 + 5x + 4 = (x + 4)(x + 1) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-2.5</td>
<td>-2.25</td>
</tr>
<tr>
<td>-5</td>
<td>4</td>
</tr>
</tbody>
</table>

\( x \)-intercepts: \((-4, 0), (-1, 0)\)

\( y \)-intercept: \((0, 4)\)

Axis of symmetry: \( x = -2.5 \)

Vertex: \((-2.5, -2.25)\)
2. \[ y = x^2 - 5x + 6 = (x - 3)(x - 2) \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>3</td>
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<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>-0.25</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

x-intercepts: \((3, 0)\) \((2, 0)\)

y-intercept: \((0, 6)\)

Axis of symmetry: \(x = 2.5\)

Vertex: \((2.5, -0.25)\)
Squares: Part 2

Name: ____________________________

For each equation:

- Enter the equation into your graphing calculator and graph it
- Fill in at least five x and y values in the table
- Sketch the graph
- Find the x-intercepts and the y-intercept
- Find the equation for the axis of symmetry
- Find the coordinates of the vertex

1. \( y = x^2 + 7x + 6 = (x + 1)(x + 6) \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-6</td>
<td>0</td>
</tr>
<tr>
<td>-3.5</td>
<td>-6.25</td>
</tr>
<tr>
<td>-7</td>
<td>6</td>
</tr>
</tbody>
</table>

x-intercepts: \((-1, 0) (-6, 0)\)
y-intercept: \((0, 6)\)
Axis of symmetry: \(x = -3.5\)
Vertex: \((-3.5, -6.25)\)
2. \( y = x^2 - 6x + 8 = (x - 2)(x - 4) \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

- x-intercepts: \((2, 0), (4, 0)\)
- y-intercept: \((0, 8)\)
- Axis of symmetry: \(x = 3\)
- Vertex: \((3, -1)\)
Quadratic Formula

Name: ____________________

General form of the quadratic equation:

\[ y = Ax^2 + Bx + C \]

Quadratic formula:

\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

For each quadratic equation:

- Write the values of \( A \), \( B \), and \( C \) from the general form
- Use the quadratic formula to solve the equation for the \( x \)-intercepts
- Use the \( x \)-intercepts to write the equation in factored form

1. \( y = x^2 - 9x - 10 = (x + 1)(x - 10) \)
   
   \[ A = 1 \quad B = -9 \quad C = -10 \]
   
   \[ x = 10 \quad x = -1 \]
   
   \( x \)-intercepts: \((10, 0) (-1, 0)\)

2. \( y = 2x^2 - 16x - 128 = (\quad)(\quad) \text{ No integer factors.} \)
   
   \[ A = 2 \quad B = -16 \quad C = -128 \]
   
   \[ x = 12.9443 \quad x = -4.9443 \]
   
   \( x \)-intercepts: \((12.9443, 0) (-4.9443, 0)\)

3. \( y = 3x^2 + 8x - 10 = (\quad)(\quad) \text{ No integer factors.} \)
   
   \[ A = 3 \quad B = 8 \quad C = -10 \]
   
   \[ x = 0.9274 \quad x = -3.5941 \]
   
   \( x \)-intercepts: \((0.9274, 0) (-3.5941, 0)\)
Vertical Motion

Name: __________________

A cannonball is shot straight up into the air from the side of a mountain. The flight of the cannonball can be modeled by this equation for vertical motion:

\[ y = -16x^2 + vx + h \]

In this equation, \( y \) is the number of feet above the valley, \( x \) is the time in seconds from the moment the ball is shot, \( v \) is the initial velocity (or speed) in feet per second, and \( h \) is the initial height in feet of the cannonball above the valley floor.

The initial velocity of the cannonball is 600 feet per second, and the cannon is 100 feet above the valley floor.

1. Write the equation for flight of the cannonball.

\[ y = -16x^2 + 600x + 100 \]

Use complete sentences to answer the following questions.

2. How high is the cannonball:
   a. 1 second after it is shot?
      \[ 684 \text{ feet} \]
   b. 20 seconds after?
      \[ 5,700 \text{ feet} \]
   c. 50 seconds after?
      \[ -9,900 \text{ feet} \]
   d. 5 seconds before it is shot?
      \[ -3,300 \text{ feet} \]
   e. 50 seconds before it is shot?
      \[ -69,900 \text{ feet} \]

3. Do all the answers to question 2 make sense in this problem situation?

   \textit{No, all of these responses do not make sense. The times in (d) and (e) before the cannonball was shot do not make sense. Before the ball was shot, it was not in motion. Also (c) does not make sense since the cannonball was shot 100 feet above the floor of the valley.}
4. Use the quadratic formula to find when the cannonball is on the ground. How do you know?

\[-0.1659 \text{ seconds} \quad \text{or} \quad 37.6659 \text{ seconds}\]

5. When is the cannonball at its highest point? How high does it get?

\[18.75 \text{ seconds} \quad \text{or} \quad 5,725 \text{ feet}\]

6. Use the quadratic formula to find when the cannonball will be back at its original level. How do you know?

\[-16x^2 + 600x + 100 = 100\]

\[-16x^2 + 600 = 0\]

\[x = \frac{-600 \pm \sqrt{600^2 - 4(-16)(0)}}{2(-16)}\]

\[x = \frac{-600 \pm \sqrt{360000}}{-32}\]

\[x = 0\]  \hspace{2cm}  \[x = -1200\]

\[\text{Answer: 0 seconds} \hspace{2cm} \text{Answer: 37.5 seconds}\]

7. When is the cannonball 3000 feet above the valley floor?

\[-16x^2 + 600x + 100 = 3000\]

\[-16x^2 + 600x - 290 = 0\]

\[x = \frac{-600 \pm \sqrt{600^2 - 4(-16)(-2900)}}{2(-16)}\]

\[x = \frac{-600 \pm \sqrt{360000 - 185600}}{-32}\]

\[x = \frac{-600 \pm 17440}{-32}\]  \hspace{2cm}  \[x = \frac{-600 - 17440}{-32}\]

\[x = -182.3877\]  \hspace{2cm}  \[x = 1017.6123\]

\[\text{Answer: 5.6996 seconds} \hspace{2cm} \text{Answer: 31.8004 seconds}\]
8. Use the information you have found so far about the problem situation to fill in this table.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Time</th>
<th>Height above Valley</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expression</td>
<td></td>
<td></td>
</tr>
<tr>
<td>seconds</td>
<td></td>
<td>feet</td>
</tr>
<tr>
<td>( x )</td>
<td></td>
<td>(-16x^2 + 600x + 100)</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>684</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>5,700</td>
</tr>
<tr>
<td>18.75</td>
<td></td>
<td>5,725</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>37.5</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

9. Graph your equation on the following grid.
Law of Powers: Part 1

Name: _______________________

1. Write each of the following expressions as a single power when appropriate.
   a. \(2x^4 \cdot 3x^3 = 6x^7\)
   b. \(5x^5 \cdot 2y^7 \cdot x^4 = 10x^9y^7\)
   c. \(4x^3 + 3x^3 - 6y^4 = 4x^2 + 3x^3 - 6y^4\)
   d. \(-4x^6 \cdot 5xy \cdot 8y^3 \cdot 3x^6y = -480x^{13}y^5\)
   e. \(2x \cdot 4y \cdot 5x^4 = 40x^5y\)

2. Simplify each of the following expressions, if necessary.
   a. \(\frac{3x^5}{x^2} = 3x^2\)
   b. \(\frac{42x^4}{6y^3} = \frac{7x^4}{y^3}\)
   c. \(\frac{121z^4}{11z^3} = 11z\)
Assessments

d. \( \frac{744x^6}{3x^3} = 248x^3 \)

e. \( \frac{10y^4}{20y^2} = \frac{y^2}{2} \) or \( \frac{1}{2}y^2 \)

3. Simplify each of the following expressions.

a. \( (x^3)^2 = x^6 \)

b. \( (y^3)^5 = y^{15} \)

c. \( (x^6)^2 = x^{12} \)

d. \( [(x^3)^2]^2 = x^{12} \)

e. \( (y^7)^7 = y^{28} \)
Law of Powers: Part 2

Name: _______________________

1. Simplify each of the following expressions.
   a. \((3x^2y^3)^3 = 27x^6y^9\)
   b. \((2x^3y^2z^4)^2 = 4x^{12}y^8z^{10}\)
   c. \((-4x^3y^5)^3 = -64x^{12}y^{15}\)
   d. \((2x^2y^3)^3 \cdot (-2x^2y^3)^2 = (8x^6y^9) \cdot (4x^4y^6) = 32x^{11}y^{15}\)
   e. \((2xy)^4 \cdot (5x^3y^2)^2 = (4x^6y^8) \cdot (125x^6y^4) = 500x^{11}y^{20}\)

2. Simplify each the following expressions.
   a. \(\frac{(xy)^3}{xy^3} = \frac{x^3y^3}{xy^3} = x^2\)
   b. \(\frac{(2xy)^2}{4x^6y^2} = \frac{4x^{10}y^4}{4x^{10}y^2} = y^2\)
   c. \(\frac{(2x^6y^3)^2}{2^2x^6y^2} = \frac{4x^{12}y^6}{2x^6y^2} = \frac{x^6y^4}{1024}\)
   d. \(\frac{(x^3y^2)^3}{(xy)^4} = \frac{(x^9y^6)}{x^4y^4} = \frac{x^5y^2}{1} = x^5y^2\)
   e. \(\frac{(yz)^3}{y^2z} = \frac{(y^3z^3)}{y^2z} = y^2z^2\)
Assessments